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Cardinal Sins: Utility Specification and the Measurement of Risk Aversion

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Abstract: In contrast to the ideal of allowing empirical data to reveal individual preferences, economists have frequently resorted to constructing preference functions with convenient parameters and then using data merely to estimate the magnitude of the parameters. In particular, it has become common practice for researchers seeking to measure risk aversion to select an arbitrary utility specification and assume that the same functional form applies to all individuals in a given sample. This note illustrates the prevalence of this practice and how unreliable such an approach may be. Using a recently developed data set from Thailand, empirical results for the Pratt-Arrow coefficients of risk aversion are compared across isoelastic and exponential utility functions. The latter are estimated to be, on average, more than five times larger than the former, suggesting the possibility of enormous distortions in empirical studies that assume specific cardinal utility functions.

Keywords: risk aversion, utility function, isoelastic, exponential, representative agent

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1. Introduction

Although utility is canonically described as an ordinal ranking of preferences over goods and services, economists have frequently adopted specific functional forms that yield cardinal measures (Barnett, 2003). As early as the 18th century, Bernoulli (1738) used a logarithmic utility function to illustrate the concept of the diminishing marginal utility of wealth in his explanation of risk aversion. This tendency has become increasingly prevalent since Pratt (1964) and Arrow (1965) introduced quantitative measures of risk aversion. Over the ensuing five decades, a large literature has been developed that seeks to estimate the Pratt-Arrow coefficients empirically, often by superimposing specific functional forms of utility on the preferences of the subjects under investigation.

While it is conceivable that an individual actually carries a cardinal utility function in his or her mind to evaluate goods and services (including monetary gambles), no such function is ever outwardly apparent to others. At best, external investigators can draw inferences about a subject's preferences by observing behavior; that is, the subject's preferences may be revealed through the choices that he or she makes. Unfortunately, the absence of sufficient behavioral data to infer an individual's utility function has commonly led researchers to forego a revealed preference approach in favor of its opposite, an assumed preference approach—by first assuming that utility takes a particular functional form, and then interpreting observed behavior as having been fashioned by that functional form. Moreover, the representative agent model, by which all individuals are coincidentally assumed to share exactly the same utility function, has likewise become dominant in the risk aversion literature, even where the purported objective is to investigate attitudinal differences among subjects. The resulting estimates of risk aversion may be highly inaccurate reflections of the individuals' true underlying risk preferences.

The present note illustrates how unreliable such measurements might be. We utilize two popular forms of utility and a recent data set to illustrate the potential magnitude of specification errors. The following section provides some background on the problem, and section 3 discusses the data, methodology, and findings. The paper ends with a short conclusion in section 4.

2. Background

For a simple risk, the univariate utility function $U(X)$, where X usually denotes wealth or income, reflects the individual decision-maker's satisfaction level. Because risk aversion has long been associated with diminishing marginal utility, Pratt (1964) and Arrow (1965) used the first and second derivatives of the utility function to define the point-elasticity of marginal utility with respect to wealth as the coefficient of relative risk aversion: $r(X) = -XU''(X)/U'(X)$. At the same time, they defined the coefficient of absolute risk aversion to be $a(X) = -U''(X)/U'(X)$.

On mathematical grounds, Arrow (1965) suggested that $r(X)$ should take values near one. Subsequently, a large and growing literature has sought to estimate $r(X)$ and $a(X)$ empirically, as well as to correlate the magnitude of risk aversion with demographic, educational, and other individualized variables. Previous work has already reviewed various aspects of this literature; see for example Cox and Harrsion (2008). Outreville's (2014) recent survey reported on more than two dozen empirical studies, which obtained estimates of relative risk aversion ranging from less than one to well over 1,000. In a number of such studies, a range of values was reported, often as the result of confidence interval estimates or variations in estimation methods.

In order to compute numeric values for $r(X)$ and $a(X)$, it is necessary for the utility function to be continuous and twice differentiable; easily differentiable functions are therefore frequently adopted for convenience. One of the most popular is the isoelastic utility function, for which $U(X) = (X^{1-\beta})/(1-\beta)$ when $\beta \neq 1$ and $U(X) = \ln X$ when $\beta = 1$; this function exhibits constant relative risk aversion (CRRA), because $r(X) = \beta$ regardless of the magnitude of X. Retrieving $a(X)$ from this function requires dividing β by X, so an appropriate initial value is needed, and the result is sensitive to the units in which wealth is measured. Another commonly used form is the exponential utility function, $U(X) = -EXP(-\alpha X)$, often called the constant absolute risk aversion (CARA) function, because its derivatives yield $a(X) = \alpha$ regardless of the magnitude of X.[1] In this case, multiplying α by X generates a measure of relative risk aversion.

Because it is impossible for an investigator to directly observe a decision-maker's utility function, it has become common practice to impose one of these functional forms, or some other arbitrary specification—usually chosen for mathematical convenience—on the preferences of the individuals in a sample, so as to estimate $r(X)$ and/or $a(X)$. That approach has been used in a wide array of contexts, including agriculture, finance, entrepreneurship, and other applications, in developed and developing countries around the world. Table 1 lists more than two dozen such studies chronologically. As suggested by the studies in this sample, the assumed functional form is most often either isoelastic or exponential, though other functions are occasionally adopted.[2] Several researchers have employed multiple specifications, in attempts to determine which function best fits their data; but even these efforts have failed to reveal the true underlying utility functions of the individuals sampled (see, for example, Dillon and Scandizzo, 1978). Indeed, several studies have found that the same individuals could be classified as either risk averse or risk loving, depending upon the functional form by which utility is evaluated (Musser, et al., 1984; Zuhair, et al., 1992; Torkamani and Haji-Rahimi, 2001).

While assigning a particular utility function to any individual is a questionable practice, it is even less reasonable to assume that every individual in a given population or sample has precisely the same functional form of utility. Yet the assumption of a 'representative agent' is common, and ironically, it occurs even when the stated purpose of the research is to examine the heterogeneity among individuals.[3] Of course, different assumptions regarding the parameterization of utility inevitably lead to different measurements, so conflicting outcomes can be obtained from the same data set, especially as preferences diverge from risk neutrality. Saha (1993), for example, simulated production decisions and portfolio allocations under decreasing, constant, and increasing risk aversion specifications, and found that assuming the wrong functional form of utility led to biases of around 77 percent in predicted behavior. Using game show data, Fullenkamp et al. (2003) estimated parameters for exponential and isoelastic utility functions under various scenarios, then used the estimates to predict the players' certainty equivalents for a series of gambles. Not surprisingly, the different utility functions yielded substantially different certainty equivalents, especially when the stakes were large. In the case of an even-odds gamble between winning \$100,000 and \$0, for example, isoelastic utility produced an average certainty equivalent of \$28,468 among a set of players assumed to be fully rational, while exponential utility yielded an average certainty equivalent of more than \$42,000 for the same players—a difference of nearly 50 percent. It is not obvious whether either of these specifications represents the true preference structure of all, many, some, or any of the individuals in the study. We provide a further sense of the potential discrepancies resulting from different utility specifications in the following section.

3. Methodology and Results

We examine the differences in risk aversion estimates between isoelastic and exponential utility functions by using the results of a simple lottery experiment recently conducted in Thailand. Hardeweg, et al. (2013) presented 930 Thai residents with the following gamble: a 50/50 chance to win 300 Thai baht or nothing.[4] A relatively modest gamble of this type is appealing for this purpose because the Pratt-Arrow coefficients are local measures, designed for use over small intervals.[5] As an alternative to the gamble, each subject was offered safe amounts that increased (from an initial value of zero) in increments of 10 baht, until (s)he switched from favoring the gamble to favoring the certain amount. The certainty equivalent (*CE*) of the gamble is the safe monetary value at which the subject is indifferent between the two choices; for simplicity we take the midpoint of the relevant 10 baht increment as the certainty equivalent.[6] The basic expected utility equation for this simple lottery is[7]

$$U(CE) = .5U(300) + .5U(0)\,. \tag{1}$$

The mean, or expected value of the gamble is 150 baht. The responses, which were validated by survey questions, are shown in Table 2. All values of $CE < 150$ indicate risk aversion, $CE = 150$ indicates risk neutrality, and $CE > 150$ indicates a love of risk. Approximately 89 percent of the sample exhibited some degree of risk aversion, while the remaining 11 percent of the sample exhibited a love of risk.[8] The mean *CE* was 63 baht, the median was 45 baht, and the mode was 15 baht.

To estimate the potential bias caused by imposing parametric assumptions on the utility function, we compare the two most popular functions. We first calculate $r(X)$ by assuming that the utility function for each individual is isoelastic. Thus, equation (1) becomes

$$\frac{CE^{1-\beta}}{1-\beta} = \frac{.5(300^{1-\beta})}{1-\beta} + \frac{.5(0^{1-\beta})}{1-\beta}, \qquad (2)$$

which reduces to $2CE^{1-\beta} = 300^{1-\beta}$. Taking natural logarithms and rearranging yields the coefficient of relative risk aversion,

$$\beta = 1 - \frac{\ln(2)}{\ln(300/CE)} = r(X). \qquad (3)$$

To obtain absolute risk aversion, we divide relative risk aversion by the expected value of the gamble, to get $a(X) = r(X)/150$. Table 2 gives the estimated $r(X)$ and $a(X)$ for each respondent, under the assumption that every individual has isoelastic utility.

For comparison, we next recalculate $r(X)$ and $a(X)$ under the alternative parameterization of exponential utility. Now equation (1) becomes

$$-EXP(-\alpha CE) = -.5EXP(-300\alpha) + -.5EXP(0). \qquad (4)$$

Inserting the observed certainty equivalents into equation (4) and solving for α gives an entirely different set of absolute risk aversion values from those determined above. To obtain relative risk aversion under the assumption of exponential utility, we multiply α by the expected value of the gamble (150 baht).

As an example of the difference, consider the median individual, who preferred the gamble to 40 baht but preferred the safe 50 baht to the gamble; we take the certainty equivalent in this case to be 45. If this individual is assumed to have processed the choice on the basis of an isoelastic (CRRA) utility function as in equation (2), then his or her relative risk aversion is calculated to be 0.63463. But if we were to assume that this subject makes decisions on the basis of exponential utility (as in equation (4)), then relative risk aversion would be calculated as 2.28, roughly 3.6 times the magnitude of the CRRA estimate. The absolute percentage difference is 72 percent if the exponential estimate is used as the denominator, and nearly 260 percent if the difference is divided by the isoelastic value. The same result is obtained if the two estimates of absolute risk aversion are compared for this individual.

The differences are even larger for the modal response, where the certainty equivalent is 15. For such individuals, relative risk aversion is 0.7686 under CRRA utility, but 6.93 if CARA is assumed; the absolute percentage difference is more than 800 percent of the CRRA value.

For the data set as a whole, the calculations based on CARA are, on average, more than 5.4 times as large as those based on CRRA. Viewed another way, the mean absolute percentage difference in risk aversion estimates between the isoelastic and exponential forms is more than 440 percent if the difference is calculated over the isoelastic value, and the differences are statistically significant below the one percent level. Thus, if it were the case that all subjects in the study, unbeknownst to an investigator, made their decisions on the basis of one functional form of utility, but the investigator inadvertently assumed a different functional form for the purpose of calibrating risk aversion, the cardinal values that resulted would be significantly distorted by specification bias.

4. Conclusion

Because the subjects' true preference functions are no more obvious in the present study than in prior research, the results above are intended to be cautionary in nature rather than definitive measurements of bias. Indeed, one cannot say with certainty that any prior research has adopted incorrect functional forms of utility or mismeasured risk aversion. But the very reason why no such definitive statement can be made is that we simply cannot know what the correct functional form of utility is for any individual, let alone for broad samples of individuals. And for that reason, the long-standing practice of imposing a mathematically convenient functional form of utility, and then assuming that exactly the same functional form applies to all individuals in a sample, even as the differences in the subjects' demographic profiles, educational levels, incomes, and other characteristics are recognized and taken into account, is a dubious methodology, unlikely to yield accurate measurements. Yet remarkably, the practice persists, and seems to be widely accepted: a search of Google Scholar shows that the 25 studies in Table 1 have been cited more than 13,000 times.

Given that quantitative estimates of risk aversion are widely sought in the literature, an alternative to imposing specific functional forms of utility is needed. As suggested by Pratt (1964), one alternative has been to apply a Taylor series expansion to the expected utility equation, yielding approximations of the risk aversion coefficients. The Taylor series approach does not require a specific functional form, but still requires that the utility function be differentiable; and even this method has been shown to generate substantial approximation bias (Eisenhauer, 2003, 2012).

Although some scholars dispute the legitimacy of measurable utility when defined over multivariate bundles of diverse goods and services, cardinality is less objectionable when comparing bundles consisting entirely of monetary values, as in the gambles described above. Presumably, an individual can determine the certainty equivalent of a risky financial outcome, which is all that the expected utility equation (1) requires. But the notion that the individual's determination of that certainty equivalent is calculated from a continuous, twice-differentiable mathematical function as in (2) or (4) is far less plausible, and even if it existed, any such function would be inherently impossible for external observers to decipher. Imposing specific cardinal functions on the preferences of research subjects therefore requires eminently unreasonable assumptions. Moreover, the imposition of an identical function on otherwise heterogeneous individuals' preferences is even less acceptable. The results of this study suggest that an incorrect functional form can yield risk aversion estimates that miss the mark by orders of magnitude; and consequently, any business decisions (e.g., insurance premiums) or policy recommendations (e.g., criminal penalties) based on such estimates may be misguided.

Endnotes

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1. An affine transformation also yields $a(X) = \alpha$.

2. Among the studies in Table 1, roughly half of the utility applications are isoelastic, roughly one-quarter are exponential, and roughly one-quarter are quadratic or other functional forms. As noted above, a logarithmic utility function is a subset of the isoelastic form.

3. For critiques of the representative agent concept, see Kirman (1992), Carroll (2002), and Harrison and Rutstrom (2009).

4. At the time of the study, 300 baht were worth approximately U.S. $17.46.

5. Fellner and Maciejovsky (2007) demonstrate that simple lottery choices accurately reflect both individual behavior and aggregate market behavior.

6. Our estimates of relative risk aversion using the menu midpoints and an isoelastic function are consistent with the ranges originally calculated by Hardeweg et al. (2013) using the upper and lower bounds of the certainty equivalent.

7. Despite criticisms that its axioms are often violated, expected utility theory remains the dominant paradigm in the economics of risk. The major alternative, prospect theory, has been similarly criticized; see Birnbaum (2006) and Rieger and Bui (2011).

8. Indeed, eight percent still preferred the gamble to 190 baht, and were offered no further alternatives. These latter individuals may have been infinitely risk loving, but for illustrative purposes, we set their CE to 195 baht; the results are not significantly altered if we exclude them from the analysis and reduce the sample size to 855.

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Eisenhauer, International Journal of Applied Economics, 13(2), September 2016, 19-29

Table 1. Selected Studies of Risk Aversion using Specific Functional Forms of Utility

Author(s)	**Date**	**Application(s)**	**Utility function(s)**	**Estimates**
Wiens	1976	Agriculture	Exponential	$0.0085 \le a(X) \le 0.091$
Hansen & Singleton	1982	Investments	Isoelastic	$0.6795 \le r(X) \le 0.9737$
Buccola	1982	Agriculture	Quadratic, Exponential	$0.0012 \le a(X) \le 0.00196$
Musser, et al.	1984	Scholarships	Quadratic, Semi-log, Isoelastic	$-0.00014 \le a(X) \le 0.00113$
Mehra & Prescott	1985	Equity holding	Isoelastic	$10 < r(X)$
Zuhair, et al.	1992	Agriculture	Quadratic, Cubic, Exponential	$-0.0019286 \le a(X) \le 0.00407888$
Nummelin	1994	Asset returns	Isoelastic	$2.269 \le r(X) \le 5.202$
Blake	1996	Portfolio insur	Isoelastic	$7.88 \le r(X) \le 47.60$
Barsky, et al.	1997	Employment	Isoelastic	$0.7 \le r(X) \le 15.8$
Senkondo	2000	Agriculture	Quadratic, Cubic, Log, Exponential	$-0.00563 \le a(X) \le 0.00547$
Torkamani & Haji-Rahimi	2001	Agriculture	Quadratic, Exponential, Cubic, Expo-power	$-0.02931 \le a(X) \le 0.007712$
Beetsma & Schotman	2001	Game shows	Isoelastic, Exponential	$a(X) = 0.12$, $0.42 \le r(X) \le 13.08$
Hanna, et al.	2001	Employment	Isoelastic	$5.61 \le r(X) \le 7.76$
Gourinchas & Parker	2002	Lifetime saving	Isoelastic	$0.514 \le r(X) \le 1.3969$
Fullenkamp, et al.	2003	Game shows	Isoelastic, Exponential	$4.8E\text{-}6 \le a(X) \le 9.7E\text{-}6$, $0.64 \le r(X) \le 1.76$
Belzil & Hansen	2004	Education	Isoelastic	$0.1073 \le a(X) \le 0.1469$, $r(X) = 0.9282$
Dohmen, et al.	2005	Lotteries	Isoelastic	$0 < r(X) \le 35$
Azar	2006	Equity holdings	Isoelastic	$4.2 \le r(X) \le 5.4$
Kan	2006	Entrepreneurs	Isoelastic	$r(X) = -0.0478$
Botti, et al.	2008	Game shows	Isoelastic, Exponential	$a(X) \sim 0.01$, $r(X) \sim 0.4$
Kimball, et al.	2008	Employment	Isoelastic	Mean $r(X) = 8.2$
Anderson & Mellor	2009	Jobs, Inheritance	Isoelastic	$-0.49 < r(X)$
Syndor	2010	Insurance	Isoelastic, Exponential	$r(X) \le 15{,}283.0$
Von Gaudecker, et al.	2011	Lotteries	Exponential	$-0.12 \le a(X) \le 0.18$

Bombardini & Trebbi	2012	Game shows	Isoelastic	$0 \leq r(X) \leq 5$

Table 2. Absolute and Relative Risk Aversion under Isoelastic and Exponential Utility*

		Assuming Isoelastic (CRRA) Utility		Assuming Exponential (CARA) Utility		CARA/ CRRA
CE	**Frequency**	***a*(*X*)**	***r*(*X*)**	***a*(*X*)**	***r*(*X*)**	**Ratio**
5	40	.005538	.830697	.138600	20.79	25.03
15	234	.005124	.768605	.046200	6.93	9.02
25	98	.004807	.721061	.027717	4.16	5.77
35	68	.004516	.677368	.019727	2.96	4.37
45	65	.004231	.634633	.015170	2.28	3.59
55	63	.003943	.591409	.012131	1.82	3.08
65	39	.003645	.546775	.009892	1.48	2.71
75	35	.003333	.499999	.008125	1.22	2.44
85	36	.003002	.450355	.006657	1.00	2.22
95	34	.002648	.397227	.005391	.81	2.04
105	73	.002265	.339738	.004262	.64	1.88
115	10	.001847	.277099	.003230	.48	1.75
125	20	.001388	.208258	.002265	.34	1.63
135	5	.000879	.131920	.001342	.20	1.53
145	7	.000311	.046652	.000445	.07	1.43
155	17	-.000331	-.049681	-.000445	-.07	1.34
165	4	-.001063	-.159385	-.001342	-.20	1.26
175	4	-.001907	-.286000	-.002265	-.34	1.19
185	3	-.002892	-.433804	-.003230	-.48	1.12
195	75	-.004060	-.609010	-.004262	-.64	1.05

*Source: Hardeweg et al. (2013) and author's calculations