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Trend and Cycle Integration of World Output

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Abstract: In this study, we explore the evolution of integration of real GDP. We apply principal component analysis on trend and cycle components of real output data to reveal the existence of common global trend and cycle factors, governing the evolution of country trend and cycle components. A central contribution of our work is the construction of quantifiable measures of world integration in trend and cycles referred to as indices of trend and cycle integration. Our indices suggest that the evolution of integration follows a volatile and unstable course; yet the level of integration in trend in 2015 is similar to the level in 1997, and the level of integration in cycles is only slightly higher. A key finding of this work is that the integration in cycles is always weaker than integration in trend, thus emphasizing that the extent of integration among countries' levels of output would be understated if only cycle fluctuations are considered. We find that the US trend component exhibits strong and positive correlations with the primary global trend factors and weaker, often negative, correlations with the minor factors. The US cycle component displays extremely volatile correlations with all global cycle factors.

Keywords: International Business Cycles Co-movement, Business Cycle Fluctuations, GDP decomposition, Real Output Integration, Index of Integration, Principal Component Analysis, Random Walk, unobserved component model

JEL Classification: F02, F15, F44, C01, E32

1. Introduction

Is globalization increasing the co-movement and integration among national output levels? The answer to this question has long been a topic of interest for academics and practitioners alike. The issue has been considered as early as in Mitchel (1927) and Kuznets (1956); yet, assessing the scope of integration has been mostly limited to exploring the business cycle synchronization between pairs of countries (Papageoriou et al., 2010; De Haan et al., 2008).

Synchronization of business cycles, however, is neither the only, nor the most dominant source of national output integration. Like Blonigen et al. (2014), my study utilizes quarterly GDP data to reveal that the bulk of real output for 44 countries is due to the permanent effects of shocks captured by stochastic trend components. Consequently, considering the permanent nature of the trend shocks, synchronization in trends across countries is of primary importance for national output integration, especially in the long run. Hence, to fully explore the scope of real output integration, my work studies the integration in trends as well as in cycles.

A key feature of this study is the use of the principal component analysis (PCA) for quantifying the level of overall integration within a data set. The principal component analysis is a form of factor analysis. It is a non-parametric approach used to describe the common features of sample data. It is robust to the presence of outliers and heavy-tailed distributions (Stevens, 1996) and therefore particularly useful in the analysis of national output. The technique transforms the observed variables into new variables, called principal components, where the goal is several components to account for the majority of observed data variability. In many cases, most of the observed data variation is summerized by the first principal component.

Factor analysis is not new to the analysis of economic activity. Kose et al. (2003) uses a latent factor model to identify a common world factor governing the volatility of macroeconomic aggregates for selected countries, thus suggesting the existence of a world business cycle. Kose et al. (2008) uses a similar methodology and finds that the fraction of variability of macroeconomic aggregates of G-7 countries that can be attributed to a common world factor increases as a result of globalization. Kose et al. (2012) also uses factor analysis to detect convergence in cycles among industrial and emerging market economies, but divergence between the two groups. Cruccini et al., (2011) uses a dynamic factor model to study the drivers of business cycle. Their study concludes that productivity is the main driving force of cycles.

Principal component analysis (PCA) has been used extensively in the studies of financial integration. Analyses by Pukthuanthong and Roll (2009) and Berger, Pukthuanthong, and Yang (2011) use market returns and PCA to develop a new indicator of financial integration. They find that simple correlations among country indices may not reflect properly the level of integration and that PCA better captures potential benefit from portfolio diversification. Volosovych (2011) uses bond returns and PCA to create an index of integration and finds that the level of integration by the end of the 20th century was higher than any previous period in history.

Some studies assign meaning to the first component. Meric et al. (2011) and Meric et al. (2012) analyze national market returns and refer to the first component as an indicator of common sources of variability. Volosovych (2011), in a similar manner, suggests that the fraction of total variability in data accounted for by the first component reflects the extent of market integration.

In this study, principal component analysis is used to separately identify the principal components defining the trend and cycle GDP components in each year. Similar to Volosovych (2011), the fraction of data variability explained by the first component is framed into indices of trend and cycle integration. Our indices take values between zero and one and quantify the level of integration in trend and cycles. The fraction of variability in each data set explained by the second and third principal components is also used to construct secondary and tertiary indices of trend and cycle integration.

Our analysis identifies both trend and cycle components as quantitatively significant and similar to Kose et al. (2003), reveals the existence of global common factors governing their evolution. The evidence suggests that the integration in trend is always stronger than integration in cycles and thus the extent of integration among countries' level of output would be understated if only cycle fluctuations are considered.

Our indices of integration display significant volatility, thus illustrating that the evolution of integration is not a steady and consistent process, but rather follows a volatile and unstable course. Our findings suggest that the level of integration in trend in 2015 is similar to the level in 1997, while the integration in cycles is only slightly higher.

The reminder of this paper is organized as follows. The following section briefly describes data and outlines the methodology used in this study. Section 3 presents the results from our empirical work; lastly, section 4 concludes.

2. Data and Methodology

In this section, we review the data and empirical methods utilized in this study. First, we begin with describing the data in subsection 2.1; next, we proceed in subsection 2.2 with explaining how the trend and cyclical components are derived; lastly, in subsection 2.3, weoffer details on the principal component analysis and the construction of trend and cycle integration indices.

2.1. Data

Our study is motivated by the desire to explore the evolution of the level of integration among world economies as suggested by the synchronization of trend and cycle GDP components. Therefore, we use quarterly real GDP data for 44 OECD and Non-OECD countries[1]. Our data spans from 1996: Q1 to 2015: Q4 and comes from OECD Main Economic Indicators (database). The duration of the study is chosen such that the longest series of data are available for the greatest number of countries. Quarterly data allows me to capture patterns of fluctuation in trend and cycles that may be obfuscated (averaged away) by a longer frequency data. All data is seasonally adjusted, denominated in constant 2010 US dollars, and thus particularly useful for cross-country analysis from the prospective of an US stakeholder.

2.2. Trend and Cycle Components

Our study of integration explores the synchronization of country trend and cycle GDP components. Since these two components are not directly observed, they need to be estimated. Here, similar to Blonigen et al. (2014), we estimate the two components using an unobserved-components (UC) model.

Consistent with literature, the UC model identifies the trend component as the accumulation of permanent, long-run effects of shocks on output. This is equivalent to saying that the trend component represents the stochastic trend of the real GDP series.
The cycle component is identified as the transitory, short run deviation of GDP around its stochastic trend and represents the accumulation of temporary, short run effects of shocks on output.

Within the UC framework, the log GDP ($Y_{i,t}$) for each country I in period t, is additively divided into trend ($T_{i,t}$) and cycle ($C_{i,t}$) components:

$$Y_{i,t} = T_{i,t} + C_{i,t} \tag{1}$$

with the trend component specified as a random walk with a drift process:

$$T_{i,t} = \alpha_i + T_{i,t-1} + v_{i,t} \tag{2}$$

and the cycle component is specified as an AR 1 process:

$$C_{i,t} = \beta_i + \beta_{1i} C_{i,t-1} + \varepsilon_{i,t} \tag{3}$$

where $v_{i,t}$ $\sim i.i.d.\ N(0,\sigma^2_{Vi})$, $\varepsilon_{i,t}$ $\sim i.i.d.\ N(0,\sigma^2_{\varepsilon i})$ are independent trend and cycle shocks.

The UC model described by Eqs. *(1)- (3)* is estimated via maximum likelihood and the trend and cycle components are derived using the Kalman Filter.

The estimated trend and cycle components are used in the construction of two separate trend and cycle data sets which are subsequently explored using the principal component analysis.

2.3. Principal Component Analysis

As laid out in Todorov (2016), the Principal Component Analysis (PCA) is a completely non-parametric statistical technique that is used to decrease dimensionality and identify patterns in data (Smith 2000, Pearson 1901, Hotelling 1933, Rencher and Christensen 2012). Its objective is to derive linear combinations of uncorrelated, optimally-weighted observed variables, called principal components (PCs), such that each PC explains the maximum amount of variation remaining in the data subject to it being uncorrelated with all previous PCs and subject to the restriction that

$$\sum_{i=1}^{n} \alpha_{ik}^{\;2} = 1$$

where *n* is the number of original variables and α_{ik} is the loading (or weight) on variable *i* in PC number *k*.

The principal components are constructed using the correlation matrix of the original variables. The eigenvectors of the correlation matrix provide the weights for the observed variables and the eigenvalues measure the variance accounted for by the PCs. The number of components derived equals the number of observed variables. A key feature of the PCA method is that all components are pairwise uncorrelated and together explain the total variance of all variables (Shlens 2009, Stevens 1996). Those components with an eigenvalue greater than one are referred to as significant (Kaiser, 1960). The number of significant components suggests the number of common factors guiding the variability of the data set (Jolliffe 2002).

The principal components are ranked by the variation explained. The first component is the one with the largest variance. It accounts for the greatest possible fraction of the total variation in the original dataset. Each remaining component is constructed such that it accounts for the maximum possible fraction of the total variation that remains unexplained by all previous components.

Consequently, each successive component accounts for a progressively smaller amount of the total data variation. In practice only the first few components, and often only the first one, are kept for further analysis. (Flury 1997, Marida 1979, Rencher and Christensen 2012).

Using the PCA method, the principal components defining the trend and cycle data sets in each year are extracted separately. For each data set, the portions of total variability explained by the first principal component in each year are stacked together to construct dynamic measures of integration, representing our primary indices of trend and cycle integration. The portions of total variability explained by the second and third components are also stacked to construct secondary and tertiary indices, thus exploring possible additional, but less prominent patterns of integration. The trend and cycle indices of integration, are plotted in Figure 2 to provide a graphic illustration of the evolution of integration across countries during the period of study.

Further technical details on PCA and the construction of the indices of integration are offered in Appendices A and B. Comprehensive analysis is available in Jolliffe (2002) and Jackson (2003) among others.

3. Empirical Results

In this part, we report the results from our empirical work. Subsection 3.1 begins by describing the quantitative importance of trend and cycle components. Subsection 3.2 introduces the results from the estimation of the indices of trend integration. Subsection 3.3 presents the correlations between the US trend component and the common trend factors. Subsection 3.4 offers the results from the estimation of the indices of cycle integration and lastly, subsection 3.5 explains the correlations between the US cycle component and the common cycle factors.

3.1. Trend and Cycle Components

The panels of Figure 1 plot the total GDP, as well as the trend and cycle components for the US and the first 3 countries in the sample (Argentina, Australia, and Austria). First and foremost, the plots illustrate that the quantitative importance of the trend component far surpasses that of the cycle component. This underlines the importance of the trend component and implies that the evolution of GDP is primarily governed by permanent shocks. For the US, the trend is mostly upward sloping, with a notable decline caused by the 2008-2009 crash. The change in trend as a result of the decline suggests that the 2008-2009 downturn should not be viewed strictly as a temporary cycle event, but also as a permanent shock altering the long run path of output. The US cycle component is characterized with great variability and particularly steep decline in 2008-2009, again illustrating the dual, both temporary and permanent, nature of the downturn.

Selected results from the estimation of the trend and cycle components are reported in Table 1. The second and third columns provide the estimated standard deviations of the stochastic trend and cycle shocks. Nearly all values are statistically significant. Comparing across countries, we note the two types of shocks have similar magnitudes. Therefore, variability in both components is important for generating variability in GDP. The shocks to the trend component, however, tend to bear greater significance due to the permanent nature of their effects. For the US, the relevant values are 0.00647 and 0.0057, respectively. This implies that the permanent shocks to the US level of output in addition to having long-run effects are also greater in magnitude.

The final column of Table 1 reports the estimated values of the AR (1) parameter from the estimation of the cycle component. The estimates are statistically significant for most countries and negative for eight of them. For the US the value is 0.3252, implying moderate strength of the relationship between past and current period transitory shocks.

In summary, the analysis in this subsection suggests considerable quantitative importance of both the trend and cycle components. The variability of shock to both components is mostly significant and thus important for generating variability in GDP. The shocks captured by the trend component, however, tend to dominate in importance due to the permanent nature of their effects.

3.2. Indices of Trend Integration

In this subsection, we explain the use of the trend component data set and the application of the principal component analysis in the derivation of the indices of trend integration.

The principal component analysis of the trend data set reveals three significant principal components in all years, except for 2015, when only two components are detected. In all years, the significant components capture more than 99% of the total variation in the data set. This implies that there were three major common factors (two in 2015) guiding the variability in country trend components in all years.

Selected results from the estimation of the principal components are reported in Table 2. The second column of the table reports the values of the primary index of trend integration. Each value represents the fraction of total variability in country trend data in a particular year, explained by the first principal component. For most years, this fraction is greater than 0.7. This suggests relatively strong integration among country trend GDP components, as in most years, more than 70 % of total variability of trend components can be related to a single common factor, referred henceforth as a primary trend factor.

Besides the primary trend factor, represented by the first principal component, there seem to be two minor, yet significant common factors driving the global trend integration. Those minor factors are captured by the second and third principal components. The fraction of total trend data variability captured by the second and third principal components is employed in the construction of the secondary and tertiary indices of trend integration. The two supplementary indices are described by the third and fourth column of Table 2.

The results presented in Table 2 suggest that for all three trend indices, the beginning (1997) and ending (2015) values are relatively similar, thus suggesting that the level trend synchronization in 2015 is not much different from what it was back in 1997.

The Indices of trend Integration are plotted in Figure 2. The primary index displays notable peaks in 2004 (0.8732), 2006 (0.9267), 2010 (0.8719) and 2015 (0.8444). In these years, the influence of the primary trend factor was the strongest and the level of integration between country trend components was the highest.

Similarly, notable troughs are observed in 2003 (0.6645), 2008 (0.6461) and 2009 (0.69). In these years, the influence of the primary trend factor was the weakest, and the level of integration between country trend components was the lowest.

The relative variability of the index suggests that the influence of the primary trend factor is not steady and consistent, but rather volatile and unpredictable, thus causing the level of trend integration to follow an unstable path with many ups and downs.

Along with the primary index of trend integration, Figure 2 also plots the secondary and tertiary trend indices. While the secondary and tertiary indices are relatively similar in magnitude, their values are much lower than the values of the primary index. This illustrates the weaker impact of the common factors captured by the second and third principal components relative to the impact of the primary trend factor.

In most instances, the plots of the primary and the secondary and tertiary indices seem to be moving in opposite direction. Most notably, the primary index rises during the mid-2000s, thus implying an increasing level of global integration in trend. At the same time, the secondary and tertiary indices decrease during mid-2000s, thus suggesting decreasing integration. One possible reason could be that the second and third principal components may be capturing minor common, possibly regional, effects opposing to those captured by the primary component. Last but not least, the plots of all three indices illustrate relatively similar levels of trend integration in 1997 and 2015.

In summary, our analysis in this subsection clearly reveals the existence of a primary trend factor causing high degree of global integration among country GDP trend components. In addition, my analysis also reveals the existence of two minor factors also having impact, albeit in opposing direction. My findings suggest that the level of trend integration in 2015 is similar to what it was in 1997.

In the next subsection we proceed with describing the correlations between the US trend component and the common trend factors.

3.3. Correlations of US Trend Component with Common Trend Factors

The second column of Table 3 describes the correlations of the US trend component with the primary trend factors. Here, each correlation value can be interpreted as a measure (although imperfect) of the level of integration between the US trend component and the primary trend factor in a particular year. In most years, the correlations are greater than 0.9, thus implying that the US trend component is strongly synchronized with the global economic environment.

A notable exception is observed in 2008, when the correlation equals -0.577. One possible explanation is that in 2008 the domestic shock was so strong that the US component negatively affected the global economic environment, thus causing a sharp decrease in the global level of trend integration (Figure 2). The domestic downturn not only reversed the direction of the relationship between the domestic component and the primary trend factor, but also decreased the level of integration between the two.

Another exception is observed in 2011, when the correlation equals 0.668. Here again, a minor domestic downturn decreased the level of integration between the US trend component and the primary trend factor, although it did not reverse the direction of the relationship.

Columns 3 and 4 in Table 3 describe the correlations between the US trend component and the secondary and tertiary trend factors. The values are mostly lower in absolute value, relative to correlations between the trend component and the primary factors, and negative on a few occasions. The implication is that while the US trend component is strongly correlated with the primary factor, it is only weakly correlated with the additional common factors.

The correlations between the US trend components and the common trend factors are plotted in Figure 3. The correlations with the primary factor are relatively stable until the huge dip in 2008; then after 2 years of relative stability follows a smaller dip in 2011, followed by relative stability for the duration of the period.

The correlations with the secondary and tertiary factors are lower, with the correlations with the tertiary factor displaying a spike up in 2008, when the correlations with the other factors spiked down.

In summary, the US trend component is positively and strongly correlated with the primary trend factos. A notable exception is the downturn in 2008. The correlations with the secondary and tertiary common factors are weaker and sometimes negative.

3.3. Indices of Cycle Integration

In this subsection, we explain the use of the cycle component data set and the application of the principal component analysis in the derivation of the indices of cycle integration.

The principal component analysis of the cycle data set reveals three significant principal components in all years. This implies that there were three major common factors guiding the variability in country cycle components in all years.

Selected results from the estimation of the principal components are reported in Table 2. The fifth column of Table 2 describes the values of the primary index of cycle integration. Each value represents the fraction of total variability in country cycle components explained by the first principal component in a particular year. For most years, this fraction is between 0.4 and 0.6, meaning that in most years, between 40% and 60% of the variability in data can be attributes to a single common factor, referred henceforth as a primary cycle factor.

The results offered in Table 2 suggest that the values of the primary cycle index are always lower than the values of the primary trend index, thus suggesting that the integration in cycles is always lower than the integration in trend. It then follows that the synchronization of temporary shocks is weaker than the synchronization of permanent shocks. This further implies that the level of integration among countries' level of output would be understated if only cycle fluctuations were considered.

Besides the primary cycle factor, represented by the first principal component, there seem to be two additional common factors driving the global cycle integration. Those additional factors are captured by the second and third principal components. The fraction of total cycle data variability captured by the second and third principal components is used in the construction of the secondary and tertiary indices of cycle integration. These two supplementary indices are described in the sixth and seventh columns of Table 2.

The results presented in Table 2 suggest that for the primary and secondary indices, the ending (2015) values slightly exceed the beginning (1997) values, suggesting that the level of temporary shock synchronization in 2015 slightly exceeds the 1997 level. The level of integration implied by the tertiary index is the same in 1997 and 2015.

Figure 4 plots the Indices of cycle integration. Notable peaks in the primary index are observed in 2009 (0.65), and 2015 (0.5494). In these years, the influence of the primary cycle factor was the strongest and the level of harmonization between country cycle components was the highest. The lowest level of trend integration is observed in 2009. Thus the 2008-2009 downturn caused trend integration to decrease while cycle integration to increase.

Figure 4 also plots the secondary and tertiary indices along with the primary index. The primary, secondary, and tertiary indices seem to be moving in a similar direction, thus implying that the second and third principal components may be capturing independent effects acting in a similar direction as those captured by the primary component. A notable exception is observed in 2008-2009, when the primary index of cycle integration increased sharply, while the secondary and tertiary indices sharply decreased. This is suggestive of a possible decoupling of the global cycle factors caused by the financial crisis. Last but not least, the plots of the primary and secondary indices illustrate slightly higher level of cycle integration in 2015 than in 1997.

In summary, our analysis in this subsection clearly reveals the existence of three global cycle factors driving the integration among country cycle GDP components. The three factors are independent, yet similar in magnitude and driving the global integration in cycles in similar directions. My analysis suggests that the integration in cycles is always lower than the integration in trend and thus considering only cycle integration may severely be understating the level of global output integration.

3.5. Correlations of the US Cycle Component with Global Cycle Factors

The fifth column of Table 3 describes the correlations of the US cycle component with the primary cycle factor. Here, each correlation value can be interpreted as a measure (although imperfect) of the level of integration between the US cycle component and the primary cycle factors.

In most years the US cycle is positively correlated with the primary cycle factors. The correlation values are particularly high in 2002 (0.97), 2009 (0.954), and 2014 (0.93). In these years, the US cycle component was synchronized with the primary cycle factor. Notable exceptions are 1998 (-

0.81), 1999 (-0.74), 2005 (-0.67), 2006 (-0.193), and 2010 (-0.035). One possible explanation is that in these years, the US cycle moved in a different direction from the primary factors due to domestic policies, like quantitative easing, that limited the cross-border impact.

Columns 6 and 7 in Table 3 describe the correlations between the US cycle component and the secondary and tertiary common cycle factors. The associated plots are presented in Figure 5. All correlations are highly volatile, with alternating peaks and troughs, thus displaying a complex relationship between the US cycle component and the common cycle factors. It can also be inferred, that while the primary global factor is the one that matters the most for the US cycle, the secondary and tertiary factors are important as well.

4. Conclusion

In this paper we analyze the evolution of integration of real GDP. We utilize principal component analysis on trend and cycle components of real output data to reveal the existence of common global trend and cycle factors, governing the evolution of country trend and cycle components.

A central contribution of our work is the construction of quantifiable measures of world integration in trend and cycles referred to as indices of trend and cycle integration. Our indices suggest that the evolution of integration follows a volatile and unstable course; yet the level of integration in trend in 2015 is similar to the level in 1997, and the integration in cycles is only slightly higher.

A key finding of this work is that the integration in cycles is always weaker than integration in trend, thus emphasizing that the extent of integration among countries' levels of output would be understated if only cycle fluctuations are considered. We find that the US trend component exhibits strong and positive correlations with the primary global trend factors and weaker, often negative, correlations with the minor factors. The US cycle component displays extremely volatile correlations with all global cycle factors.

A logical extension of the research presented in this article is exploring the factors affecting the levels of integration, as well as the dynamics of their effects. Investigation of integration benefits further by exploring the effects of integration on national income may be also worthwhile. Last but not least, this analysis could be extended to a variety of economic aggregates of interest.

Endnotes

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[1] Australia, Austria, Belgium, Canada, Chile, Czech. Rep., Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Israel, Italy, Japan, S. Korea, Luxemburg, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Slovak Rep., Slovenia, Spain, Sweden, Switzerland, Turkey, UK, US, Argentina, Brazil, Costa Rica, India, Indonesia, Latvia, Lithuania, Russia, South Africa.

References

Berger D, Pukthuanthong K, Jimmy Yang J. 2011. "International diversification with frontier markets" *Journal of Financial Economics*, 101(1):227-242.

Blonigen, B. A., Piger, J., Sly, N. 2014. "Comovement in GDP trends and cycles among trading partners" *Journal of International Economics*, 94(2), 239-247.

Crucini, Mario J., Kose, M. Ayhan, Otrok, C. 2011. "What are the driving forces of international business cycles?" *Review of Economic Dynamics*, 14.1, 156-175.

De Haan, J., Inklaar, R., Jong-a Pin, R. 2008. "Will business cycles in the euro area converge? A critical survey of empirical research" *Journal of Economic Surveys*, 22,234–73.

Flury, B. 1997. "A First Course in Multivariate Statistics" *Springer Texts in Statistics*.

Hotelling, H. 1933. "Analysis of a complex of statistical variables into principal components" *Journal of Educational Psychology"* 24, 417–441, 498–520.

Huber, P. J., Ronchetti, E.M. 2009. "Robust Statistics" *Wiley Series in Probability and Statistics*, 2nd ed., John Wiley & Sons, New York.

Jackson, J. E. 2003. "A User's Guide to Principal Components" *Wiley*, New York.

Jolliffe, I. T. 2002. "Principal Component Analysis" *Springer*, New York, 2nd ed.

Shlens, J. 2009. "A Tutorial on Principal Component Analysis" Unpublished manuscript.

Kaiser, H.F. 1960. "The application of electronic computers to factor analysis" *Educational and Psychological Measurement*, 20, 141-151.

Kose, M. Ayhan, Otrok, C., Whiteman., C.H. 2003. "International business cycles: World, region, and country-specific factors" *American Economic Review*, 1216-1239.

Kose, M. Ayhan, Otrok, C., Whiteman., C.H. 2008. "Understanding the evolution of world business cycles" *Journal of international Economics*, 75.1, 110-130.

Kose, M. Ayhan, Otrok, C., Eswar, P. 2012. "Global business cycles: Convergence or decoupling?" *International Economic Review*, 53.2.

Kuznets, S. 1958. "Long swings in the growth of population and in related economic variables" *Proceedings of the American Philosophical Society*, 102, 25–57.

Marida, K.V., Kent, J.T., and Bibby, J.M. 1979. "Multivariate Analysis" *Academic Press*, London.

Meric, L., Eichhorn, B., McCcall, C., Meric, G. 2011. "The co-movements of national stock markets and global portfolio diversification: 2001-2010" *Review of Economics and Business Studies*, 4(22), 87-98.

Meric, L., Kim, J., Gong, L., Meric, G. 2012. "Co-movements of and linkages between Asian stock markets" *Business and Economics Research Journal*, 3(1), 1-15.

Mitchell, W. 1927. "Business Cycles: The problem and its setting" *National Bureau of Economic Research Books.*

Papageorgiou, T.,Michaelides, G., Milios, J. G. 2010. "Business cycles synchronization and clustering in Europe (1960–2009)" *Journal of Economics and Business*, 62,419–70.

Pukthuanthong, K., Roll, R. 2009. "Global Market Integration: An Alternative Measure and Its Application" *Journal of Financial Economics,* 94, 214–32.

Rencher, A. C., Christensen, W. 2012. "Methods of Multivariate Analysis" *Wiley* 3rd ed. Hoboken.

Shlens, J. 2009. "A Tutorial on Principal Component Analysis" Unpublished manuscript.

Smith L. 2002. "A tutorial on Principal Component Analysis" Unpublished manuscript.

Solnik, B. 1974. "The International Pricing of Risk: An Empirical Investigation of the World Capital Market Structure" *The Journal of Finance*, 29(2), 365-378.

Stevens, J. 1996. "Applied Multivariate Statistics for the Social Sciences" *Lawrence Erlbaum Associates*, NJ, 3rd edition.

Todorov, G. 2017. "Are international portfolio diversification opportunities decreasing? Evidence from Principal Component Analysis" *International Journal of Economic and Financial Issues*, Vol. 7, No.3, 1-23

Volosovych, V. 2011. "Financial Market Integration over the Long Run: Is there a U-shape?" *Journal of International Money and Finance*, 30, 1535–1561.

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Table 1. Selected estimates from the estimation of the trend and cycle GDP components

Country	St. Dev. Of Trend Shock	St. Dev of Cycle Shock	AR (1)
Australia	0.00537*	0.00556*	-0.11289
Austria	0.00744*	0.00720*	0.19840*
Belgium	0.00559*	0.00433*	0.59890*
Canada	0.00635*	0.00540*	0.46828*
Chile	0.01192*	0.01189*	0.22419*
Czech Rep.	0.00356*	0.00670*	0.66080*
Denmark	0.00288*	0.00893*	0.11070
Estonia	0.02106*	0.01880*	0.41860*
Finland	0.01265*	0.01200*	0.19530
France	0.00508*	0.00417*	0.53267*
Germany	0.00846*	0.07935*	0.37040*
Greece	0.01517*	0.01358*	0.27710*
Hungary	0.00914*	0.00752*	0.56000*
Iceland	0.02651*	0.02550*	-0.32010*
Ireland	0.02021*	0.01948*	-0.19497*
Israel	0.00933*	0.09330*	0.20869*
Italy	0.00744*	0.00598*	0.56711*
Japan	0.01072*	0.01060*	0.22545*
Korea	0.01414*	0.01380*	0.34737*
Lux	0.01910*	0.01898*	-0.19660*
Mexico	0.00956*	0.00840	-0.44960*
Netherlands	0.00728*	0.00600*	0.41280*
New Zealand	0.00913*	0.00940	0.03343
Norway	0.01179*	0.01165	-0.36760*
Poland	0.01049*	0.11345*	-0.18178*
Portugal	0.00840*	0.00729*	0.23790*
Slovak Rep.	0.01807*	0.01837*	-0.07800
Slovenia	0.01176*	0.01039*	0.43660*
Spain	0.00688*	0.00308*	0.86445*
Sweden	0.00948*	0.00900*	0.36460*
Switzerland	0.00600*	0.00528	0.50848*
Turkey	0.02090*	0.02046*	0.25590*
United Kingdom	0.00600*	0.00440*	0.66513*
United States	0.00647*	0.00570*	0.32520*
Argentina	0.53768	0.01700	0.05218
Brazil	0.01280*	0.01300*	0.21310*
Costa Rica	0.01225*	0.01306	0.03290
India	0.09690*	0.02180*	0.96820*
Indonesia	0.01782	0.01720*	0.37890*
Latvia	0.02142*	0.02000*	0.37438*
Lithuania	0.02095*	0.02060*	0.27570
Russia	0.01816*	0.01600*	0.51124*
South Africa	0.00593*	0.04800*	0.60066*

*Indicates significance at 5% level of significance

Table 2. Estimates from the principal component analysis. The table provides estimates of the primary, secondary, and tertiary indices of trend and cycle integration

Year	Primary Index of Trend Integration	Secondary Index of Trend Integration	Tertiary Index of Trend Integration	Primary Index of Cycle Integration	Secondary Index of Cycle Integration	Tertiary Index of Cycle Integration
1997	0.8503	0.0875	0.0621	0.4354	0.3141	0.2505
1998	0.7082	0.1796	0.1123	0.3931	0.3485	0.2584
1999	0.827	0.093	0.08	0.464	0.2826	0.2534
2000	0.8633	0.0814	0.0553	0.4048	0.3516	0.2436
2001	0.7131	0.1642	0.1227	0.4424	0.3093	0.2483
2002	0.7744	0.1623	0.0633	0.4018	0.3565	0.2417
2003	0.6645	0.1926	0.1429	0.4356	0.363	0.2015
2004	0.8732	0.0776	0.0493	0.4638	0.3236	0.2126
2005	0.8947	0.0714	0.0338	0.4483	0.3326	0.2191
2006	0.9267	0.0393	0.034	0.3635	0.3314	0.305
2007	0.8624	0.099	0.0387	0.3971	0.3349	0.268
2008	0.6461	0.2615	0.0923	0.4637	0.3519	0.1844
2009	0.69	0.2452	0.0645	0.6502	0.2636	0.0863
2010	0.8719	0.0954	0.0327	0.4329	0.3594	0.2076
2011	0.7651	0.159	0.0759	0.4197	0.3269	0.2534
2012	0.7179	0.1518	0.1303	0.397	0.3424	0.2605
2013	0.7539	0.1312	0.1149	0.4391	0.3424	0.2184
2014	0.7563	0.1779	0.0658	0.4122	0.3267	0.2611
2015	0.8444	0.1556		0.5494	0.4506	0.2505

Note: No value is available for the tertiary trend index in 2015 as in 2015 there were only two significant principal components detected.

Primary Index of Trend Integration – each value represents the fraction of total variability in country Trend GDP components explained by the first principal component in each year.

Secondary Index of Trend Integration - each value represents the fraction of total variability in country Trend GDP components explained by the second principal component in each year.

Tertiary Index of Trend Integration - each value represents the fraction of total variability in country Trend GDP components explained by the third principal component in each year.

Primary Index of Cycle Integration - each value represents the fraction of total variability in country Cycle GDP components explained by the first principal component in each year.

Secondary Index of Cycle Integration - each value represents the fraction of total variability in country Cycle GDP components explained by the second principal component in each year.

Tertiary Index of Cycle Integration - each value represents the fraction of total variability in country Cycle GDP components explained by the third principal component in each year.

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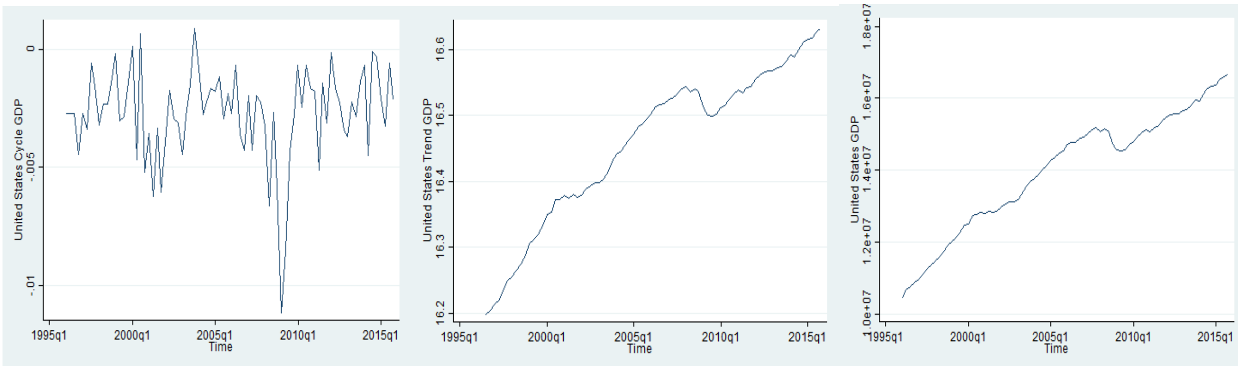
Table 3. Correlations between the US GDP components and global trend and cycle factors. The table provides estimates of the correlations between the US Trend and Cycle components and the primary, secondary and tertiary trend and cycle factors

Year	Correlations between US GDP Trend component and the Primary Trend Factor	Correlations between US GDP Trend component and the Secondary Trend Factor	Correlations between US GDP Trend component and the Tertiary Trend Factor	Correlations between US GDP Cycle component and the Primary Cycle Factor	Correlations between US GDP Cycle component and the Secondary Cycle Factor	Correlations between US GDP Cycle component and the Tertiary Cycle Factor
1997	0.9983	-0.0237	0.0569	0.8701	0.1986	0.4418
1998	0.9833	-0.1415	-0.1108	-0.8131	-0.5776	0.0670
1999	0.9928	-0.0102	0.0833	-0.7468	-0.3108	-0.5766
2000	0.9053	-0.4261	0.0172	0.1318	0.9189	-0.3709
2001	0.9424	-0.1890	-0.0216	0.4418	0.8967	-0.0129
2002	0.9652	0.2701	0.0921	0.9743	-0.0333	-0.2227
2003	0.9348	0.3314	0.1261	0.3055	0.3213	0.9133
2004	0.9723	0.2199	-0.0643	0.4434	-0.8798	-0.0878
2005	0.9694	0.1373	-0.1968	-0.6796	-0.5401	0.5002
2006	0.9139	0.3494	-0.2052	-0.1937	0.9526	0.2755
2007	0.9280	0.3467	-0.1340	-0.1223	-0.6596	0.7533
2008	-0.5771	-0.4829	0.6563	0.5251	-0.7179	0.4654
2009	0.9762	0.2131	0.0319	0.9542	-0.2985	-0.0248
2010	0.9944	-0.1028	0.0167	-0.0353	0.9438	0.3365
2011	0.6682	-0.3276	0.6651	-0.9019	0.3786	-0.2046
2012	0.9606	-0.2821	0.0327	0.8879	0.4064	-0.2168
2013	0.9912	0.0033	0.1302	0.4618	0.8817	0.0670
2014	0.9420	0.3266	-0.0784	0.9396	-0.0553	0.3373
2015	0.9062	0.4227	0.0000	0.0286	0.9949	0.0000

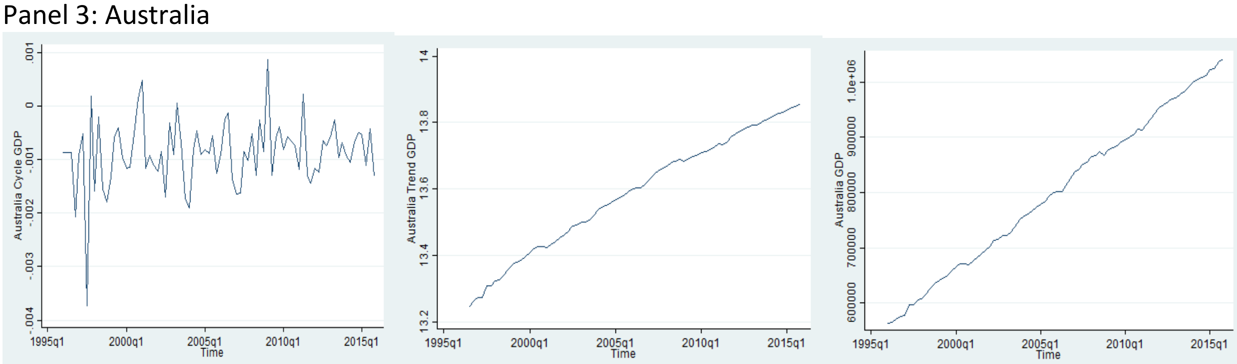
Figure 1. Trend and Cycle Components by Country

Panel 1: USA



Panel 2: Argentina

Panel 3: Australia



Panel 4: Austria

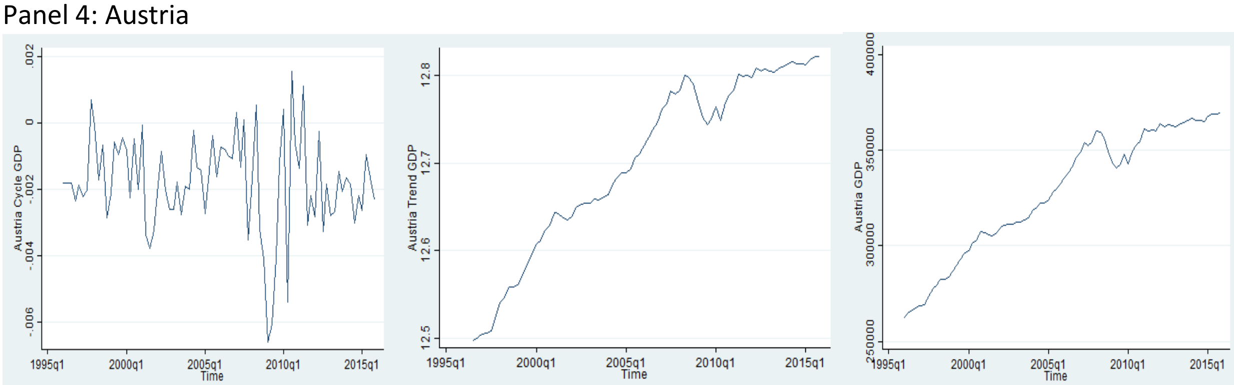


Figure 2. Indices of Trend Integration. Estimated yearly values of the primary, secondary, and tertiary indices of Trend Integration from 1997 to 2015

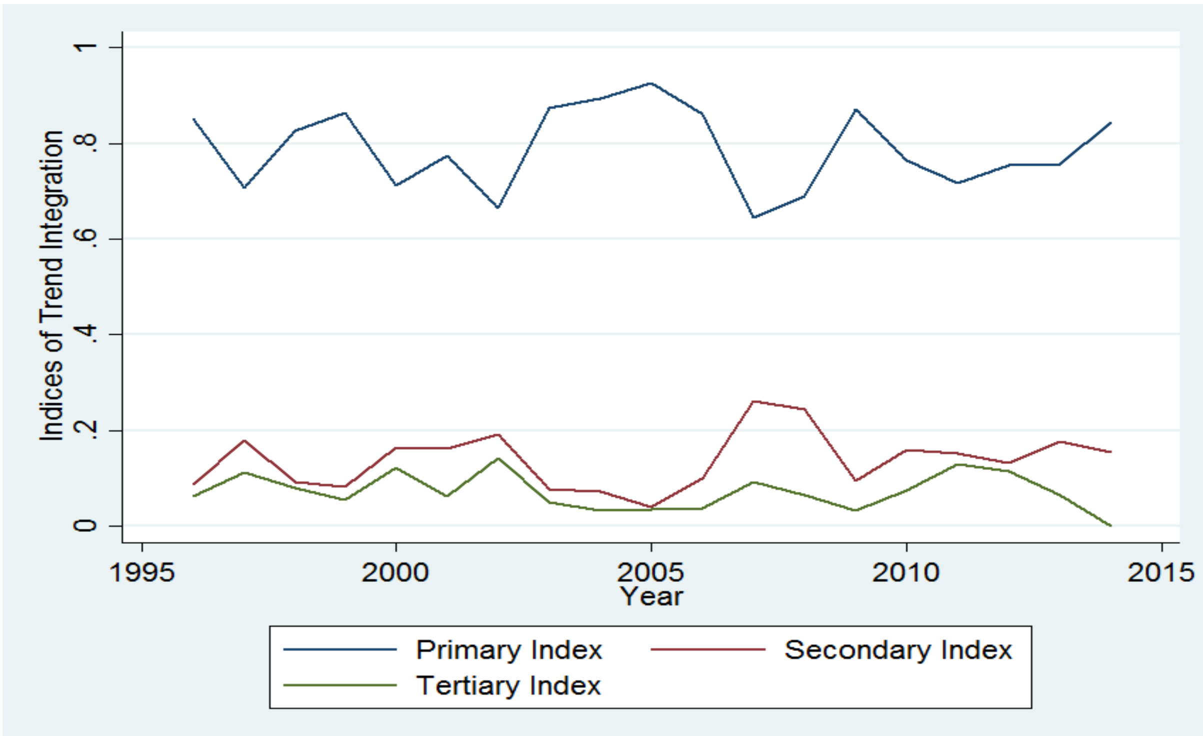


Figure 3. Correlation of the US Trend Component with the Primary, Secondary, and Tertiary Trend Factors; estimated yearly values between 1997 and 2015

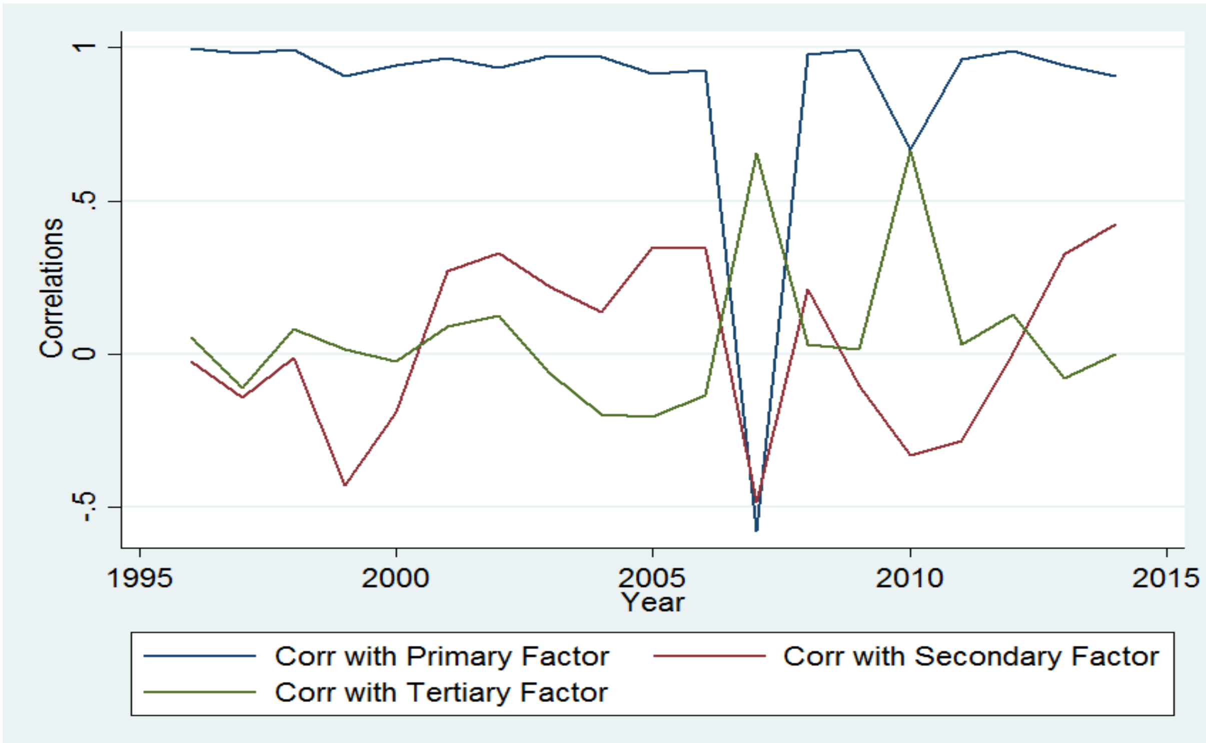


Figure 4. Indices of Cycle Integration. Estimated yearly values of the primary, secondary, and tertiary indices of Cycle Integration from 1997 to 2015

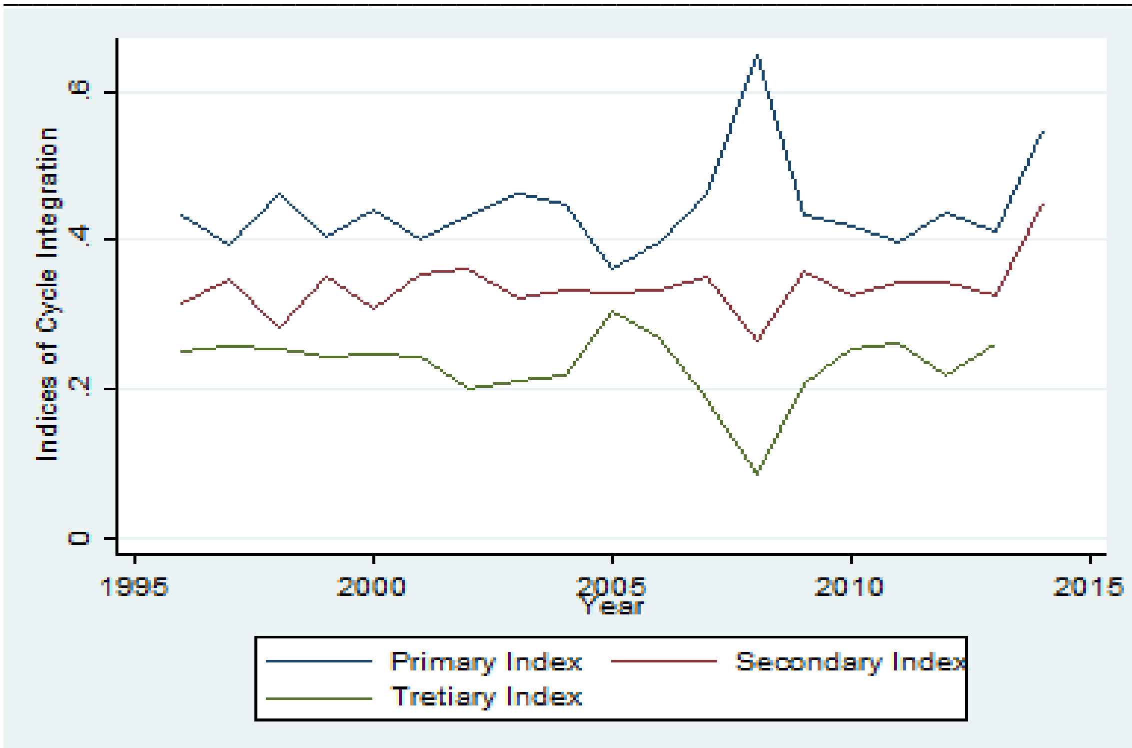
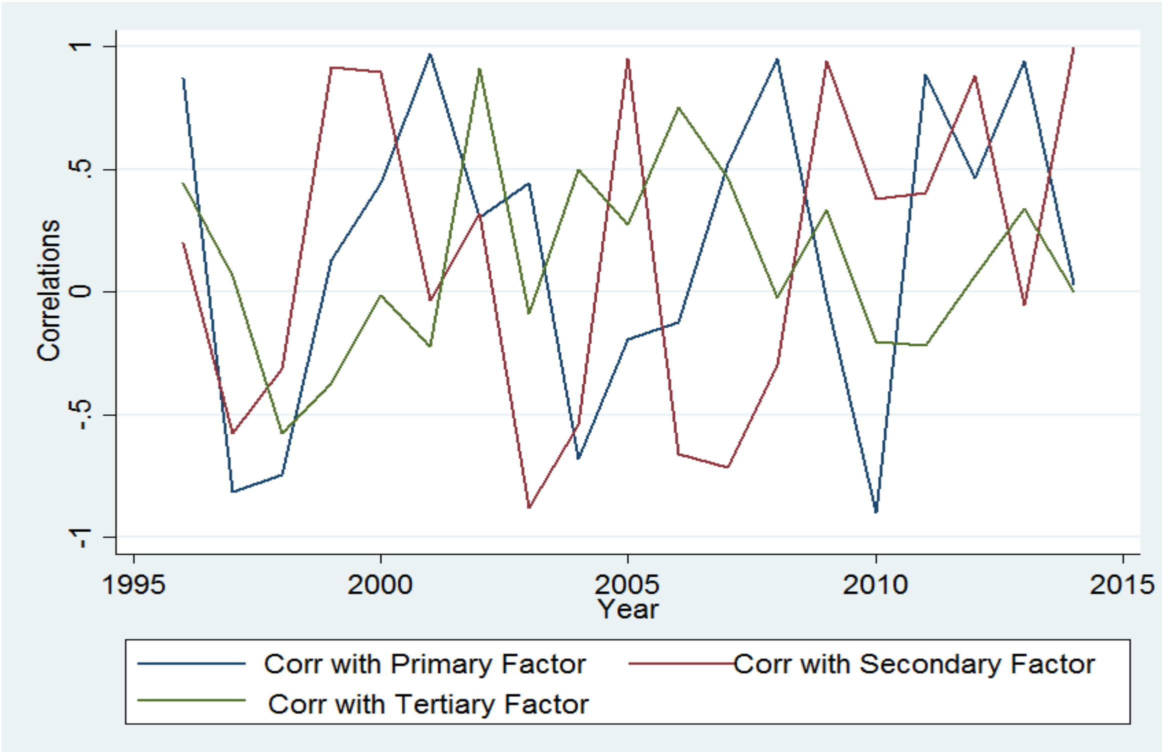


Figure 5.Correlation of the US Cycle Component with the Primary, Secondary, and Tertiary Cycle Factors; estimated yearly values between 1997 and 2015



Appendix A

Principal Component Analysis

As laid out in Todorov (2016), principal component analysis (PCA) is a variable reduction statistical method that is often used to identify patterns in data and describe possible underlying data structure. It is especially useful in the analysis of datasets containing a relatively large number of variables, where those variables are believed to be imperfect measures of one or more underlying constructs. This implied redundancy in variables allows for the reduction the observed variables into a smaller number of principal components (artificial unobserved variables) that account for most of the variance in the observed variables.

In the process of data reduction, PCA extracts the eigenvectors from the eigen decomposition of the correlation matrix of the original variables. The eigenvectors are then used to create a series of uncorrelated linear combinations of the variables (principal components) that explain the total variance in the dataset. The number of extracted principal components is equal to the number of original variables and the sum of the variances of all components is equal to the sum of the variances of the original variables. The use of a correlation matrix results in the observed variables being standardized with a variance equal to 1. Thus, the total variance in the dataset is equal to the number of the variables analyzed. In practice, only those components with relatively high variance are kept for further analysis.

PCA is founded on a set of simple assumptions and requires no probability distribution specified for the observed data. Shlens (2009) outlines those assumptions as follows:

1. Linearity: The relationship between the observed variables is linear
2. PCs are orthogonal. This assumption makes PCA soluble with linear algebra decomposition techniques.
3. Large variances have important structure - PCs with larger associated variances represent interesting structure, while those with lower associated variances represent noise
4.

As an illustration of how principal components are derived, consider a set of variables X_j (e.g. national stock market indices), such that j=1…K. Let X_1, X_2, X_3… X_k are measured on even observational intervals (monthly returns) and are put together to form a linear combination such that

$$F_1=\alpha_1X_1 + \alpha_2X_2 + \alpha_3X_3 + \alpha_4X_4 + \ldots\ldots + \alpha_kX_k$$

Where F_1 is referred to as the first principal component of the K observed variables X.

The coefficients of the component F_1, summarized by the vector $A_1' = (\alpha_1^1, \alpha_2^1, \alpha_2^1 \ldots \alpha_k^1)$, are called variable loadings. A_1' is selected such that the sample variance of F_1 is maximized:

$$Var\ (F_1) = A_1'Z_{xx}\ A_1$$

Where Z_{xx} represents the sample correlation matrix.

The coefficients contained in A_1' are elements of an eigenvector of the sample correlation matrix Z_{xx} selected such that A_1' A_1=1. This allows for the variance of the component F_1 to be represented by the eigenvalue λ_1 corresponding to the eigenvector A_1'.

In PCA, the number of components is equal to the number of originally observed variables. If there are K observed variables, then there are K principal components and the variance of each F_j, j=1….K; is represented by the eigenvalue λ_j corresponding to the eigenvector A_j'.

Each successive component is derived such that it is orthogonal to the preceding one(s) and explains the maximum possible fraction of the total variance that remains unexplained by the previous components. For example, F_3 explains the maximum possible fraction of total variance, that remains unexplained after F_1 and F_2 have been derived.

Each component F_j, j=1…K, can be determined from the sample correlation matrix Z_{xx} by solving the following characteristic equation:

$$| Z_{xx} - \lambda I| = 0$$

This equation has K ordered roots, called eigenvalues such that:

$$\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \ldots . \geq \lambda_k \geq 0$$

A distinct property of the eigenvalues is that λ_1 =Var(F_1), λ_2 = Var(F_2), λ_3 +Var(F_3) etc. The total variance in the dataset is then equal to the sum of the eigenvalues such that:

$$\lambda_1 + \lambda_2 + \lambda_3 + \ldots . + \lambda_k = K$$

The proportion of the total variance explained by the first principal component is given by λ_1/K, the proportion of the variance explained by the second component is given by λ_2 /K etc.

Principal components are ranked according to the variance they explain. Keiser (1960) advises that only components with a variance greater than the variance of a single variable, those with eigenvalues greater than one, are considered for further analysis. According to this criterion, other components are considered less significant and constitute noise.

Last but not least, if the variable loadings in a component are multiplied by the square root of the respective component's eigenvalue, the product will produce estimates of the correlations between the variables and the principal component.

More in-depth analysis and detailed discussions of PCA are offered in Stevens (1996), Smith (2002), Marida et.al. (1979), and Jolliffe (2002) among many others.

Appendix B

Construction of Trend and Cycle Indices

Here I describe my approach to applying the PCA on GDP trend and cycle components and quantifying the trend and cycle indices. The procedure is repeated for each of the trend and cycle components separately.

The steps in the process are outlined as follows:

1. I perform PCA on country component values separately for each year. This provides for as many principal components as there are countries in the data set. Each principal component is based on an eigenvector with a respective eigenvalue.

2. Rank PCs according to size of eigenvalues. Each eigenvalue measures the variation explained by a particular PC and the sum of all eigenvalues equals the total variation in the data set.

3. Obtain the proportion of total variation explained by those principal components that are significant. This is done by dividing the respective eigenvalue by the sum of all eigenvalues.

4. Repeat this procedure separately for each year from 1996 to 2015. Obtain the fraction of total variation explained by each significant components for each year, and stack the corresponding values in vectors to form indices. An index only takes values between 0 and 1.