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Heckscher-Ohlin Model with Assignment Problem with Skilled and Unskilled Labor

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Abstract: In this work, I construct an extended model of the Heckscher-Ohlin model, in which there is an assignment problem of how to assign workers to two types of endowments, skilled and unskilled labor. In the comparative statics for a small open economy, when product prices change, the outputs change more than in the Heckscher-Ohlin model. In the comparative statics for a whole economy, when consumer tastes or production technologies change, the product prices change less than in the Heckscher-Ohlin model. Furthermore, a greater elasticity of substitution between endowments causes a greater extent of change in the outputs for a small open economy and a lesser extent of change in the product prices for a whole economy. These results explain why we see different kinds of relationships between wage changes and other economic changes.

Keyword: Heckscher-Ohlin model, assignment model, heterogeneous agents, income differentials

JEL Classification: D33, F11, F61, I31, J31

1. Introduction

In the early 20th century, Heckscher and Ohlin developed the Heckscher-Ohlin model [see Heckscer-Ohlin (1991)]. The Heckscher-Ohlin model assumes that differences in factor endowments among countries cause international trade, and the pattern of international trade depends on the differences in the factor endowments. This insight is different from the Richardian model [see Richardo (1817)]. In the Richardian model, the causes of international trade are technological differences across countries, and comparative advantages determine the patterns of international trade.

The Heckscher-Ohlin model has been extended, modified, and examined by many economists. Stolper and Samuelson (1941) formulated the Stolper-Samuelson Theorem, which states that an increase in a product price increases the factor price used intensively for that good and reduces the other factor price. Rybczynski (1955) found that an increase in a factor endowment increases the output of the product using the factor intensively and reduces the other output. Vanek (1968) presented and examined the multigood and multifactor Heckscher-Ohlin model. The empirical research on the Heckscher-Ohlin model includes Leontief (1953), Leamer (1980), Bowen, Leamer, and Sveikauskas (1987), Trefler (1993), and others.[1]

In this paper, I extend the Heckscher-Ohlin model using concepts of assignment models. Assignment models are models in which heterogeneous resources are assigned to tasks based on characteristics of the resources and the tasks, and the outputs are then produced. Roy (1951) considered models in which workers choose their jobs from among different kinds of jobs based on which job is best suited for them. Tinbergen (1951, 1956, and 1970) considered models in which the preference of each worker determines which job each worker chooses. Sattinger (1979) introduced a one-to-one matching model in which workers have different levels of ability and machines have different sizes. This model is extended in many ways, including by Teulings (1995 and 2005), Costrell and Loury (2004), Tervio (2008) and Sasaki (2012).

Martin (1976), Chen (1995), and others have discussed the effects of endogenous factor supply on the Heckscher-Ohlin model. In these papers, the supply of factors depends on the factor prices. For example, when workers determine how long they work, an increase in wage increases their working hours and reduces their leisure time. Thus, the supply of labor depends on the wage for labor.

Here, I consider an assignment problem between two factors, say skilled labor and unskilled labor. Workers choose their jobs based on the factor price ratio between skilled labor and unskilled labor. As the ratio changes, workers alter their jobs. As a result, the supply of both skilled labor and unskilled labor depends on the wages for both skilled labor and unskilled labor. Thus, this paper proposes another aspect of endogenous factor supply on the Heckscher-Ohlin model.

In international trade literature, concepts of assignment models are used by Leamer (1999), Grossman and Maggi (2000), Ohnsorge and Trefler (2007), and others. Leamer (1999), assuming that effort levels affect productivity, analyzes income distribution in developing countries and developed countries. Grossman and Maggi (2000) consider a model in which countries have similar aggregate factor endowments, but different diversity and ability relationships. These assumptions are what create types of trade patterns. Ohnsorge and Trefler (2007) assume that there are heterogeneous workers, and they analyze what types of trade pattern and income distributions are created in the economy by sorting mechanism.

In this paper, my main focus is the relationship between assignment and prices of factor and product. Heterogeneous workers choose to work as skilled workers or unskilled workers. When product prices or factor endowments change, the changes cause factor prices and outputs to change. However, the factor prices and output changes also alter how factor endowments are assigned to sectors optimally because a change in factor prices might alter which job each worker is most suited for. In the previous studies, these relationships are not examined extensively.

Today, as globalization evolves, developed countries are exposed to extensive competition with developing countries, which have cheap labor. Some have said that globalization and competition with developing countries cause wages, especially for unskilled work, to drop, widening income differentials. One explanation is the Factor Price

Equalization Theorem of the Heckscher-Ohlin model. On one hand, the competition with developing countries in unskilled labor might cause the wages of unskilled labor to decrease and cause loss of jobs. On the other hand, competition within developed countries also has consequences and might reduce the wages of others. Thus, when considering the effect of competition with developing countries, one must take into consideration not only competition with developing countries but also competition within developed countries. In this paper, I attempt to extend the Heckscher-Ohlin model to include both effects and explain the reality.

2. The Model

2.1 Assumption

In a Heckscher-Ohlin model, countries have fixed amounts of factor endowments. For example, assuming that there are two countries and two resources, labor and capital, the first country has 500 units of labor and 300 units of capital and the other has 400 units of capital and 250 units of capital. More generally, we suppose that a country has A of labor and B of capital. Using those endowments, countries produce products and trade a portion of those products.

I think that one of the natural extensions of the Heckscher-Ohlin model is cases in which endowments are given in a function form, not fixed amounts, and are, to an extent, interchangeable. Considering labor, if one wants to produce a machine, more efficient skilled workers are preferable. However, less efficient skilled workers might be also acceptable to a extent. If the wage difference between the less and more efficient skilled workers is large enough, less efficient skilled workers are more adequate from cost-efficiency standpoint. Thus, with wages changing, there is a possibility that the amounts of effective labor that each industry can use are changing.

This type of relationship is also true between labor and capital. If the capital price is too high, producing capital from labor might be more efficient. Thus, the amount of capital with which a country is endowed might not be considered a fixed amount.

Figure 1 is the Heckscher-Ohlin model's two-factor case, where x, on the horizontal axis, is an amount of one of the two factors and y, on the vertical axis, is an amount of the other factor. The figure depicts the endowment possibility frontier. The amounts of each factor are fixed.

Conversely, Figure 2 depicts cases in which endowments are given in a function form. The figure depicts the endowment possibility frontier. If one wants to increase the amount of one of the factors, one can do that in exchange for a decrease in the amount of the other factor. As the relative price changes, the amounts of factors change. The fact that endowments are not fixed gives additional flexibility to the model.

In the Heckscher-Ohlin model case, Figure 1, whatever the factor prices are, the amounts of the endowments are fixed. Conversely, in Figure 2, with factor prices changing, the amounts of the endowments change too. This means that, as the factor prices change, a portion of one of the factors transforms into the other factor.

For simplicity, we assume that we are able to add up different levels of worker ability and that the sum of the abilities is the amount of a factor. For example, assume that there are three workers, Mike, John, and Alice. Mike has three of skilled work abilities, John has two, and Alice has four. They have the same unskilled work ability, say one. If all three are utilized as skilled workers, there are nine skilled work factors. If all three are utilized as unskilled workers, there are three unskilled work factors. In more complex models, combinations of workers' abilities and jobs' difficulties must affect outputs. However, this simplified assumption largely reduces the difficulty of analysis.

For example, let's consider skilled workers and unskilled workers. For skilled work, more competent people are probably preferable to get a job done. However, if less competent workers need much lower wages than more competent workers, less competent workers might be preferable to companies. Considering the economy as a whole, as the wage ratio of skilled workers to unskilled workers grows, more workers are assigned to skilled work, which is done more efficiently. However, as more and more workers are assigned to skilled work, the marginal increase of efficiency unit of knowledge worker per newly assigned worker decreases. Since, as more and more workers are assigned to knowledge work, the competence of newly reassigned workers decreases. As a result, the shape of the function of factor endowments has to be like that shown in Figure 2.

Putting workers' wages on the vertical axis and workers' ability on the horizontal axis, Figure 3 shows workers' wages. Assume that all workers have the same ability in unskilled work and receive the same wage, w_u, and that workers have different ability levels, x, in skilled work and are compensated on a pay-for-ability basis, xw_s. Each worker chooses the optimal work, skilled work or unskilled work, based on how much he or she gets. As Figure 3 shows, workers with lower ability levels work as unskilled workers and earn the wages of unskilled work, w_u. Workers who have higher ability levels work as skilled workers and earn the wages of skilled work, xw_s. The wage line of unskilled workers and the wage line of skilled workers are connected because the point where the wage lines meet is where workers can earn the same wage on skilled and unskilled work, and it divides the workers into skilled workers and unskilled workers.

Next, we turn our attention to a comparison of the endowment functions between two countries. Assume that there are two factors, X and Y. Let $w_i, i = x, y$ be the factor prices and w be the relative factor price, $\frac{w_y}{w_x}$. Let $x_i(w), i = 1, 2$ be the endowment of factor X that i country has at the relative factor price w and $y_i, i = 1, 2$ be the endowment of factor Y that i country has at the relative factor price, w.

Definition 2. 1.

Assume that there are two factors, x and y, and two countries. When the relative factor price is w, if the relative factor endowment of Y factor to X factor in country 1, $\frac{y_1(w)}{x_1(w)}$, *is larger than* *the relative factor endowment of Y factor to X factor in country 2,* $\frac{y_2(w)}{x_2(w)}$*, then country 1 is Y abundant at the relative price w.*

This definition is straightforward. For example, given a relative factor price, if country 1 possesses 100 units of factor Y and 150 units of factor X and country 2 possesses 150 units of factor Y and 200 units of factor X, the relative factor amount of country 1 is $\frac{2}{3}$ and that of country 2 is $\frac{3}{4}$. Then, country 2 is Y abundant at the given relative price and country 1 is X abundant at the given relative price.

Definition 2. 1. is true when a relative price is given. Next, we generalize Definition 2. 1. to a property that holds true at any relative price.

Definition 2. 2.

Assume that there are two factors, x and y, and two countries. Whatever the relative price is, if the relative factor endowment of Y factor to X factor in country 1, $\frac{y_1(w)}{x_1(w)}$*, is larger than the relative factor endowment of Y factor to X factor in country 2,* $\frac{y_2(w)}{x_2(w)}$*, then country 1 is completely Y abundant.*

Definition 2. 2. is a very useful property because, as explained later, being completely *i* abundant guarantees that a country is abundant in a factor whatever the relative price is, and it simplifies the analysis.

For example, assume that there are two types of jobs, skilled labor and unskilled labor. For unskilled labor, each worker produces the same amount. Conversely, when engaging in skilled work, each worker produces different amounts. Assume that there are two countries and that each country has a continuum of labor, $i \in [0,1]$. The continuum of labor is indexed by relative ability in skilled work. The *0*-th worker is the least competent and the *1*-th worker is the most competent. Let $g_j(i)$ be the ability level that the *i*-th worker in country *j* has. The ability level is addable in production. We get the next proposition.

Proposition 2. 1.

Assume that there are two countries and two types of jobs, unskilled and skilled, and that every worker produces the same amount in the first type of job, an unskilled job. If $g_1(i)$ *is higher than* $g_2(i)$ *at any i, then country 1 is completely skilled labor abundant.*

Proof 2. 1.

At a given relative factor price, country 2 has more workers assigned to unskilled work. Because $g_1(i)$ *is higher than* $g_2(i)$ *at any i, the number of workers who have less than a* *certain ability level is larger in country 2 than in country 1. The ability of a worker at any i is higher in country 1, and more workers are assigned to skilled work in country 1 than in country 2. Thus, the sum of the ability level assigned to skilled work in country 1 is higher than in country 2. Therefore, the relative factor amount of skilled labor to unskilled labor in country 1 is larger than in country 2.*

2. 2 Existence of Equilibrium

Assume that there are two countries. The countries have the same production technologies. The production functions are assumed to be increasing, concave, and homogeneous of degree one in inputs, skilled labor, L_s, and unskilled labor, L_u. There are two products. Producing the first product, say X_1, requires more skilled labor compared to the other, X_2. FIRs (factor intensity reversals) do not occur. The market is completely competitive. We assume that skilled labor and unskilled labor are fully mobile within countries but not mobile between countries. Both countries have the identical homothetic taste. Endowments, skilled labor and unskilled labor, are given in a functional form. The endowment functions are decreasing and concave, and one of the countries is completely skilled labor abundant. Let $p_i, i = 1, 2$ be the prices of products and $w_j, j = s, u$ be the prices of factors, skilled labor and unskilled labor.

First, we consider whether equilibrium exists. Let $p = \frac{p_1}{p_2}$ be the relative price and X^s and X^d, $\frac{X_1}{X_2}$, be the relative supply and demand of products. With the relative price on the vertical axis and the relative supply and demand on the horizontal axis, the world demand curve is a downward slopping curve. The relative supply curve is an upward slopping curve because the endowment functions and the production functions are concave. Therefore, the intersection is the equilibrium.

Proposition 2. 2.

If both countries produce both goods, the factor prices are equalized across countries.
As in a basic Heckscher-Ohlin model, the Factor Price Equalization Theorem holds true. Given prices, if a country produces both goods, the factor prices are uniquely determined to be where the unit-cost lines intersect because FIRs do not occur, the factor prices are stable because the endowment functions are concave.

Proposition 2. 3.

Suppose that two countries produce two goods and engage in free trade and that FIRs do not occur. If a country is completely X abundant, then the country exports the product using X intensively.
When both countries produce both goods, according to Proposition 2. 2, the factor prices are equalized between countries. This implies that the country that is completely skilled labor abundant produces more skilled labor intensive goods relative to the other country. Taking into consideration that the countries have the identical homothetic taste, the skilled labor abundant country exports skilled labor intensive goods. When specialization

occurs, the only possibility is that the skilled labor abundant country will specialize in producing the skilled labor intensive goods and the unskilled labor abundant country will specialize in producing the unskilled labor intensive goods. Thus, each country exports goods that use the abundant factor intensively.

Proposition 2. 4.

Assume that there are two countries, one is completely X abundant relative to the other, and two goods, a good using X intensively and a good using Y intensively, that FIRs do not occur, and that the countries change their trade policy from a closed economy to a open economy. Then, in the completely X abundant country, the prices of X and a good using X intensively increase, and the prices of Y and a good using Y intensively decrease. Furthermore, in the completely Y abundant country, the prices of Y and a good using Y intensively increase, and the prices of X and a good using X intensively decrease.

As Proposition 2. 2 and 2. 3 state, the product prices and the factor prices are equalized across countries, and each country exports a good using each country's abundant factor. This means that, comparing the situation before and after the trade policy change, each country increases the amount of a good using each country's abundant factor and exports them. Thus, if there is no trade between the countries, there will be a surplus of the good using each country's abundant factor at the prices in the open economy. As a result, in each country, the price of the good using each country's abundant factor in the closed economy is lower than in the open economy. As explained later in equation (6) of the next subsection, when the price of a good using a factor intensively increases, the price of the factor increases.

2. 3 Model Structure

2. 3. 1 Functions

As in the previous subsections, let $w_j, j = s, u,$ be the prices of labor (s represents skilled and u represents unskilled) and $p_i, i = 1, 2,$ be the prices of products. Let $L_j, j = s, u,$ be the amount of labor employed as skilled workers, s, and unskilled workers, u, and $L_{j,i}, i = 1, 2$ and $j = s, u,$ be the amount of labor employed in industry i as skilled and unskilled labor. Thus, the following holds.

$L_s = L_{s,1} + L_{s,2}, L_u = L_{u,1} + L_{u,2}.$

Assume that the production functions for a good *i, i=1,2,* are $F_i(L_{s,i}, L_{u,i})$ and have constant return to scale, $F_i(aL_{s,i}, aL_{u,i}) = aF_i(L_{s,i}, L_{u,i})$. Let $y_i, i = 1, 2,$ be the output of good *i,* $F_1(L_{s,1}, L_{u,1}) = y_1, F_2(L_{s,2}, L_{u,2}) = y_{2.}$

Let $l_{j,i}, j = s, u$ and *i=1, 2* be the amount of skilled and unskilled labor to produce one unit of good *i*. Thus, each industry needs $l_{s,i} = \frac{L_{s,i}}{y_i}, i = 1, 2,$ of skilled labor and $l_{u,i} = \frac{L_{u,i}}{y_i}, i = 1, 2,$ of unskilled labor.

The market is completely competitive. Thus, profits of firms are zero. As a result, the prices of products times the derivatives of the production functions are equal to the prices of labor:

$$p_i \frac{\partial F_i(L_{s,i}, L_{u,i})}{\partial L_{j,i}} = w_j, i = 1, 2, j = s, u.$$

And the prices of products must be equal to the costs of production:

$$p_i = w_s l_{s,i} + w_u l_{u,i}, i = 1, 2.$$

Assume that all consumers have the same utility function of Constant Elasticity of Substitution (CES) form:

$$u(x_1, x_2) = \left\{(\tau_1 x_1)^{\frac{\mu-1}{\mu}} + (\tau_2 x_2)^{\frac{\mu-1}{\mu}}\right\}^{\frac{\mu}{\mu-1}},$$

where $\tau_i, i = 1, 2$, is the parameter discribing how a consumer likes good I and μ_u is the elasticity of substitution with respect to consumer demand. For notational purposes, we omit subscript u here. Each consumer maximizes their utility, subject to their budget constraints:

$$\max_{x_1, x_2} \; u(x_1, x_2), \qquad s.t. \; p_1 x_1 + p_2 x_2 = B_i, i \in C,$$

where B_i is a budget of a consumer and C is set of all consumers. Since the utility functions are the CES form, the following equation holds for all consumers:

$$\left(\frac{\tau_1 x_1}{\tau_2 x_2}\right)^{\mu} = \frac{p_1}{p_2}.$$

Taking into consideration $\frac{x_1}{x_2} = \frac{y_1}{y_2}$, we get

$$(\tau\, y)^{\mu} = p, \tag{1}$$

where $\tau = \frac{\tau_1}{\tau_2}$ and $y = \frac{y_1}{y_2}$.

Assume that the implicit endowment function is written as

$$M = m(L_s, L_u). \tag{2}$$

The implicit endowment function means that L_s and L_u can take on values which satisfy the implicit endowment equation. Arranging (2), we get

$$L_s = g(L_u), \tag{3}$$

where *g(.)* is the explicit endowment function. The explicit endowment function means that, when the economy has L_u of unskilled labor, the economy can have L_s of skilled labor in maximum. Since labor is divided into skilled and unskilled labor based on a relative wage, the explicit endowment function satisfies the following equations:

$$\frac{dg}{dL_u} < 0, \frac{d^2g}{dL_u^2} < 0, \quad s.t.\ L_s \geq 0, L_u \geq 0.$$

The derivative of (3) must be equal to the ratio of the wage of unskilled labor to the wage of skilled labor; otherwise, the equilibrium is not attained:

$$\frac{dg(L_u)}{dL_u} = \frac{w_u}{w_s} \Longrightarrow g' = \frac{1}{w},$$

where $g' = \frac{dg(L_u)}{dL_u}$ and w is the ratio of the factor price of skilled labor to the factor price of unskilled labor, $\frac{w_s}{w_u}$. When the ratio of factor prices, w, and the amount of labor, L_s and L_u, change by dw, dL_s, dL_u, the following equation must hold true:

$$dL_u = -\frac{g'dw}{g''w}, \tag{4}$$

where $g'' = \frac{d^2g(L_u)}{dL_u^2}$.

2. 3. 2 Relationship between Product Prices and Factor Prices

We turn our attention to relationships among the product prices, the factor prices, the outputs, and others.[2] Let

$$c_i(w_s, w_u) = min\{w_s l_{s,i} + w_u l_{u,i}\ |F_i(l_{s,i}, l_{u,i}) = 1\}, i = 1, 2$$

be the unit-cost functions which is the minimum costs produce a unit of good *i*. The zero profit conditions are

$$p_i = c_i(w_s, w_u), i = 1, 2.$$

Totally differentiating the above, we get

$$dp_i = a_{i,s}dw_s + a_{i,u}dw_u$$
$$\Longrightarrow \frac{dp_i}{p_i} = \frac{w_s a_{i,s}}{c_i}\frac{dw_s}{w_s} + \frac{w_u a_{i,u}}{c_i}\frac{dw_u}{w_u}, i = 1, 2, \tag{5}$$

where $a_{I,j}, i = 1, 2, j = s, u,$ are the optimal amount of skilled labor or unskilled labor given $\mathrm{w_s}$ and $\mathrm{w_u}$.[3] Rearranging (5), we get

$$\hat{p}_i = \theta_{i,s}\widehat{w}_s + \theta_{i,u}\widehat{w}_u, i = 1, 2,$$

where $\hat{p}_i = \frac{dp_i}{p_i}$ and $\widehat{w}_j = \frac{dw_j}{w_j}, j = s, u$, and $\theta_{i,j}$ are the cost share of *j* labor in the *i* industry. Thus, $\theta_{i,j}$ satisfies $\theta_{i,s} + \theta_{i,u} = 1$.

These equations are rewritten in matrix form and solved as

$$\begin{pmatrix}\hat{p}_1\\ \hat{p}_2\end{pmatrix} = \begin{pmatrix}\theta_{1,s} & \theta_{1,u}\\ \theta_{2,s} & \theta_{2,u}\end{pmatrix}\begin{pmatrix}\widehat{w}_s\\ \widehat{w}_u\end{pmatrix}$$
$$\Rightarrow \begin{pmatrix}\widehat{w}_s\\ \widehat{w}_u\end{pmatrix} = \frac{1}{|\theta|}\begin{pmatrix}\theta_{2,u} & -\theta_{1,u}\\ -\theta_{2,s} & \theta_{1,s}\end{pmatrix}\begin{pmatrix}\hat{p}_1\\ \hat{p}_2\end{pmatrix}, \quad (6)$$

where $|\theta|$ denotes the determinant of the matrix on the upper equations and can be expressed as

$$|\theta| = \theta_{1,s}\theta_{2,u} - \theta_{1,u}\theta_{2,s} = \theta_{1,s} - \theta_{2,s} = \theta_{2,u} - \theta_{1,u}.$$

(6) means that changes in the product prices have a magnified effect on the factor prices. Assuming that $\hat{p}_1 - \hat{p}_2$, we get

$$\widehat{w}_s > \hat{p}_1 > \hat{p}_2 > \widehat{w}_u.$$

Jones (1965) called this equation the magnification effect.

2. 3. 3 Relationship between Amounts of Factors and Outputs

The full employment conditions for the economy are written as

$$L_s = a_{1,s}y_1 + a_{2,s}y_2$$
$$L_u = a_{1,u}y_1 + a_{2,u}y_2,$$

where $y_i, i = 1, 2$ are the outputs in *i* good. Totally differentiating the above, we get

$$dL_j = a_{1,j}dy_1 + a_{2,j}dy_2$$
$$\Rightarrow \frac{dL_j}{L_j} = \frac{y_1 a_{1,j}}{L_j}\frac{dy_1}{y_1} + \frac{y_2 a_{2,j}}{L_j}\frac{dy_2}{y_2}$$
$$\Rightarrow \hat{L}_j = \lambda_{1,j}\hat{y}_1 + \lambda_{2,j}\hat{y}_2, j = s, u,$$

where $\lambda_{i,j}$ is the fraction of the skilled or unskilled labor employed in industry *i*.[4] These equations are rewritten in matrix form and solved as

$$\begin{pmatrix}\hat{L}_s\\ \hat{L}_u\end{pmatrix} = \begin{pmatrix}\lambda_{1,s} & \lambda_{2,s}\\ \lambda_{1,u} & \lambda_{2,u}\end{pmatrix}\begin{pmatrix}\hat{y}_1\\ \hat{y}_2\end{pmatrix}$$
$$\begin{pmatrix}\hat{y}_1\\ \hat{y}_2\end{pmatrix} = \frac{1}{|\lambda|}\begin{pmatrix}\lambda_{2,u} & -\lambda_{2,s}\\ -\lambda_{1,u} & \lambda_{1,s}\end{pmatrix}\begin{pmatrix}\hat{L}_s\\ \hat{L}_u\end{pmatrix}, \quad (7)$$

where |λ| is the determinant of the matrics on the upper equations and can be expressed as

$$|\lambda| = \lambda_{1,s}\lambda_{2,u} - \lambda_{2,s}\lambda_{1,u} = \lambda_{1,s} - \lambda_{1,u} = \lambda_{2,u} - \lambda_{2,s}.$$

2. 3. 4 Substitution between Factors

Let w be the ratio of the factor price of skilled labor to the factor price of unskilled labor, $\frac{w_s}{w_u}$, and $\hat{w}$ is $\frac{dw}{w}$. Thus, $\hat{w}$ means $\hat{w}_s - \hat{w}_u$. From (4), the transformation equations for the factors, skilled labor and unskilled labor, are written as

$$\hat{L}_s = b\eta(\hat{w}_s - \hat{w}_u)$$
$$\hat{L}_u = \eta(-\hat{w}_s + \hat{w}_u), \quad (8)$$

where b is $\frac{w_u L_u}{w_s L_s}$ and η is $\frac{g'}{g'' L_u}$ and a parameter that measures how much a change in the factor prices affects the endowments of labor, both skilled and unskilled. In (8), when $\hat{w}_s$ and $\hat{w}_u$ change by the same percentages, $\hat{L}_s$ and $\hat{L}_u$ must not change. Why b is $\frac{w_u L_u}{w_s L_s}$ is because the values of increased labor force and decreased labor force have to be the same. This implies that b is $\frac{w_u L_u}{w_s L_s}$.

2. 3. 5 Relationship between Outputs and Factor Ratios

Let a_1 be the ratio of $a_{1,s}$ to $a_{1,u}$, $\frac{a_{1,s}}{a_{1,u}}$, and a_2 be the ratio of $a_{2,s}$ to $a_{2,u}$, $\frac{a_{2,s}}{a_{2,u}}$. Let L denote the ratio of the endowment of skilled labor to the endowment of unskilled labor, $\frac{L_s}{L_u}$. Assuming $a_2 < L < a_1$, the fraction of the skilled and unskilled labor employed in industry i, $\lambda_{i,j}, i = 1, 2, j = s, u$, is rewritten as

$$\lambda_{1,s} = \frac{a_1(L - a_2)}{L(a_1 - a_2)}, \lambda_{1,u} = \frac{L - a_2}{a_1 - a_2}$$
$$\lambda_{2,s} = \frac{a_2(a_1 - L)}{L(a_1 - a_2)}, \lambda_{2,u} = \frac{a_1 - L}{a_1 - a_2}.$$

Therefore, the amount of the outputs, y_1 and y_2, are rewritten as

$$y_1 = \frac{L - a_2}{a_1 - a_2}\{a_1 b_{1,s} + b_{1,u}\}L_u$$
$$y_2 = \frac{a_1 - L}{a_1 - a_2}\{a_2 b_{2,s} + b_{2,u}\}L_u$$

where $b_{i,j}$ is the marginal productivity of the skilled and unskilled labor measured by the product, which can be rewritten as $\frac{w_j}{p_i}$. Differentiating and arranging the above, we get

$$\hat{y}_1 = \zeta_{1,1}\hat{a}_1, \hat{y}_1 = \zeta_{1,2}\hat{a}_2, \hat{y}_2 = \zeta_{2,1}\hat{a}_1, \hat{y}_2 = \zeta_{2,2}\hat{a}_2, \quad (9)$$

Where

$$\zeta_{1,1} = \left\{\frac{-a_1}{a_1 - a_2} + \frac{a_1 w}{a_1 w + 1}\right\}, \zeta_{1,2} = \left\{\frac{a_2}{a_1 - a_2} - \frac{a_2}{L - a_2}\right\},$$
$$\zeta_{2,1} = \left\{\frac{-a_1}{a_1 - a_2} + \frac{a_1}{a_1 - L}\right\}, \zeta_{2,2} = \left\{\frac{a_2}{a_1 - a_2} + \frac{a_2 w}{a_2 w + 1}\right\},$$

and $\hat{y}_i$ and $\hat{a}_i$ are $\frac{dy_i}{y_i}$ and $\frac{da_i}{a_i}$, respectively.

Since $\left\{\frac{-a_1}{a_1-a_2} + \frac{a_1 w}{a_1 w+1}\right\} < 0$, $\left\{\frac{a_2}{a_1-a_2} - \frac{a_2}{L-a_2}\right\} < 0$, $\left\{\frac{-a_1}{a_1-a_2} + \frac{a_1}{a_1-L}\right\} > 0$, and $\left\{\frac{-a_1}{a_1-a_2} + \frac{a_1}{a_1-L}\right\} > 0$

hold true, a change to more skilled labor intensive production decreases the output using skilled labor intensively and increases the output using unskilled labor intensively.

2. 3. 6 Substitution in Production and Consumer Demand

The relationships between the factor prices, w_1 and w_2, and the ratios between the optimal amounts of skilled or unskilled labor, $a_{i,j}$, is written as

$$\hat{a}_1 = -\mu_1\hat{w}, \hat{a}_2 = -\mu_2\hat{w}, \quad (10)$$

where $\mu_i, i = 1, 2$ are the elasticity of substitution with respect to production. Totally differentiating (1), the relationship between the product prices, p_1 and p_2, and the outputs, y_1 and y_2, is written as

$$\hat{y} = -\mu_u\hat{p} \quad (11)$$

in cases in which a change in consumer taste does not happen, and

$$\hat{y} + \hat{\tau} = -\mu_u\hat{p} \quad (12)$$

in cases in which a change in consuer taste happens, where $\hat{y}$,$\hat{\tau}$, and $\hat{p}$ are $\frac{dy}{y}$, $\frac{d\tau}{\tau}$, and $\frac{dp}{p}$, respectively, and μ_u is the elasticity of substitution with respect to consumer demand.

2. 4 Comparative Statics for a Small Open Economy

We turn our attention here to the comparative statics. In this subsection, we examine the comparative statics for a small open economy. We analyze the comparative statics for a whole economy in the next subsection.

In this subsection, we consider a small open economy that is too small to affect the international prices of goods and thus behaves as a price taker. We do not assume that the small open economy and the other countries have the same technologies. Thus, the small open economy maximizes the total value of the products they produce under the conditions under which the product prices are given.

2. 4. 1 Effect of Product Price Changes

Proposition 2. 5.

Assume that endowments are given in a functional form. For a small open economy, when the relative price of a product increases, the output of the product increases more than when endowments are given by fixed amounts.

When the product prices, p_1 and p_2, change by $\hat{p}_i, i = 1, 2$ percentage, from (6), (7), (8), (9), and (10), the effects are written as

$$\widehat{w}_s = \frac{\theta_{2,u}}{|\theta|}\hat{p}_1, \widehat{w}_u = \frac{-\theta_{2,s}}{|\theta|}\hat{p}_1, \hat{L}_s = \frac{b\eta}{|\theta|}\hat{p}_1, \hat{L}_u = \frac{-\eta}{|\theta|}\hat{p}_1,$$
$$\hat{a}_1 = \frac{-\mu_1}{|\theta|}\hat{p}_1, \hat{a}_2 = \frac{-\mu_2}{|\theta|}\hat{p}_1,$$
$$\hat{y}_1 = \left\{-\zeta_{1,1}\mu_1 - \zeta_{1,2}\mu_2 + \frac{\lambda_{2,u}b\eta + \lambda_{2,s}\eta}{|\lambda|}\right\}\frac{\hat{p}_1}{|\theta|},$$
$$\hat{y}_2 = \left\{-\zeta_{2,1}\mu_1 - \zeta_{2,2}\mu_2 - \frac{\lambda_{1,u}b\eta + \lambda_{1,s}\eta}{|\lambda|}\right\}\frac{\hat{p}_1}{|\theta|}$$

in a $\hat{p}_1$ change case, and

$$\widehat{w}_s = \frac{-\theta_{1,u}}{|\theta|}\hat{p}_2, \widehat{w}_u = \frac{\theta_{1,s}}{|\theta|}\hat{p}_2, \hat{L}_s = \frac{-b\eta}{|\theta|}\hat{p}_2, \hat{L}_u = \frac{\eta}{|\theta|}\hat{p}_1,$$
$$\hat{a}_1 = \frac{\mu_1}{|\theta|}\hat{p}_2, \hat{a}_2 = \frac{\mu_2}{|\theta|}\hat{p}_2,$$
$$\hat{y}_1 = \left\{\zeta_{1,1}\mu_1 + \zeta_{1,2}\mu_2 - \frac{\lambda_{2,u}b\eta + \lambda_{2,s}\eta}{|\lambda|}\right\}\frac{\hat{p}_2}{|\theta|},$$
$$\hat{y}_2 = \left\{\zeta_{2,1}\mu_1 + \zeta_{2,2}\mu_2 + \frac{\lambda_{1,u}b\eta + \lambda_{1,s}\eta}{|\lambda|}\right\}\frac{\hat{p}_2}{|\theta|}$$

in a $\hat{p}_2$ change case. This result is similar to the Stolper-Samuelson Theorem in Stolper and Samuelson (1941). The differences are that, in this paper's model, the changes in the factor prices, caused by product price changes, make the factor endowments change. This is because, when the factor prices change, it might be beneficial for some skilled or unskilled workers to change from one job to the other. As a result, the product price changes have a greater effect on the outputs. Thus, Proposition 2. 5 occurs.

2. 4. 2 Effect of Endowment Function Changes

Assume that the factor endowment function changes, and, as a result, the amounts of the factors change by $\hat{L}_j, j = s, u$ percentage. From (7), the effects are written as

$$\hat{y}_1 = \frac{\lambda_{2,u}}{|\lambda|}\hat{L}_s, \hat{y}_2 = \frac{-\lambda_{1,u}}{|\lambda|}\hat{L}_s$$

in the $\hat{L}_s$ change case, and

$$\hat{y}_1 = \frac{-\lambda_{2,s}}{|\lambda|}\hat{L}_u, \hat{y}_2 = \frac{\lambda_{1,s}}{|\lambda|}\hat{L}_u$$

in the $\hat{L}_u$ change case. This result is analougous with the Rybczynski Theorem in Rybczynski (1955). However, in this paper's model, when a change in endowments happens, both skilled labor and unskilled labor might change. For example, we consider cases like that of Proposition 2. 1. If the change is that the later endowment function is completely skilled labor abundant relative to the initial one, and the endowment of skilled labor increases, the endowment of unskilled labor decreases.

2. 4. 3 Effect of Technological Changes

We turn our attention here to technological changes.
Assume that the technologies of industry i change, and the factor prices and the optimal amount of skilled or unskilled labor change by $\hat{w}_j$ and $\hat{a}_{1,j}, j = s, u$. Since the unit cost line of industry i is a downward slopping curve, if $\hat{w}_s > 0$ holds true, then $\hat{w}_u < 0$; conversely, if $\hat{w}_u > 0$ holds true, then $\hat{w}_s < 0$. From (7), (8), (9), and (10), the subsequent effects that the technological changes make are written as

$$\hat{a}_j = -\mu_j \hat{w}, \hat{L}_s = b\eta\hat{w}, \hat{L}_u = -\eta\hat{w},$$
$$\hat{y}_1 = \zeta_{1,i}\hat{a}_i - \zeta_{1,j}\mu_j\hat{w} + \frac{\lambda_{2,u}b\eta + \lambda_{2,s}\eta}{|\lambda|}\hat{w},$$
$$\hat{y}_2 = \zeta_{2,i}\hat{a}_i - \zeta_{2,j}\mu_j\hat{w} - \frac{\lambda_{1,u}b\eta + \lambda_{1,s}\eta}{|\lambda|}\hat{w}, (i,j) = (1,2),(2,1).$$

The increase in w causes the output of product 1 to increase through increasing skilled labor and changing the factor mix in another industry to use less skilled labor. The decrease of a_1 directly increases the output of product 1 and decreases the output of product 2. Assuming that w increases and a_1 decreases or w decreases and a_1 increases, the changes expands industry 1 in functional endowments cases more than in fixed factor endowments cases.

2. 5 Comparative Statics for a Whole Economy

In what follows, we consider the comparative statics for a whole economy. In the previous subsection, the product prices are supposed to be fixed or given externally. In this subsection, the equilibrium determines the product prices. For simplicity, the economy is supposed to have the same technologies across countries.

Proposition 2. 6.

Assume that endowments are given in a functional form. For a whole economy, when a change in consumer's taste or production technologies happens, the change in the outputs is less than in cases in which endowments are given by fixed amounts.

In this subsection, we verify Proposition 2. 6.

2. 5. 1 Effect of Demand Change

We consider a change of consumer demand or taste. When the consumers' taste changes, the outputs or the product prices have to change to compensate for the change in consumer taste. Writing again (12), we get

$$\hat{y} + \hat{\tau} = -\mu_u \hat{p}.$$

If the consumers' taste changes, to satisfy the equation, the relative output, y, and the relative product price, p, change accordingly. Solving (12) by using (6), (7), (8), (9), (10), and (11), the changes in the relative product price and the relative output are written as[5]

$$\hat{p} = \frac{-1}{\phi + \mu_u}\tau, \hat{y} = \phi \hat{p} \quad (13)$$

$$\phi = \frac{1}{|\theta|}\left\{\left(-\frac{a_1 w}{a_1 w + 1} + \frac{a_1}{a_1 - L}\right)\mu_1 + \left(\frac{a_2}{L - a_2} + \frac{a_2 w}{a_2 w + 1}\right)\mu_2 + \frac{b\eta + \eta}{|\lambda|}\right\}.$$

The other effects are written as

$$\hat{w} = \frac{1}{|\theta|}\hat{p}, \hat{L}_s = \frac{b\eta}{|\theta|}\hat{p}, \hat{L}_u = \frac{-\eta}{|\theta|}\hat{p},$$
$$\hat{a}_1 = \frac{-\mu_1}{|\theta|}\hat{p}, \hat{a}_2 = \frac{-\mu_2}{|\theta|}\hat{p},$$
$$\hat{y}_1 = \left\{-\zeta_{1,1}\mu_1 - \zeta_{1,2}\mu_2 + \frac{\lambda_{2,u} b\eta + \lambda_{2,s}\eta}{|\lambda|}\right\}\frac{\hat{p}}{|\theta|},$$
$$\hat{y}_2 = \left\{-\zeta_{2,1}\mu_1 - \zeta_{2,2}\mu_2 - \frac{\lambda_{1,u} b\eta + \lambda_{1,s}\eta}{|\lambda|}\right\}\frac{\hat{p}}{|\theta|}.$$

In (13), $\left(\frac{a_2}{L - a_2} + \frac{a_2 w}{a_2 w + 1}\right)$ is clearly more than one, and $\left(-\frac{a_1 w}{a_1 w + 1} + \frac{a_1}{a_1 - L}\right)$ is more than one, too, because

$$-\frac{a_1 w}{a_1 w+1}+\frac{a_1}{a_1-L}=\left(\frac{\frac{1}{a_1 w}}{\frac{1}{a_1 w}+1}-1\right)+\left(1+\frac{\frac{1}{a_1}}{\frac{1}{L}-\frac{1}{a_1}}\right)>1.$$

Although changes in the product prices have a magnified effect on the factor prices in the Hechscher-Ohline type models including this paper's model, the outputs also change greatly when the product prices change, and the changes in the factor prices change the ratio between the optimal amounts of skilled and unskilled labor. The changes in the factor prices have a greater effect on the outputs in the Heckscher-Ohlin type models than in one product models because $\left(\frac{a_2}{L-a_2}+\frac{a_2 w}{a_2 w+1}\right)$ and $\left(-\frac{a_1 w}{a_1 w+1}+\frac{a_1}{a_1-L}\right)$ are more than one, too. As a result, smaller changes in the product prices are able to offset the change in consumer taste.

This analysis also applies to cases in which the productivity of one industry increases because a decrease in the product price, caused by the increase in productivity, has a similar effect to that of a consumer taste change.

2. 5. 2 Effect of Endowment Function Changes

Next, we consider changes in the endowment function. When the endowment function changes, assuming that the relative factor price is constant, the amounts of skilled and unskilled labor at the equilibrium factor prices change. Let $\hat{L}_j^I$ be the initial effect of a change in the endowment function on the amounts of skilled and unskilled labor. Then, the economy has to satisfy the equations,

$$\hat{y}+\mu_u\hat{p}=\frac{\hat{L}_s^I}{|\lambda|}, \hat{y}+\mu_u\hat{p}=-\frac{\hat{L}_u^I}{|\lambda|}. \tag{14}$$

From (6), (7), (8), (9), (10), (11), (13), and (14), the total effects on the outputs and the product prices are written as

$$\hat{y}=\frac{\mu_u}{\mu_u+\phi}\frac{\hat{L}_s^I}{|\lambda|}, \hat{p}=\frac{-1}{\mu_u+\phi}\frac{\hat{L}_s^I}{|\lambda|}$$

in the $\hat{L}_s^I$ change case, and

$$\hat{y}=\frac{-\mu_u}{\mu_u+\phi}\frac{\hat{L}_u^I}{|\lambda|}, \hat{p}=\frac{1}{\mu_u+\phi}\frac{\hat{L}_s^I}{|\lambda|}$$

in the $\hat{L}_u^I$ change case. The amounts of skilled and unskilled labor finally change to

$$\hat{L}_s=\frac{A_1|\lambda|\mu_1+A_2|\lambda|\mu_2+\eta}{A_1|\lambda|\mu_1+A_2|\lambda|\mu_2+b\eta+\eta}\ \hat{L}_s^I, \tag{15}$$
$$\hat{L}_u=\frac{\eta}{A_1|\lambda|\mu_1+A_2|\lambda|\mu_2+b\eta+\eta}\ \hat{L}_s^I,$$

$$A_1 = -\frac{(a_1 w)}{a_1 w + 1} + \frac{a_1}{a_1 - L}, A_2 = \frac{a_2}{L - a_2} + \frac{a_2 w}{a_2 w + 1}$$

in the $\hat{L}_s^I$ change case, and

$$\hat{L}_s = \frac{b\eta}{A_1|\lambda|\mu_1 + A_2|\lambda|\mu_2 + b\eta + \eta}\hat{L}_u^I,$$
$$\hat{L}_u = \frac{A_1|\lambda|\mu_1 + A_2|\lambda|\mu_2 + b\eta}{A_1|\lambda|\mu_1 + A_2|\lambda|\mu_2 + b\eta + \eta}\hat{L}_u^I \quad (16)$$

in the $\hat{L}_u^I$ change case. The other effects are written as

$$\hat{w} = \frac{1}{|\theta|}\hat{p}, \hat{a}_1 = \frac{-\mu_1}{|\theta|}\hat{p}_1, \hat{a}_2 = \frac{-\mu_2}{|\theta|}\hat{p},$$
$$\hat{y}_1 = \left\{-\zeta_{1,1}\mu_1 - \zeta_{1,2}\mu_2 + \frac{(\lambda_{2,u}b\eta + \lambda_{2,s}\eta)}{|\lambda|}\right\}\frac{\hat{p}}{|\theta|},$$
$$\hat{y}_2 = \left\{-\zeta_{2,1}\mu_1 - \zeta_{2,2}\mu_2 - \frac{\lambda_{1,u}b\eta + \lambda_{1,s}\eta}{|\lambda|}\right\}\frac{\hat{p}}{|\theta|}.$$

In (15) and (16), the final effects are less than the initial effects because an increase in the skilled labor increases the output using skilled labor intensively, which reduces the price of the skilled labor intensive goods and the factor price of the skilled labor, and this leads to a decrease in the skilled labor.

2. 5. 3 Effect of Technological Changes

We turn our attention here to technological changes. Assume that a technological change happens and that the initial effects of the change are $\hat{w}^I$ and $\hat{a}_i^I$. Then the economy has to satisfy the equations:

$$\hat{y} + \mu_u\hat{p} = \psi, \quad (17)$$
$$\psi = -A_i\hat{a}_i^I + A_j\mu_j\hat{w}^I + \frac{b\eta + \eta}{|\lambda|}\hat{w}^I, (i,j) = (1,2), (2,1).$$

From (6), (7), (8), (9), (10), (11), (13), and (17), the total effects on the outputs and the product prices are written as

$$\hat{y} = \frac{\mu_u}{\mu_u + \phi}\psi, \hat{p} = \frac{-1}{\mu_u + \phi}\psi.$$

The relative factor price, the ratios between the optimal amounts of skilled and unskilled labor, and the amounts of skilled and unskilled labor are written as

$$\hat{w} = \frac{A_i|\lambda|\mu_i\hat{w}^I - A_i|\lambda|\hat{a}_i^I}{A_1|\lambda|\mu_1 + A_2|\lambda|\mu_2 + b\eta + \eta},$$

$$\hat{a}_i = \frac{(A_j|\lambda|\mu_j + b\eta + \eta)\hat{a}_i^I + (A_j|\lambda|\mu_j + b\eta + \eta)\mu_i \hat{w}^I}{A_1|\lambda|\mu_1 + A_2|\lambda|\mu_2 + b\eta + \eta},$$
$$\hat{a}_j = \frac{-A_i|\lambda|\mu_j \hat{a}_i^I - A_i|\lambda|\mu_i \mu_j \hat{w}^I}{A_1|\lambda|\mu_1 + A_2|\lambda|\mu_2 + b\eta + \eta},$$
$$\hat{L}_s = \frac{b\eta A_i|\lambda|\mu_i \hat{w}^I + b\eta A_i \hat{a}_i^I}{A_1|\lambda|\mu_1 + A_2|\lambda|\mu_2 + b\eta + \eta},$$
$$\hat{L}_u = \frac{-\eta A_i|\lambda|\mu_i \hat{w}^I - \eta A_i \hat{a}_i^I}{A_1|\lambda|\mu_1 + A_2|\lambda|\mu_2 + b\eta + \eta}.$$

The final effects of the initial effect part on the relative factor price, the ratios between the optimal amounts of skilled and unskilled labor, and the amounts of skilled and unskilled labor are less than the initial effects, respectively.

2. 6 Specialization

In the following, we consider specialization in this paper's models. Assume that there are two countries, two factors of production, skilled labor and unskilled labor, and two goods, skilled labor intensive goods and unskilled labor intensive goods.[6] In the Heckscher-Ohlin model, when the difference in the relative endowment between two countries is sufficiently large, each country specializes in what each country is suited for. The skilled labor abundant country mainly specializes in goods using skilled labor intensively, while the unskilled labor abundant country mainly specializes in goods using unskilled labor intensively.

Suppose that endowments are given in a functional form, that one of two countries is completely skilled labor abundant, and that the difference in the relative endowment between the two countries is sufficiently large. As in the Heckscher-Ohlin model, one of following three cases occurs. First, the completely skilled labor abundant country specializes in goods using skilled labor intensively, and the other in goods using unskilled labor intensively. Second, the completely skilled labor abundant country produces both goods using skilled labor intensively and goods using unskilled labor intensively, and the other specializes in goods using unskilled labor intensively. Third, the completely skilled labor abundant country specializes in goods using skilled labor intensively, and the other produces both goods using unskilled labor intensively and goods using skilled labor intensively. Because, as in the Heckscher-Ohlin model, considering the Production Possibility Frontier (PPF) of the countries, given the relative product price, the relative output of skilled labor intensive goods is larger in the skilled labor abundant country than in the unskilled labor abundant country. As explained before, a completely skilled (unskilled) labor abundant country means that the country is skilled labor abundant at any relative factor price.

When the specialization of one or both countries occurs, the relative factor prices differ between countries. The relative price of skilled labor in the completely skilled labor abundant country is lower than in the completely unskilled labor abundant country. The relative price of unskilled labor in the completely skilled labor abundant country is higher than in the completely unskilled labor abundant country. This happens intensively because the technologies in each country change to technologies under which production uses the respective abundant factor more to use endowments fully. This leads to changes in the relative factor prices.

When the relative factor prices change, factor endowments also change. The lower relative factor price of skilled labor in the completely skilled labor abundant country relative to the completely unskilled abundant country means that, in the completely skilled labor abundant country, more competent workers are used as a unskilled worker. In other words, to assign more competent workers as unskilled labor, the relative price of unskilled labor has to increase. This explains why the next proposition occurs.

Proposition 2. 7

When specialization occurs, the relative price of skilled (unskilled) labor in the completely skilled (unskilled) labor abundant country is lower than in the other. In the completely skilled labor abundant country, more competent workers are used as unskilled labor relative to the completely unskilled labor abundant country.

Thus, the income differentials are larger in the completely unskilled labor abundant country than in the completely skilled labor abundant country.

In addition, when a completely unskilled abundant country is catching up with a completely skilled labor abundant country in education -- in other words, the amount of skilled labor increases relative to the amount of unskilled labor --the relative price of skilled labor in the completely skilled labor abundant country increases, and the relative price of skilled labor in the completely unskilled labor abundant country drops. Thus, catching up in education in developing countries widens the income differentials of developed countries.

2. 7 Summary and Effects of Functional Form Endowments

In this subsection, we summarize and analyze what we found. In the comparative statics for a small economy, we found results similar to the Stolper-Samuelson Theorem and the Rybcynski Theorem. The differences are that, when one of the factor prices increases, the endowment whose factor price increases also increases. This means that an increase in the product price using skilled labor intensively increases the endowment in skilled labor by increasing the factor price of skilled labor and decreasing the factor price of unskilled labor. As a result, the increase in the product price using skilled labor intensively increases the output using skilled labor intensively more than in the Heckscher-Ohlin models.

Generally, functional form endowments increase the extent of changes in the outputs when the relative product price or the relative factor price move by a certain percentage. Two substitutional effects, one of which is between the products and the other between the factors, also increase the extent of changes in the outputs when the relative product price or the relative factor price moves by a certain percentage. Of course, changes in product prices have a magnified effect on the factor prices. As a result, a change in the product prices has a large effect on the outputs, and this effect in this paper's model is larger than in the basic Heckscher-Ohlin model.

The magnified effects of the functional form endowments, the two substitutional effect and the effect of changes in the product prices on the factor prices means that, in the comparative statics for a small open economy, a change in the product prices or the production technologies has a large effect on the outputs. Therefore, a change in the product prices or production technologies causes greater specialization.

Conversely, in the comparative statics for a whole economy, a change in product prices has a large effect on the outputs, which means that a change in consumer taste, the endowment function, or the production technologies has a smaller effect on the product prices because, if the magnified effects have a larger effect on the outputs, even a small change in the product prices facilitates the achievement of the equilibrium conditions. Thus, in the comparative statics for a whole economy, the magnified effects do not increase the extent of the changes in the product prices but decrease the extent of the changes in the product prices.

As for functional form endowments, as explained before, this increases the extent of change in the outputs for a small open economy and decreases the extent of change in the product prices for a whole economy. For a small and whole economy, the functional form endowments increase the extent of specialization in production in the economy. As η increases, which means that a smaller change in the factor prices has a larger effect on the endowments, the extent of change in the outputs for a small open economy increases given a change in the product prices and the production technologies. For a whole economy, as η increases, the extent of change in the product prices decreases given a change in consumer taste, the production technologies, and the endowments. As η increases, more and more effects caused by a given change in consumer taste and so on are absorbed into a change in the outputs. As a result, as η approaches infinity, all effects except the initial effects will disappear for a whole economy. This means that, in cases that have large η, the features of the Heckscher-Ohlin model do not hold much. Thus, when analyzing the economy, one must take into consideration how much the endowments change given a change in the factor prices.

In an extreme case, when the elasticity of substitution between the factors of production is infinite, the relative factor price is constant. Assuming that the infinite elasticity of substitution between the factors and that each production needs only each one of the two factors, such as $a_{1,s} > 0, a_{1,u} = 0$, $a_{2,s} = 0$, and $a_{2,u} > 0$, this is similar to the Richarian model.[7] Thus, by controlling the parameters, this paper's model can derive many different solutions and analyses.

3. Implications

Invention, technological change, innovation, and other factors change the structure of production, how to produce goods, which goods are produced, and what type of factors are used. These changes affect how each factor is utilized in an industry, how much the factors are compensated, how to divide the whole production into labor, capital, land, and so on, and then the distribution between members of the economy. However, every invention, technological change, innovation and so on is not the same. One technological change might alter what workers do dramatically but not change the distribution of earnings. Conversely, another change might has a little effect on the economy but create workers who earns a lot of money.

One of the reasons is that, as this paper has explained, there are differences in the extent to which one factor is substituted for another. When the elasticity of substitute in a factor is large, a small change in the relative factor price compensates for a given technological change. Conversely, when the elasticity of substitute in a factor is very small, even a small increase in a factor supply leads to a very large increase in the factor price. For instance, since the early nineteenth century, we have experienced a lot of changes in the structure of the economy. New technologies and innovations have created new products, and the shares of each good produced in the economy have been changing. The share of agricultural products in the economy has been decreasing, the share of industrial products had been increased until the mid-twentieth century, and the share of education has been increasing steadily. Within agricultural and industrial products, what is produced has been changing. Furthermore, what kind of job workers do has been changing, too.

However, every change does not have an effect on relative wages in the same way. This is because what jobs require from workers is similar between jobs in some cases but not in others. When jobs are similar, workers are interchangeable between jobs, but when jobs are not similar, new types of workers are needed. As a result, in some cases, the emergence of an industry created high earners. For example, in the sports industry, Floyd Mayweather earned 85 million dollars, Manny Pacquiao earned 62 million dollars, and Tiger Woods earned 59.4 million dollars (Forbes 2012). Even though the size of the sports industry is not very large, the sports industry creates millionaires.

Now, we turn our attention to a more specific topic. Income inequity within developed countries has increased since the early 1980s. From the mid-1980s to the late-2000s, inequality rose in 15 out of 19 countries (OECD 2013). Among the causes of this phenomenon, skill biased technological change, outsourcing, and computer technologies have been discussed much and relate to the Heckscher-Ohlin model. Feenstra and Hanson (1999) found that computers explained about 35 percent of the increase in the relative wage of nonproduction workers (compared to that of production workers), while outsourcing explained 15 percent.[8] Cheung and Fan (2002) and Fan and Cheung (2004) analyzed the effects of trade between Hong Kong and China on the relative wage between skilled and unskilled workers.

When analyzing income differences between workers, using the Heckscher-Ohlin model poses some problems. The Heckscher-Ohlin model assumes that the factors of production are completely distinctive and given by fixed amounts. Even though nonproduction workers might have more productive ability than production workers, workers assigned to production work could be used in nonproduction work in a less effective way. Thus, if the relative wage between nonproduction and production workers changes, the reallocation of some production workers to nonproduction work might be optimal and increase the total value of outputs. This reallocation has to offset the initial effects of skill biased technological change, outsourcing, and computer technologies to an extent. Thus, using the Heckscher-Ohlin model under the assumption that each factor is completely different has a problem.

There are three elements in the relationship between factor endowments and factor prices. First, technologies determine the structure of an economy. Thus, a technological change affects how goods are produced and how outputs are distributed between factors. Second, the elasticities of substitutes between factors determines how much each technological change affects factor prices and the extent to which substitution between factors occurs. Finally, changes in endowments, whether in a functional form or not, affect factor prices, product prices, and others. Thus, skill biased technological change, outsourcing, and computer technologies cannot entirely determine factor prices.

Particularly in Japan and Europe, there is an excess supply of educated workers. McGuinness (2006) examined over-education in developed countries and concluded that the evidence on wages is in line with assignment theory, not human capital theory nor job competition model, given the lower returns on surplus education and the over-education penalty. However, since there are overeducated workers, there should be some kind of over-supply of highly educated workers. This means that, when international competition reduces the wages of unskilled workers, indirect competition between skilled workers and unskilled but educated workers reduces the wages of skilled workers, too. Thus, competition from low wage countries does not necessarily widen inequality. As a result, the more general assumption under which skilled workers and unskilled workers are not distinctive probably produces more realistic results.

Developed countries have unemployment problems and difficulty in finding employment.[9]This might mean that the market is imperfect, and the market imperfection widens the wage differences because if market works efficiently, the existence of unemployed or overqualified workers reduces the wages of workers who could be replaced with others.

As Pritchett (2001) said, education might be like piracy. People with more education get hired as skilled workers and have higher wages. At the same time, Pritchett found that the cross-national data suggested negative externalities of education. This means that it is rational for individuals to get an advanced degree in education, but it is not for nations.
This might suggest that market imperfection causes the wages of unskilled workers to drop and the wages of skilled worker remain relatively stable, even though some of the unskilled workers have the ability to work as skilled workers. If so, the larger wage

differentials between skilled workers and unskilled workers impair efficiency and reduce output. This is only a hypothesis, but the above facts and the model in this paper suggest that this might be the case. Thus, further research is needed.

4. Conclusion

In this paper, I attempt to extend the Heckscher-Ohlin model and introduce an assignment problem between two factors in the model. In the original Heckscher-Ohlin model, endowments are given by fixed amounts. In the model proposed in this paper, endowments are given in a functional form and are, to an extent, interchangeable. This extension enables us to examine trade patterns, factor prices, and product prices more thoroughly. In section 2, I construct the model and analyze it using comparative statics. According to the model, if both countries produce both goods, the factor prices are equalized across countries, as in the Heckscher-Ohlin model. In comparative statics for a small open economy, if endowments are given in a functional form, then when the relative price of a good increases, the output of the good increases more than when endowments are given by fixed amounts. In comparative statics for a whole economy, if endowments are given in a functional form, then when a change in consumer tastes or production technologies occurs, the change in outputs is less than when endowments are given by fixed amounts. In specialization, the completely skilled labor abundant country uses more competent workers as unskilled labor, whose wages are higher than in the completely unskilled labor abundant country. These results partially explain why observers have seen many patterns in which technological changes affect the structure of the economy.

Endnotes

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1. Feenstra (2004) is a good textbook for graduate students. The textbook thoroughly explains and examines empirical and theoretical works of the Heckscher-Ohlin model.
2. We use the dual approach here. See Jones (1965).
3. From the first part to the second part, the envelope theorem is used, $\frac{\partial \mathrm{c_i}}{\partial \mathrm{w_s}} = a_{i,s} + \left(w_s \frac{\partial a_{i,s}}{\partial w_s} + w_u \frac{\partial a_{i,u}}{\partial w_s}\right) = a_{i,s}$.
4. Here, we use the envelop theorem, too.
5. In the comparative statics for a whole economy, the product prices and the factor prices are not determined uniquely because, as long as the relative prices are the equilibrium prices, the product prices can move freely.
6. Here, we restrict our discussion to cases of two goods, but the result can be applied to cases of more than two goods.
7. However, comparative statics cannot be done in this paper's method, because the Richadian model has corner solutions.
8. Here, Feenstra and Hanson use nonproduction workers as a proxy for more skilled workers, and production workers as a proxy for less skilled workers.
9. For example, Pedersen et al. (2011) in Europe and Ohta et al. (2008) in Japan.

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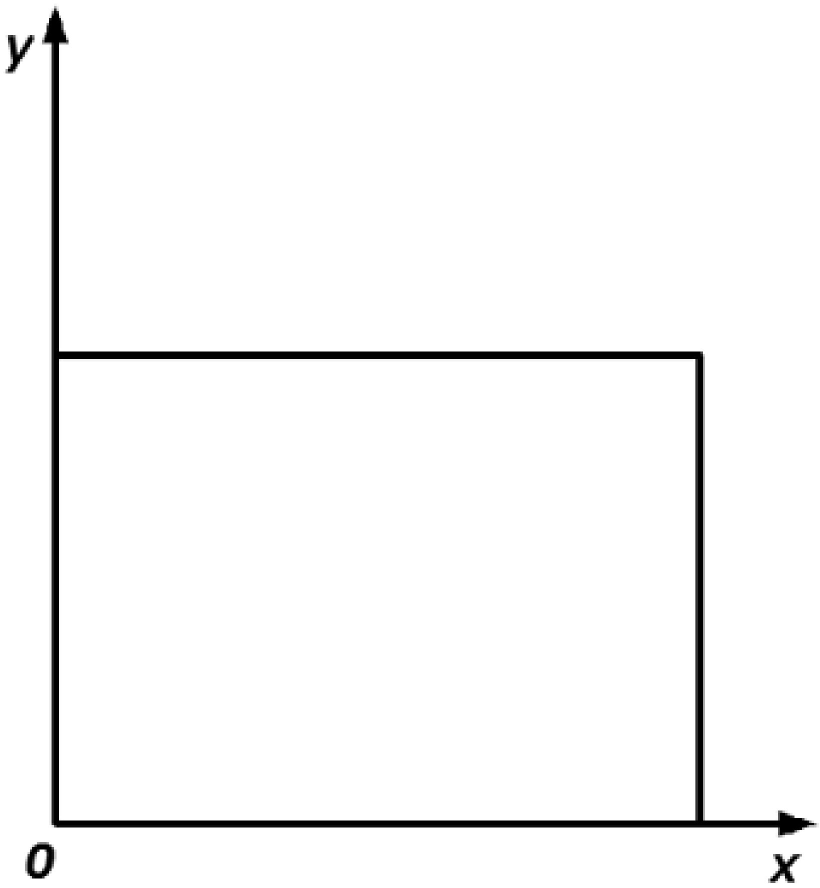

Figure 1. Heckscher-Ohlin Model

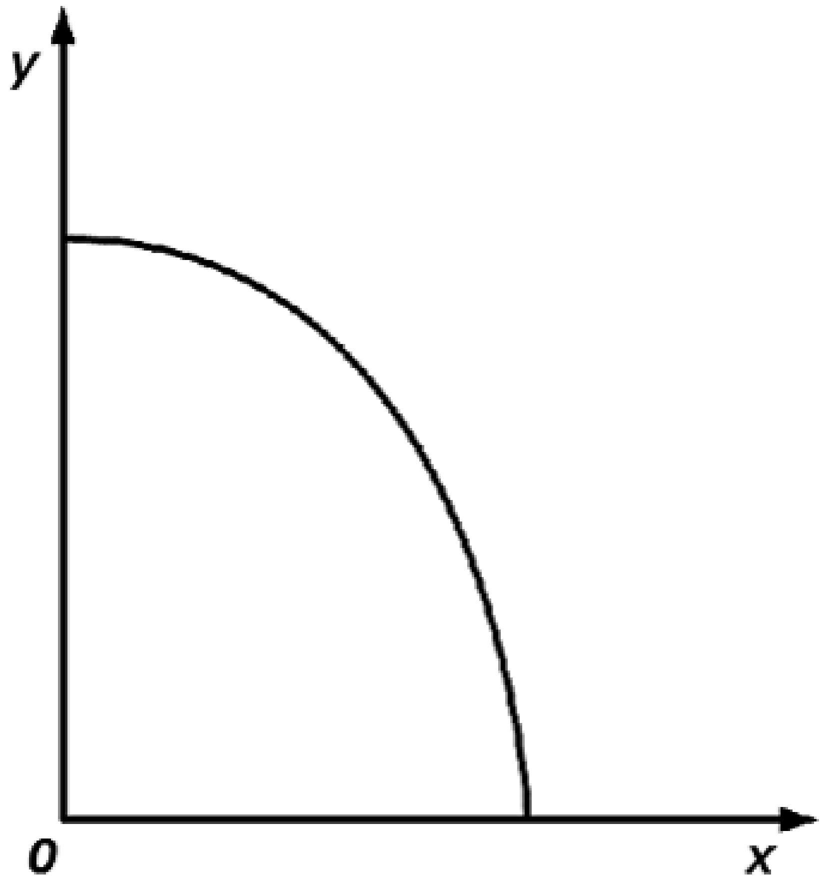

Figure 2. Generalized Case

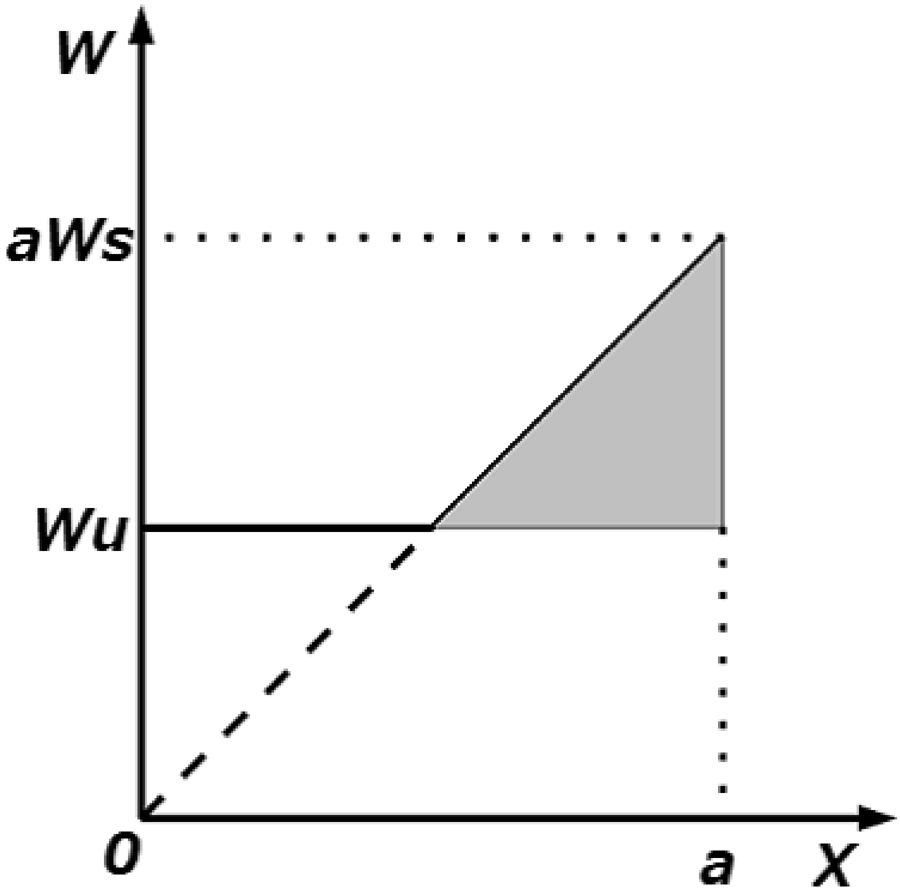

Figure 3. Worker's Wages