The S-Curve Revisited: The Terms, Volume and Balance of Trade over the Business Cycle

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Abstract This paper examines the ability of a dynamic general equilibrium model to replicate trade balance movements seen in the data. Specifically, the S-curve pattern found in the cross-correlation between the terms of trade and the trade balance is evaluated. This paper extends earlier work in this area by examining the behavior of the trade balance in greater detail, focusing on the cross-correlation between output, the terms of trade, the volume of trade, and the trade balance at different leads and lags. Models driven by productivity shocks are found to replicate most of the cross-correlation functions estimated from the data. These results support the proposition that the S-curve and J-curve patterns seen in the data are generated by the endogenous responses of both the terms of trade and the balance of trade to productivity shocks rather than by the response of the trade balance to exogenous terms of trade innovations.

Keywords: S-curve, trade balance, terms of trade

JEL Classification: F31, F32, F41

1. Introduction

The interaction between the terms of trade and the trade balance is an important topic in the field of international economics. In the popular press, trade deficits are generally described as unfavorable and are assumed to be a primary cause of domestic job loss. Politicians and other policy makers face pressure to reduce trade deficits either by increasing the barriers to trade or by changing policy variables. Allowing or encouraging the exchange rate to depreciate is a solution often employed to worsen the terms of trade in order to improve the trade balance. Because trade deficits can influence important policy decisions, a greater understanding of the causes of trade fluctuations is desirable.

Much of the analysis of the interaction between the terms and balance of trade has taken place through an examination of whether or not a J-curve pattern exists in the data. The J-curve pattern is characterized by an initial worsening and later improvement of the trade balance following a terms of trade depreciation. Following a worsening of the terms of trade, the decreased value of exports and increased value of imports tends to cause the trade balance to deteriorate. However, over time, the higher relative price of imports should cause the volume of exports to increase and the volume of imports to decrease. If the goods are sufficiently elastic, these volume movements eventually produce a net improvement of the balance of trade.
Over the years, the J-curve has been analyzed for a wide variety of countries and data sets using an assortment of econometric techniques. This area of research has met with decidedly mixed results. Examples of papers finding support for the J-curve include Marwah and Klein (1996), Bahmani-Oskooee and Als (1994) and Hacker and Hatmani (2003). Evidence of a weak or ‘delayed’ J-curve has also been found by several authors such as Moffet (1989), Rosensweig and Koch (1988), Yusoff (2007) and Bahmani-Oskooee and Bolhasani (2011). Other authors such as Rose and Yellen (1989), Rose (1991), Hsing (2009), Hsing and Savvides (1996), and Mehmet and Mushtag (2010) have not found any evidence of a J-curve in the data.

A closely related area of research, focused on in this paper, has been directed at capturing the cross correlation between the terms of trade and the trade balance at various leads and lags. Backus et al. (1994) find that the trade balance is “…negatively correlated, in general, with current and future movements in the terms of trade, but positively correlated with past movements.” They describe this cross-correlation pattern as resembling an S-curve, similar to the J-curve pattern commonly described in the literature. They also find that this S-curve pattern is replicated by a general equilibrium model driven by productivity shocks. Research into the S-Curve continues to be an active area of research. Recent papers such as Senhadji (1998), Parikh and Shibatha (2004), Bahmani-Oskooee et al (2008), and Bahmani-Oskooee and Ratha (2007, 2008, 2010), have continued the discussion. A comprehensive review of the empirical investigations into the existence of both the J-curve and the S-curve can be found in Bahmani-Oskooee and Hegerty (2010).

In this paper, I revisit the S-curve pattern and its original interpretation as the endogenous movements of both the terms and balance of trade generated by productivity shocks. The area under investigation is expanded to include an estimation of the cross-correlation functions between output, the terms of trade, the balance of trade, and the volume of imports and exports at different leads and lags. The estimated relationships are then compared to those obtained from a general equilibrium model. This allows the data to be evaluated within a theoretical framework and serves as an informal test of the model’s economic relevance.

Examination of the data from the original G-6 countries over a thirty year period results in empirical regularities similar to those reported in previous research. The trade balance is found to be negatively correlated with output. Additionally, an S-curve pattern is found in the cross-correlation function between the trade balance and terms of trade. Empirical analysis also reveals several empirical relationships beyond those previously reported. Both the volume of exports and the volume of imports are positively correlated with output. The terms of trade is positively correlated with the volume of exports and negatively correlated with future import volumes. Also, increases in output are associated with terms of trade depreciations.

A two-country general equilibrium model driven by productivity innovations is able to replicate most of the regularities found in the trade data. A positive productivity innovation leads to an increase in output and a deterioration of the terms of trade. Because the productivity innovation is persistent, there is an inflow of investment goods leading to an initial trade balance deficit. Later, the trade balance improves as goods are exported to the foreign country. This allows the model to replicate the countercyclical behavior of the trade balance. Following a productivity
shock, the increased volume of both imports and exports leads to positive correlations between output and both of these variables. The terms of trade depreciation coupled with the initial trade balance deficit generates the \textit{S-curve} pattern in the relationship between those variables. Also, the model replicates the positive correlation between the terms of trade and import volume.

The remainder of the paper is organized as follows. Section 2 introduces the data and reports the empirical findings. Section 3 introduces a two-country general equilibrium model and section 4 reports impulse response functions and cross-correlation functions generated by the model simulations. Section 5 concludes.

2. Properties of the Data

Quarterly data from the International Monetary Fund’s \textit{International Financial Statistics} over the period 1980.i through 2010.i are used to examine the relationships between output and the various components of trade for the original G-6 countries.\textsuperscript{1} The trade balance, denoted by $TB$, is defined as the ratio of the net export of goods and services to output. The terms of trade, denoted by $TOT$, is defined as the ratio of import prices to export prices, where an increase in this variable corresponds to a depreciation of the terms of trade. Output is defined as real gross domestic product and is denoted by $Y$. The volume of imports and exports are symbolized by $VEX$ and $VIM$ respectively. In order to focus on the behavior of these variables over the business cycle, the data is detrended using the Hodrick and Prescott filter with a smoothing parameter of 1600.

The focus of this paper is on the correlation between the various trade variables and output at different leads and lags. Figures 1 through 7 contain the cross-correlation functions from the G-6 countries at leads and lags of up to eight quarters. The correlations are reported on the vertical axis and $k$, the lead or lag of the second variable, is reported on the horizontal axis. The cross-correlation between the terms of trade and the trade balance, $TOT_t$ and $TB_{t+k}$, for $k$ between -8 and 8 replicates the S-curve pattern described in Backus \textit{et al.} (1994). The results reported in Backus \textit{et al.} (1994) are found to be consistent with those reported in this paper using an addition 15 years of data. Lagged and contemporaneous values of the trade balance are negatively correlated with the terms of trade while future trade balance values are positively correlated with the terms of trade. This cross-correlation pattern is similar to the J-curve pattern commonly discussed in the literature.

The cross-correlation function between output and the trade balance, $Y_t$ and $TB_{t+k}$, is generally negative, reaching a minimum at contemporaneous values of both output and trade. Over the business cycle, increases in output are associated with a worsening of the trade balance. The cross-correlation between output and the terms of trade, $Y_t$ and $TOT_{t+k}$, is positive, indicating that increases in output are associated with a depreciation of the terms of trade. Both the cross-correlation functions between output and the volume of exports, $Y_t$ and $VEX_{t+k}$, and between output and the volume of imports, $Y_t$ and $VIM_{t+k}$, are positive and hump-shaped, reaching a maximum at $k=0$. A rise in output is associated with both an increased outflow and inflow of goods and services.
The cross-correlation functions between the terms of trade and the volume of exports and between the terms of trade and import volume are both positive, reaching a maximum at $k=0$. A terms of trade depreciation is associated with increases in the volume of both imports and exports. The terms of trade appears to be negatively correlated with future import volumes and positively correlated with current and past import volumes.

### 3. A General Equilibrium Model

In this section, a two-country, two-good international real business cycle model similar to that used in Backus et al. (1994) is described. In each country, goods are produced using capital and labor as inputs. A representative agent in each country maximizes utility under complete markets by consuming and investing an aggregate good created by combining goods from both countries. The representative agent in the domestic country chooses leisure, $L_t$, and consumption, $c_t$, in order to maximize expected lifetime utility, subject to the constraints listed below. The momentary utility function faced by the domestic agent is given by,

$$U(c_t, L_t) = \left(\frac{1}{1-\sigma}\right)c_t^{1-\sigma}L_t^{\sigma}$$

where $(1/\sigma)$ is the intertemporal elasticity of substitution. In each country, final goods, $y_t$, are produced using capital, $k_t$, and labor, $N_t$, according to the Cobb-Douglas production function,

$$y_t = a_t k_t^{1-\alpha} N_t^{\alpha}$$

where $a_t$ is a stochastic productivity shock and $\alpha$ is labor’s share in production. The representative agent in each country consumes and invests an aggregate good composed of final goods from both countries. In the domestic country, the aggregate good is produced by

$$x_t = [\eta(y_{ht})^{-\mu} + (1-\eta)(y_{ft})^{-\mu}]^{1/\mu}$$

where $x_t$ is the amount of aggregate final goods available to the domestic agent for consumption and investment, $y_{ht}$ is the amount of domestic final goods which remain in the domestic country and $y_{ft}$ is the amount of foreign final goods which are traded to the domestic country. Corresponding variables representing the foreign country are denoted with an asterisk.

The agent in each country faces several constraints on resources. The agent must allocate a fixed amount of time between labor and leisure, the production and use of final goods must be equal, and total investment and consumption by the domestic agent must be equal to the amount of aggregate goods available. These constraints are, respectively,

$$1-L_t-N_t=0 \tag{4}$$

$$y_t = y_{ht} + y_{ft} \tag{5}$$
where $i_t$ is investment in domestic country capital goods and $g_t$ represents government consumption. Also, new capital is created by investing aggregate final goods,

\((\gamma)k_{t+1} = (1-\delta)k_t + i_t\)  

where $\delta$ is the capital stock depreciation rate. The parameter values used to calibrate the standard model are similar to those used in other international real business cycle papers and are summarized in Table 1.4

4. Properties of the Theoretical Economy

In this section, the pattern of movement generated from the theoretical economy is compared to the previously discussed empirical regularities. Impulse response functions are computed in order to understand more fully the interaction between the model variables. The model is simulated to obtain theoretical cross-correlation functions. These simulated functions are then compared to those estimated from the data. This comparison is used to evaluate the model’s overall ability to replicate the data and serves to highlight any components of the model which fail to accurately reflect reality.5

4.1 Impulse Response Function

The pattern of movement caused by a one-percent innovation in productivity as variables deviate from and return to their steady-state levels is graphed in Figure 8. Following such a shock, output increases, both directly due to the productivity increase and to increased labor participation. This high level of production causes a depreciation of the domestic terms of trade that reach its maximum point several quarters after the productivity shock. The substantial increase in investment demand following the shock moderates the initial depreciation of the terms of trade, creating a lag between the shock and the point of maximum terms of trade depreciation.

Because there is a high degree of persistence in the driving process, following a productivity shock, investment goods flow into the country in order to take advantage of the expected high future level of productivity. This leads to an increased level of imported goods. This inflow, combined with the depreciated terms of trade, causes the balance of trade to worsen. In later periods, the balance of trade improves as the volume of imports decreases and the volume of exports begins to increase. The increased production in the domestic country is eventually exported once demand for new domestic capital has been reduced. Initially, however, export volume does not increase and it is only after the huge increase in investment demand has waned that the increased output is exported abroad.

4.2 Model Simulation

The model is simulated in order to generate theoretical cross-correlation functions.6 Figures 9 through 15 contain the cross-correlation functions generated from the model. The mean cross-correlations from the G-6 data are also included for comparison.
The model is able to replicate many aspects of the cross-correlation functions estimated from the data. The terms of trade is negatively correlated with current and lagged values of the trade balance and positively correlated with future values. This general pattern is also seen in the data and has been described as resembling an S-curve. Following a productivity innovation, the terms of trade depreciation reaches its largest level a few quarters after the maximum deterioration of the balance of trade. This leads to a negative correlation between the terms of trade and current and past values of the trade balance. In later periods, the balance of trade improves as exports increase, leading to a positive correlation between the terms of trade and future values of the balance of trade.

The model is also able to replicate the negative cross-correlation function between output and the trade balance, $Y_t$ and $TB_{t+k}$. Increases in output caused by productivity shocks are associated with trade balance deteriorations. Following a productivity shock, the inflow of investment goods initially outweighs the increase in exports. However this cross-correlation becomes positive for $k$ larger than two or three due to the eventual trade balance improvement caused by persistent productivity increases. Eventually, as investment demand decreases, the volume of goods exported surpasses imports, leading to a trade surplus.

The model generates positive cross-correlation functions between output and both the volume of exports and imports similar to those seen in the data. In the case of export volume, however, the model predicts a lag of two quarters, with a maximum correlation occurring between output and future export volume. In the model, an increase in output generally precedes the maximum increase in exports by a few quarters, causing future export volume to remain positively correlated with current output.

The positive correlation seen in the data between the terms of trade and past import volume and the negative correlation between the terms of trade and the future volume of imports are also generated by the model. The cross-correlation function between $TOT_t$ and $VIM_{t+k}$ reaches a minimum at about $k=4$. The timing of the correlation between the terms of trade and export volume predicted by the model does not replicate that seen in the data. In the model, the terms of trade is negatively correlated with the volume of exports and positively correlated with future export volumes. This correlation reaches its maximum at $k=4$ quarters. This lag is similar to that seen between output and export volume. In both instances the lag predicted by the model is not replicated in the data.

The model is also able to replicate the relationship between output and the terms of trade, $Y_t$ and $TOT_{t+k}$. In the data this cross-correlation function is positive, particularly for negative values of $k$. Increases in output are correlated with terms of trade depreciations. In the model, productivity shocks cause sustained increases in output and depreciations of the terms of trade, leading to a positive correlation between these variables. This positive correlation would generally not be found in models driven primarily by demand shocks or external, exogenous terms of trade innovations.
5. Conclusions

The basic dynamic general equilibrium model examined in this paper is able to generally replicate the pattern of comovement found between output and the terms, balance and volume of trade. Consistent with earlier work, an S-curve is found in the data. Additionally, most of the other cross-correlations between output and the various components of the trade balance are predicted by the model. The finding that both the terms and balance of trade are driven by productivity shocks and that the subsequent behavior generates an S-curve pattern, reminiscent of the J-curve, is given additional support. The fact that productivity shocks would generate a pattern similar to the J-curve underscores the importance of properly identifying the shocks generating movements in the trade balance in the broader J-curve literature.

The results suggest two avenues for further research. This paper has identified several new patterns exhibited by the cross-correlation functions generated by the international trade data. Additional empirical work, looking at the more detailed correlations for a wider set of countries, including lesser developed countries is suggested. It is possible that trade in smaller economies is driven more by external shocks, such as terms of trade shocks, rather than by internal productivity innovations. This would potentially generate a different pattern of correlations than that seen in the larger economies examined in this paper.

A second area of future research would be to refine the empirical models to better replicate the data. In recent years, a literature has grown, focused on improving the ability of dynamic general equilibrium models to replicate the international trade data. A small sample of these improvements include adding variable factor utilization rates in works such as Baxter and Farr (2004), asset market restrictions as in Baxter and Crucini (1995), and expectation shocks and asset bubbles as represented in Xiao (2004). These augmentations have improved the ability of the models to replicate the trade data along dimensions other than those examined in this paper. Future work could include an evaluation of these augmented models and their ability to replicate the cross-correlation functions discussed in this paper.

Endnotes

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1. These countries represent the largest industrialized economies over this period and include France, Germany, Italy, Japan, UK and US.

2. The stochastic productivity shock evolves according to $\mathbf{A}_t = \rho \mathbf{A}_{t-1} + \mathbf{e}_t$, where $\mathbf{A}_t = [a_t \ a_t^*]$, and the autocorrelation matrix, $\rho$, describes the degree of persistence of the technology innovations. The innovation vector $\mathbf{e}_t$ is assumed to be normally distributed with a variance-covariance matrix given by $\Omega$. The driving process is similar to that used in Backus et al. (1994), estimated using Solow residuals generated from the data.
3. This aggregation function from Armington (1969) is a standard feature of international real business cycle models. In this aggregator function, the elasticity of substitution between domestic and foreign final goods is controlled by the parameter $\mu$ such that $\sigma_x = 1/(1 + \mu)$.

4. Following Stockman and Tesar (1995), the intertemporal elasticity of substitution, $1/\sigma$, is set equal to 0.5. Government consumption is set equal to one-fifth of total production. Also, the elasticity of substitution between domestic and foreign final goods in forming the aggregate goods, $\sigma_x$, is fixed at 1.5. The method of King et al. (1988) is used to solve and simulate the model.

5. For a discussion of the ability of this type of model to match other aspects of the data such as the relative volatilities of these and other variables, see Baxter and Crucini (1995) or Stockman and Tesar (1995).

6. These functions are generated from 1000 model simulations, each of 120 periods in length.

7. As found in Backus et al (1994) and Senhadji (1998), the productivity shocks are the key to generating the S-curve pattern. As seen in these models, demand shocks will generate a positive, hump-shaped cross-correlation function between the terms of trade and the trade balance.

References

Armington, Paul S. 1969. “A Theory of Demand for Products Distinguished by Place of Production,” International Monetary Fund Staff Papers, 16(1), 159-78.


Table 1  Model Calibration

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Calibration</th>
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<tbody>
<tr>
<td>time discount factor (quarterly)</td>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>rate of technological progress (quarterly)</td>
<td>$\gamma$</td>
<td>1.004</td>
</tr>
<tr>
<td>intertemporal elasticity of substitution</td>
<td>$1/\sigma$</td>
<td>0.5</td>
</tr>
<tr>
<td>labor’s share of output</td>
<td>$\alpha$</td>
<td>0.66</td>
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<tr>
<td>capital depreciation rate (quarterly)</td>
<td>$\delta$</td>
<td>0.025</td>
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<tr>
<td>elasticity of substitution between home and foreign goods</td>
<td>$\sigma=1/(1+\mu)$</td>
<td>1.2</td>
</tr>
<tr>
<td>trade as a share of output</td>
<td>exports/gdp</td>
<td>0.15</td>
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<td>leisure preference parameter$^a$</td>
<td>$\Psi$</td>
<td></td>
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<tr>
<td>Armington aggregator parameter$^b$</td>
<td>$\eta$</td>
<td></td>
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<tr>
<td>Autocorrelation Matrix</td>
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<tr>
<td></td>
<td>$\begin{bmatrix} 0.91 &amp; 0 \ 0 &amp; 0.91 \end{bmatrix}$</td>
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<tr>
<td>Variance-Covariance Matrix</td>
<td>$\Omega$</td>
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<tr>
<td></td>
<td>$\begin{bmatrix} 7.26 &amp; 1.87 \ 1.87 &amp; 7.26 \end{bmatrix}$</td>
<td></td>
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</table>

$^a$Calibrated to set the steady-state leisure at 4/5 of total time available.

$^b$Calibrated to set trade as a share of output equal to 0.15.
Figure 1. S-Curve: Cross-Correlation Between (TOT<sub>t</sub>,TB<sub>t+k</sub>)

![Figure 1. S-Curve: Cross-Correlation Between (TOT<sub>t</sub>,TB<sub>t+k</sub>)](image1)

Figure 2. Cross-Correlation Function: (Y<sub>t</sub>,TB<sub>t+k</sub>)

![Figure 2. Cross-Correlation Function: (Y<sub>t</sub>,TB<sub>t+k</sub>)](image2)
Figure 3. Cross-Correlation Function: \((Y_t, \text{TOT}_{t+k})\)

Figure 4. Cross-Correlation Function: \((Y_t, \text{VEX}_{t+k})\)
Figure 5. Cross-Correlation Function: \((Y_t, VIM_{t+k})\)

Figure 6. Cross-Correlation Function: \((TOT_t, VEX_{t+k})\)
Figure 7. Cross-Correlation Function: (TOT_t, VIM_{t+k})

Figure 8. Model Impulse Response Function
Figure 9. S-Curve: Cross-Correlation Between \((TOT_t, TB_{t+k})\)

Figure 10. Cross-Correlation Function Between \((Y_t, TB_{t+k})\)
Figure 11. Cross-Correlation Function Between \((Y_t, TOT_{t+k})\)

![Cross-Correlation Function Between \((Y_t, TOT_{t+k})\)](image)

Figure 12. Cross-Correlation Function: \((Y_t, VEX_{t+k})\)

![Cross-Correlation Function: \((Y_t, VEX_{t+k})\)](image)
Figure 13. Cross-Correlation Function: \((Y_t, VIM_{t+k})\)

![Graph of Cross-Correlation Function: \((Y_t, VIM_{t+k})\)](image)

Figure 14. Cross-Correlation Function: \((TOT_t, VEX_{t+k})\)

![Graph of Cross-Correlation Function: \((TOT_t, VEX_{t+k})\)](image)
Figure 15. Cross-Correlation Function: (TOT_t, VIM_{t+k})