Econometric Modeling of the Interactions between Japan’s Housing Investment and GDP

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Abstract This paper pursues an econometric investigation of the interactions between Japan’s housing investment and gross domestic product (GDP). A cointegrated vector autoregressive (VAR) analysis of Japan’s recent time series data reveals two cointegrating relationships, which characterize the underlying long-run interactions of the variables in question. The cointegrated VAR model is then reduced to a vector equilibrium correction model, which is seen as a parsimonious representation of various dynamic interactions in the data.

Keywords: Housing Investment, Gross Domestic Product, Cointegrated Vector Autoregressive Model.

JEL Classification: C32, E22.

1. Introduction

This paper investigates the interactions between Japan’s housing investment and gross domestic product (GDP). A cointegrated vector autoregressive (VAR) approach is adopted for the analysis of Japan’s aggregate time series data. The introductory section briefly reviews the related literature and then describes the most significant aspects of this paper.

Macroeconomic time series data, such as the data of GDP, aggregate consumption and investment, tend to exhibit non-stationary behavior and should be treated as processes integrated of order 1 (denoted I(1) hereafter) rather than stationary processes. An I(1) cointegrated VAR model is developed by Johansen (1988, 1996) with a view to exploring various time series properties of non-stationary data. An econometric analysis using the cointegrated VAR system enables us to detect the underlying long-run economic relationships in the data. See Juselius (2006) and Kurita (2007), inter alia, for empirical research using the cointegrated VAR methodology.
The interactions between aggregate housing investment and GDP have attracted much attention in the literature on applied macroeconomics. See Green (1997), Coulson and Kim (2000), and Wen (2001), inter alia. The existing studies convey useful information about the relationships between the two aggregate variables. It seems, however, that the existing research tends to focus on the analysis of the US economy. In order to obtain a deeper understanding of empirical macroeconomics, it should be useful to pursue an econometric investigation into the interactions between Japan’s residential investment and GDP. Furthermore, it is of much importance, from the viewpoint of Japan’s economic and housing policy, to investigate both the short-run and long-run interdependent relationships between aggregate housing investment and GDP. With a view to addressing these empirical issues, this paper adopts the cointegrated VAR methodology and conducts a formal econometric analysis of Japan’s quarterly time series data. Various empirical findings revealed in this paper may be seen as quantitative information useful for the development of Japan’s policy in the future.

The organization of this paper is as follows. Section 2 reviews a cointegrated VAR model and its objectives in empirical research. Section 3 conducts a cointegrated VAR analysis of Japan’s time series data, and then proceeds to a parsimonious dynamic representation of the data. Section 4 gives concluding remarks. All the empirical analysis and graphics in this paper use CATS in RATS (Dennis, Hansen, Johansen and Juselius, 2005) and PcGive (Doornik and Hendry, 2007).

2. Review of the Cointegrated VAR Model

This section briefly reviews a likelihood-based analysis of a cointegrated VAR model based on Johansen (1988, 1996). Let us introduce a p-dimensional I(1) cointegrated VAR(k) model for $X_t$ as follows:

$$\Delta X_t = \alpha(\beta', \gamma) \left( X_{t-1} \right) + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \mu + \varepsilon_t,$$

for $t = 1, \ldots, T$, (1)

where a sequence of innovations $\varepsilon_t$ has independent and identical normal $N(0, \Omega)$ distributions conditional on $X_{-k+1}, \ldots, X_0$, and $\alpha, \beta \in \mathbb{R}^{p \times r}$ for $r < p$, $\gamma \in \mathbb{R}^r$, $\mu \in \mathbb{R}^p$ and $\Gamma_i \in \mathbb{R}^{p \times p}$. Let us define $\beta^* = (\beta', \gamma)$ and $X_{t-1}^* = (X_{t-1}', t)'$ for future reference. In Equation (1), $\alpha$ is called adjustment vectors, while $\beta^*$ is referred to as cointegrating vectors and the index $r$ denotes cointegrating rank. Note that linear trend $t$ is restricted in the cointegrating space of Equation (1),
that is, $X_t$ is subject to linear trend but free from quadratic trend, so that the mean of $\Delta X_t$ is non-zero. See Johansen (1996, Ch.5) for details of this specification.

This paper takes an interest in modeling the interactions between Japan’s residential investment and GDP, allowing for potential interdependent relationships between other macroeconomic variables. Based on the consideration of residential investment by Mankiw and Taylor (2008, Ch.18), the following variables are selected for $X_t$:

$$X_t = (h_t, y_t, p_t^h - p_t, \Delta p_t^h, i_t),$$  

(2)

where $h_t$ is the log of Japan’s aggregate housing investment in real terms, $y_t$ is the log of Japan’s real GDP, $p_t^h - p_t$ is a differential between the log of a deflator for Japan’s aggregate housing investment and that of Japan’s GDP deflator, $\Delta p_t^h$ is inflation based on a deflator for Japan’s aggregate housing investment, and $i_t$ is Japan’s long-term government bond yield. See the Appendix for details of the data. The empirical analysis using a set of variables in (2) aims to investigate the interactions between $h_t$ and $y_t$, taking account of various other influences arising from $p_t^h - p_t$, $\Delta p_t^h$, and $i_t$.

The cointegrated VAR model (1) assumes the presence of $r$ cointegrating relations in the variable set (2). The cointegrating rank $r$ is, however, unknown in practice, thus it is necessary to determine the rank based on the data analysis. A log-likelihood ratio (log-LR) test statistic for the determination of $r$ is constructed from the VAR model, and its asymptotic quantiles are provided by Johansen (1996, Ch.15). Determining the cointegrating rank in Equation (1) then allows us to inspect various restrictions on $\alpha$ and $\beta^*$ using standard likelihood-based asymptotic inferences. Cointegrating linkages of the variables may be interpreted as empirical representations of long-run economic relations. It is, therefore, important to check if a set of cointegrating combinations, $\beta^* X_{t-1}^*$, are identified such that they can be subject to long-run economic interpretations. In contrast, a group of first-order difference terms, $\sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i}$, may characterize short-run dynamics or interactions in the model.

Let us consider potential roles played by $p_t^h - p_t$, $\Delta p_t^h$ and $i_t$ in the estimated cointegrating relationships. First, $p_t^h - p_t$ represents the relative price of housing investment, which should be
influenced by the supply and demand for the existing stock of houses. In line with the argument by Mankiw and Taylor (2008, Ch.18), we may reason that, if \( p^h_i - p_r \) is high, the motive to build new houses is positively affected, thereby leading to an increase in the flow of new houses. It is, therefore, conjectured that \( h_t \) tends to move in the same direction as \( p^h_i - p_r \), and this tendency may be reflected in the structure of the cointegrating relationships. Next, \( \Delta p^h_i \) represents housing price inflation and its increase should have a negative impact on demand for new housing. As a result, \( \Delta p^h_i \) may tend to be negatively correlated with \( h_t \), playing a significant role in the cointegrating combinations. Finally, \( i_t \) could embody expected inflation based on the Fisher equation and also represent borrowing costs for households, thus it may have long-run positive and negative influences on \( y_i \) and \( h_t \), respectively. Besides these economic variables, Equation (1) contains deterministic trend \( t \), which can be seen as approximations to a smooth demographic change or potential GDP growth rate in Japan. The next section, using the cointegrated VAR methodology, investigates if these economic reasonings, some or all of them, could be reflected in the estimated cointegrating vectors.

### 3. Econometric Modeling of Japan’s Aggregate Data

This section performs a cointegrated VAR analysis of Japan’s quarterly time series data, spanning a period from the fourth quarter in 1985 to the second quarter of 2007 (denoted as 1985.4 - 2007.2 hereafter). The sample period is of interest in that it covers the era of Japan’s economic turmoil, during which an asset bubble economy took place and collapsed, then leading to long-term stagnation. The end point of this period corresponds to a structural break in the data, reflecting the fact that Japan’s building regulation was tightened for security reasons, leading to a sharp decline in the overall residential investment in the latter half of 2007. See the Appendix for details of the data and their overview.

This section is composed of three sub-sections. Section 3.1 addresses the issue of choosing the cointegrating rank, and Section 3.2 then reveals a set of identified cointegrating relations. Finally, Section 3.3 achieves a parsimonious dynamic system for Japan’s residential investment and GDP.
3.1 Determination of the Cointegrating Rank

This sub-section checks the validity of an estimated unrestricted VAR model, then proceeding to a discussion on the choice of the cointegrating rank. The unrestricted VAR model is purely a statistical representation and provides a set of estimated coefficients which are not necessarily subject to economic interpretations. Identifying cointegrating relations and conducting the model reduction, we are able to pursue such interpretations. The unrestricted VAR model should give a basis for a likelihood-based analysis of cointegration, thus its residuals need to satisfy the condition of normality and temporal independence, as in accordance with $\varepsilon_i$ in (1). The lag-length of the VAR model is set to be three for its residuals to be free from serial correlations. The estimated VAR model, however, suffers from an outlier caused by a regime change associated with the introduction of the value added tax in 1989. A dummy variable, defined as $D_{99,j} = 1$ in 1989.1 and 0 otherwise, is thus included in the model unrestrictedly, according to Doornik, Hendry and Nielsen (1998).

Table 1 displays a battery of residual diagnostic tests for the unrestricted VAR model. Most of the test results are expressed as the form $F_j(k, T - l)$, which denotes an approximate F-test against the alternative hypothesis $j$: kth-order serial correlation ($F_{ar}$: see Godfrey, 1978, Nielsen, 2006), kth-order ARCH ($F_{arch}$: see Engle, 1982), heteroscedasticity ($F_{het}$: see White, 1980). A chi-square test for normality ($\chi^2_{n}$: see Doornik and Hansen, 2008) is also provided. The diagnostic test statistics for the residuals, apart from that for some ARCH effects in the equation for $h_t$, are all insignificant at the 5% level, suggesting that the model’s residuals are close to the Gaussian distribution with no temporal dependence. Simulation studies in the literature show that Johansen’s maximum likelihood procedure tends to be affected by the presence of residual autocorrelation, while Johansen’s procedure is robust to ARCH effects in the residuals (see Cheung and Lai, 1993; Gonzalo, 1994; Rahbek, Hansen and Dennis, 2002). It is therefore justifiable, given the results in Table 1, to proceed to a standard likelihood-based cointegration analysis using this model.

The estimation of the unrestricted VAR model brings us to the determination of the cointegrating rank. Table 2 provides log-LR test statistics for the cointegrating rank. Standard log-LR test statistics for the cointegrating rank, based on Johansen (1996, Ch.6), are denoted as $-2 \log Q(H(r) | H(p))$ and displayed in the first panel of the table. In addition, Bartlett-corrected log-LR test statistics, i.e.
log-LR test statistics adjusted for the sample size, according to Johansen (2002), are also presented in the same panel as \(-2 \log Q_{bc}^{H}(H(r) | H(p))\). Both of the log-LR tests reject the null hypotheses of $r = 0$ \(\text{and} \quad r \leq 1\), but do not reject the remaining hypotheses at the 5% level; hence both of the tests are in favour of $r = 2$. Thus $r = 2$ is chosen, which paves the way for a further cointegration analysis.

### 3.2 Restricted Adjustment and Cointegrating Vectors

The determination of the cointegrating rank allows us to estimate the $I(1)$ cointegrated VAR model (1), in which we are able to examine joint restrictions on the adjustment and cointegrating vectors. As demonstrated above, cointegrating relations may represent long-run economic linkages embedded in the VAR system. It is therefore important to identify interpretable cointegrating relations and how adjustment mechanisms work in the VAR system.

The first cointegrating vector is normalized for $h_t$ and a homogeneity restriction is imposed on the coefficient for $y_t$, corresponding to unit income elasticity. The second cointegrating vector is, in contrast, normalized for $y_t$ so that the long-run influences of $h_t$ on $y_t$ can be evaluated. A series of trials of hypothetical restrictions on $\alpha$ and $\beta$ has led to the following acceptable restrictions:

\[
X_t^* = \begin{pmatrix}
  h_t \\
  \Delta P_t^b \\
  i_t \\
  y_t \\
  p_t^h - P_t
\end{pmatrix} : \hat{\alpha} = \begin{pmatrix}
  * \\
  0 \\
  * \\
  0 \\
  0
\end{pmatrix}, \quad \hat{\beta}^* = \begin{pmatrix}
  1 \\
  0 \\
  0 \\
  -1 \\
  1
\end{pmatrix},
\]

where ^ denotes estimator and * inside $\hat{\alpha}$ and $\hat{\beta}^*$ indicates a coefficient free from any restrictions, and the order of the variables in $X_t^*$ is changed according to the adjustment structure of $\hat{\alpha}$. Table 3 reports a set of restricted estimates with standard errors in the parentheses, as well as the corresponding log-LR test statistic. The p-value of the test statistic is 0.17, hence allowing us to conclude that the null hypothesis for the joint restrictions is not rejected at the 5% level. Since both of the cointegrating relations are identified, it is possible to regard them as empirical representations.
of the underlying long-run economic relationships.

First, let us consider economic interpretations of the two restricted cointegrating vectors, given in the right panel of Table 3. Solving the first cointegrating relation for $h_t$ leads to

$$h_t = y_t - \Delta p^h_t + 9.495(p^h_t - p_t) - 0.035t + \nu_{1,t},$$

where $\nu_{1,t}$ is a stationary error term. As mentioned above, the coefficient for $y_t$ is set to unity, indicating the presence of unit income elasticity in the long-run relation. The housing price inflation, $\Delta p^h_t$, holds a unit coefficient with minus sign, suggesting that an increase in the inflation rate has a negative one-for-one influence on $h_t$. In addition, the coefficient for the price differential $p^h_t - p_t$ is positive, while that for the time trend $t$ is negative; the former may reflect an increase in the flow of new houses motivated by a rise in $p^h_t - p_t$, while the latter may be interpreted as an approximation to influences of a smooth demographic change occurring in Japan.

The second cointegrating combination is, based on the right panel of Table 3, expressed as

$$y_t = 0.317h_t + 0.014i_t + 0.005t + \nu_{2,t},$$

where $\nu_{2,t}$ is a stationary error term. It should be noted that the coefficient for $h_t$ is positive and highly significant, suggesting the presence of a positive long-run impact of $h_t$ on $y_t$. This finding, together with the unit income elasticity in the first cointegrating relation, indicates that there exist long-run interactions between Japan’s residential investment and GDP. The coefficient for $i_t$ is also positive, which seems to be in contrast to the IS curve in the conventional Keynesian framework; but it could possibly indicate a long-run influence of expected inflation, embedded in the nominal interest rate, on the GDP. Finally, the deterministic trend may be treated as an approximation to the underlying potential GDP.

Figure 1 (a) and (b) record the plots of the two restricted cointegrating relations, both of which appear to be stationary processes rather than $I(1)$, in accord with the non-rejection of the null hypothesis observed in Table 3. Furthermore, Figure 1 (c) displays the recursive plots of the log-LR test for the restrictions examined in Table 3, covering a span of 10 years up to the end of the sample
period. The recursive plots all lie below the 5% critical value, indicating that the null hypothesis is not rejected at the 5% significance level over a substantial period of time.

Next, let us check the structure of restricted adjustment vectors reported in the left panel of Table 3. According to the first column of the table, the adjustment coefficients for \( h_t, \Delta p^h_t \) and \( i_t \) are all highly significant, while the remaining coefficients can be set zero; these three variables react to disequilibrium errors represented by the first cointegrating combination. The second column in the panel, in contrast, shows that the adjustment coefficient for \( y_t \) is significant with all the other coefficients being zero; \( y_t \) exclusively adjusts to the second cointegrating combination. Besides, according to Table 3, all the adjustment coefficients for \( p^h_t - p_t \) can be set to be zero. This implies that \( p^h_t - p_t \) is judged to be weakly exogenous for all the parameters in a partial model given \( p^h_t - p_t \). See Engle, Hendry and Richard (1983) and Johansen (1996, Ch.8) for details of weak exogeneity.

3.3 A Parsimonious Vector Equilibrium Correction System

Lastly, this sub-section attains a reduced equilibrium correction system for Japan’s residential investment and GDP. First, all the data are transformed to \( I(0) \) series by differencing and using the restricted cointegrating relationships, and an \( I(0) \) equilibrium correction system is then estimated. Insignificant regressors are removed from the system step by step, and it turns out that \( \Delta y_t \) acts as a significant contemporaneous regressor in the equation for \( \Delta h_t \). As a result of the model reduction, the author arrives at a parsimonious simultaneous equilibrium correction system as follows:

\[
\Delta \hat{h}_t = -0.03 \text{ecm}_{h,t-1} + 1.16 \Delta y_t + 0.51 \Delta h_{t-1} - 0.33 \Delta h_{t-2} \\
+ 0.03 \Delta^2 p^h_{t-1} + 0.02 \Delta i_{t-1} + 2.32 \Delta (p^h_{t-2} - p_{t-2}) - 0.02, \\
\]

\[
\Delta^2 \hat{p}^h_t = -0.92 \text{ecm}_{h,t-1} + 3.91 \Delta h_{t-1} + 21.17 \Delta (p^h_{t-1} - p_{t-1}) + 3.22 D_{89,t} - 0.48, \\
\]

\[
\Delta \hat{i}_t = 0.23 \text{ecm}_{h,t-1} - 0.14 \Delta^2 p^h_{t-1} + 14.77 \Delta y_{t-1} - 17.48 \Delta (p^h_{t-1} - p_{t-1}), \\
\]

\[
\Delta \hat{y}_t = -0.16 \text{ecm}_{y,t-1} + 0.003 \Delta^2 p^h_{t-1} - 0.03 D_{89,t} + 1.51, \\
\]

\[
\Delta (\hat{p}^h_t - \hat{p}_t) = 0.05 \Delta h_{t-1} + 0.002 \Delta^2 p^h_{t-1} + 0.17 \Delta (p^h_{t-2} - p_{t-2}) + 0.02 D_{89,t} + 0.002, \\
\]
Vector Tests: \[ F_{ar}(125,265) = 1.16[0.15], \quad \chi^2_{ar}(10) = 15.08[0.13], \]
\[ F_{het}(375,593) = 1.02[0.40], \]
where \( ecm_{h,t} \) and \( ecm_{y,t} \) represent the restricted cointegrating relationships, that is,

\[
ecm_{h,t} = h_t - y_t + \Delta p_t^h - 9.495(p_t^h - p_t) + 0.035t,
\]
\[
ecm_{y,t} = y_t - 0.317h_t - 0.014i_t - 0.005t.
\]

The vector tests are all insignificant at the 5% level, indicating that the reduced system represents the data well from a statistical viewpoint. Figure 2 (a) and (c) record the actual and fitted values for the two variables of interest in this paper, that is, \( \Delta h_t \) and \( \Delta y_t \), showing that the tracking of the data appears to be good. Figure 2 (b) and (d) present the scaled residuals of the equations for these two variables. The graphs indicate no clear evidence for outliers and serial correlation. The density functions of the scaled residuals are displayed in Figure 3 (a) and (c), which suggest that both of the residuals’ density functions appear to be very close to the standard Normal density functions. Finally, residual correlograms for these two equations are recorded in Figure 3 (b) and (d), indicating that there is no strong evidence for serial correlation, in accordance with Figure 2 (b) and (d). These graphic analyses, together with the results of the vector tests reported above, allow us to conclude that the reduced system is a parsimonious representation of the underlying data generation process.

According to the equation for \( \Delta h_t \) in the system above, the coefficient for \( \Delta y_t \) is positive and significant, suggesting the presence of a contemporaneous short-run income effect on \( \Delta h_t \); the equation for \( \Delta y_t \) is free from any contemporaneous influences, but affected by the past value of \( h_t \) by way of \( ecm_{y,t-1} \). The set of these empirical findings may be regarded as statistical evidence for the presence of short-run inter-dependent relationships between \( h_t \) and \( y_t \). As expected from the results of the cointegration analysis in Section 3.2, either \( ecm_{h,t-1} \) or \( ecm_{y,t-1} \) is highly significant in most of the equations, playing a key role in the parsimonious dynamic system.

4. Concluding Remarks

This paper pursues an econometric investigation of the interactions between Japan’s housing investment and GDP. A cointegrated VAR analysis of Japan’s recent time series data reveals two
cointegrating relationships, which characterize the underlying long-run interactions of the variables in question. The cointegrated VAR model is then reduced to a vector equilibrium correction model, which is seen as a parsimonious representation of the underlying data generation process. The analysis of this paper contributes to a deeper understanding of empirical aspects of macroeconomics, and also provides useful information for the development of Japan’s economic and housing policy in future.

Acknowledgement

I would like to thank the editor and anonymous referees for the appreciation of my paper.

Endnote

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References


Appendix: Data Overview and Details

(Data Overview)

\[ \log(\text{real housing investment in Japan}) \]
\[ \log(\text{real GDP in Japan}) \]
\[ \log(\text{implicit deflator for Japan's housing investment}) \]
\[ \log(\text{implicit deflator for Japan's GDP}) \]
\[ \text{10-year government bond yield in Japan} \]

[1] System of National Accounts, the webpage of Economic and Social Research Institute, Japan.

The data for \( h_t \) and \( y_t \) are seasonally adjusted. The deflators, \( p_{t}^{h} \) and \( p_{t} \), are constructed by subtracting the logged real variables from the corresponding logged nominal variables. Both of the deflators are multiplied by 100 for normalisation.
Table 1. Residual Diagnostic Tests for the Unrestricted VAR Model

<table>
<thead>
<tr>
<th></th>
<th>( h_t )</th>
<th>( y_t )</th>
<th>( p_t^h - p_t )</th>
<th>( \Delta p_t^h )</th>
<th>( i_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Autocorr. ([F_{ar}(5,64)])</td>
<td>1.03[0.41]</td>
<td>1.88[0.11]</td>
<td>0.97[0.45]</td>
<td>0.89[0.49]</td>
<td>1.16[0.34]</td>
</tr>
<tr>
<td>ARCH ([F_{arch}(4,61)])</td>
<td>3.11[0.02]*</td>
<td>1.51[0.21]</td>
<td>1.60[0.19]</td>
<td>1.53[0.20]</td>
<td>0.46[0.77]</td>
</tr>
<tr>
<td>Hetero. ([F_{het}(32,36)])</td>
<td>1.27[0.25]</td>
<td>0.92[0.60]</td>
<td>0.92[0.59]</td>
<td>0.97[0.53]</td>
<td>1.45[0.14]</td>
</tr>
<tr>
<td>Normality (\chi^2_{sd}(2))</td>
<td>1.02[0.60]</td>
<td>1.03[0.60]</td>
<td>0.05[0.98]</td>
<td>3.23[0.20]</td>
<td>5.85[0.05]</td>
</tr>
</tbody>
</table>

Note. The figures in the square brackets are p-values.
* denotes significance at the 5% level.

Table 2. Determination of the Cointegration Rank

<table>
<thead>
<tr>
<th></th>
<th>( r = 0 )</th>
<th>( r \leq 1 )</th>
<th>( r \leq 2 )</th>
<th>( r \leq 3 )</th>
<th>( r \leq 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-2 \log Q(H(r) \mid H(p)))</td>
<td>144.98[0.00]**</td>
<td>85.31[0.00]**</td>
<td>39.69[0.10]</td>
<td>9.18[0.95]</td>
<td>3.07[0.86]</td>
</tr>
<tr>
<td>(-2 \log Q^{BC}(H(r) \mid H(p)))</td>
<td>125.63[0.00]**</td>
<td>74.30[0.00]**</td>
<td>26.40[0.72]</td>
<td>5.83[1.00]</td>
<td>2.16[0.94]</td>
</tr>
</tbody>
</table>

Note. The figures in the square brackets are p-values.
** denotes significance at the 1% level.

Table 3. Restricted Adjustment and Cointegrating Vectors

\[
\begin{bmatrix}
\hat{\alpha} & 1 & 0 \\
\hat{h}_t & -0.044 & 0 \\
\hat{\Delta p_t^h} & -0.992 & 0 \\
\hat{i_t} & 0.454 & 0 \\
\hat{y_t} & 0 & -0.241 \\
\hat{p_t^h} - p_t & 0 & 0 \\
\end{bmatrix},
\begin{bmatrix}
\hat{\beta}^* & 1 & -0.317 \\
\hat{h}_t & 1 & 0 \\
\hat{\Delta p_t^h} & 1 & 0 \\
\hat{i_t} & 0 & -0.014 \\
\hat{y_t} & 0 & 1 \\
\hat{p_t^h} - p_t & -9.495 & 0 \\
\hat{t} & 0.035 & -0.005 \\
\end{bmatrix}
\]

Log LR Test Statistic for the Set of Restrictions: \(\chi^2(9) = 12.74\ [0.17]\)
Figure 1. Restricted Cointegrating Relations and Recursive log-LR Test Statistic

(a) The First Cointegrating Relation

(b) The Second Cointegrating Relation

(c) Log LR Test Statistic and 5% Critical Value

Figure 2. Fitted and Actual Values, Scaled Residuals

(a) Δ\(h_t\) and \(\Delta y_t\), Fitted

(b) Scaled Residuals

(c) Scaled Residuals

(d) Scaled Residuals

Δ\(h_t\)
Figure 3. Residual Density Functions and Correlograms

(a) Correlogram for \( \Delta h_t \)

(b) Correlogram for \( \Delta y_t \)

Density functions for \( \Delta h_t \) and \( \Delta y_t \) are shown, with distributions approximating normal distributions with different means and variances.