The J-curve Effect: An Examination Using a Structural Vector Error Correction Model

Jon Nadenichek

California State University, Northridge

Abstract Many previous studies of the J-curve phenomenon have suffered from the improper treatment of nonstationary variables and have not fully accounted for indirect linkages between variables. In this paper, a partially identified structural vector error correction model is used to correct these problems. In contrast to many earlier studies, a J-curve pattern is found in the response of the trade balance between the U.S. and the other G-7 countries.

Keywords: J-curve, trade balance, real exchange rate, SVECM

JEL Classification: F14

1. Introduction

The trade balance and the exchange rate are two of the most visible international economic variables. Therefore, gaining a better understanding of their behavior and pattern of interaction has long been an important topic in economics. This paper examines the dynamic response of the trade balance to changes in the real exchange rate. Specifically, the focus of the paper is on whether or not a J-curve pattern, an initial worsening and later improvement of the trade balance following a depreciation of the exchange rate, exists between the U.S. and the other G-7 countries.

As summarized in Bahmani-Oskooee and Ratha (2004a), previous analysis of a possible J-curve effect has met with decidedly mixed results. Support for the J-curve hypothesis has been reported in Wilson (1993), Bahmani-Oskooee and Alse (1994), Marwah and Klein (1996) and Hacker and Hatemi-J (2003). Evidence of a weak or ‘delayed’ J-curve has also been found by several authors such as Rosensweig and Koch (1988) and Moffet (1989). Other authors such as Felmingham (1988), Rose and Yellen (1989), Rose (1990, 1991), Demeulemeester and Rochat (1995), Shirvani and Wibratte (1997) and Wilson (2001), have not found evidence of a J-curve in the data.

Much of this previous work has occurred within a single-equation framework that ignores potentially important indirect linkages between the exchange rate and the trade balance. In an equilibrium analysis, the real exchange rate is generally endogenous and entangled with the other variables in the system. Hence, estimation of a J-curve effect that does not explicitly account for the possibility of feedback may give misleading results. As discussed in Demirden and Pastine
(1995), “…[in] order to test the J-curve in a flexible exchange rate environment it is necessary to specify an econometric structure that explicitly deals with feedback effects.”

In this paper, the effects of real exchange rate innovations on the trade balance are examined using a system-of-equations approach. This allows the effects of exogenous exchange rate movements to be isolated from the endogenous changes caused by other variables. A structural vector error correction model (SVECM) is partially identified and estimated. This partial identification allows the structural real exchange rate shocks to be fully recovered with a minimum of restrictions and assumptions. In this manner, the model is less subject to misspecification and is able to encompass a wider range of theoretical frameworks than a fully identified system.

A J-curve pattern is found in each of the six country pairs examined. In every case, a real exchange rate depreciation causes an initial worsening of the trade balance lasting anywhere from one quarter to almost two years in length. Later, there is a sustained and rather substantial trade balance improvement as the volume effect eventually outweighs the value effect. This general pattern of response is robust for a wide range of data treatment concerning stationarity and cointegration. Also, decomposition of the forecast error variance indicates that real exchange rate shocks are an important source of variance in the trade balance.

2. Stationarity Issues

Bilateral quarterly data between the United States and the other G-7 countries from 1974.i through 2004.i are used in the estimation. This use of bilateral data avoids some of the potential problems inherent in using aggregated exchange rate and trade data. Also, starting the estimation period after the breakdown of the Bretton-Woods system avoids the issue of a possible structural break between fixed and floating exchange rate systems. Output in each country is defined as the log of real gross domestic product. The real exchange rate is defined in terms of foreign goods per U.S. good and the bilateral trade balance is defined as foreign exports minus imports to the United States expressed as a share of total U.S. gross domestic product.

To properly estimate the system, the order of integration of the variables must be determined. In previous research, output has generally been treated as nonstationary, both on theoretical and empirical grounds. Despite some reported evidence of mean reversion, the real exchange rate is also generally treated as containing a unit root, while the trade balance, when expressed as a share of output, is usually treated as a stationary variable. In this paper, the univariate properties of the variables are examined using the commonly employed augmented Dickey-Fuller (ADF) test for unit roots. The null hypothesis of nonstationarity cannot be rejected against the alternate hypothesis of trend stationarity for real output in any of the countries examined. The results also indicate that the real exchange rate is nonstationary, although these results are not as conclusive as in the case of output. Additional ADF tests indicate that first differencing is sufficient to remove any nonstationarity in both output and the real exchange rate.

The testing is less definitive in the case of the balance of trade. In three instances, the null of nonstationarity is rejected at the ten percent level, while in the remaining three cases, the results fall short of rejection at that level of significance. The estimated persistence of the trade balance
is generally less than 0.90, consistent with a stationary variable subject to persistent innovations. The trade balance is treated as stationary in the benchmark specification, with the possibility of nonstationarity examined in a later section.

The possibility of cointegration between the nonstationary variables is formally evaluated using a version of the Johansen testing procedure. The results of the test, reported in Table 1, indicate the presence of two unit roots in the three variable system of U.S. output, foreign output and the real exchange rate, \([y \ y^* \ q]\). In every country pair, the null hypothesis of zero cointegrating vectors is rejected against the alternative of at least one cointegrating vector at least at the ten percent level.

These results support those reported in Kawai and Ohara (1997) who find that bilateral real exchange rates are cointegrated with other real variables and relative labor productivity in particular. However, these vectors are estimated with quite a bit of uncertainty and, in contrast to the results reported in Kawai and Ohara (1997), no consistent pattern of cointegrating relationships is found across the countries.\(^3\)

3. The Estimation Procedure

In order to isolate the exogenous exchange rate innovations from endogenous movements caused by productivity shocks or other factors, a system of equations is estimated. This allows the interactions between the variables to be more fully captured than in a standard single-equation approach. The main difficulty in using a system-of-equations approach is the need to impose structure in order to identify the variables of interest. This involves placing restrictions upon the empirical model that may or may not be valid. In this paper, this problem is minimized by only partially identifying the system. Only innovations to the real exchange rate and the coefficients necessary to generate the impulse response functions are identified. This estimated system is fairly general and is consistent with a variety of theoretical frameworks. The general procedure used closely follows that outlined in King and Watson (1995).

The vector of variables used in the estimation includes U.S. and foreign output, the real exchange rate and the trade balance and is denoted by \(X' = [y \ y^* \ q \ b]\). The corresponding vector of structural disturbances is given by \(\varepsilon' = [\varepsilon_y \ \varepsilon_y^* \ \varepsilon_q \ \varepsilon_b]\) and is assumed to have a diagonal covariance matrix, \(E(\varepsilon_i, \varepsilon_j) = D\) and to be independently distributed across time, \(E(\varepsilon_i, \varepsilon_s) = 0\) for \(s \neq t\).

In order to examine the effects of exchange rate shocks on the trade balance, the third row of the structural vector auto regression (SVAR),

\[
A(L)X_i = \varepsilon_i, \tag{1}
\]
must be estimated. The first step in the estimation process is to transform the data in order to render them stationary. The transformed structural vector error correction model (SVECM) is given by

\[ B(L)x_t = \eta_t, \]  

(2)

where \( x_t = T(L)X_t \) denotes the vector of transformed variables and \( \eta_t \) is the vector of transformed structural innovations. For simplicity, the stationary transformation is chosen such that it does not alter the structural innovations or the structural equations of the model. This implies that the SVECM has the same innovations as the SVAR, \( \eta_t = \varepsilon_t \). Combining Equations (1) and (2) yields the relationship between the coefficients of the transformed and untransformed models,

\[ B(L)T(L) = A(L). \]  

(3)

The restriction that the stationary transformation does not alter the structural equations of the model implies that \( T_0 = I \) and \( B_0 = A_0 \). The stationary transformation implied by the first differencing and cointegrating vector from the specification discussed in section 1 is given by

\[
S(L) = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
1 & \delta_1 & \delta_2 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix} + \begin{bmatrix}
-1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix} L.
\]

(4)

The stationary transformation \( T(L) \) that satisfies \( T_0 = I \) is generated by \( T(L) = S_0^{-1}S(L) \).

The general identification problem involves recovering the structural innovations from the reduced form vector error correction model (VECM) denoted by,

\[ F(L)x_t = e_t, \]  

(5)

where \( e \) is the vector of errors from the reduced-form estimation such that \( e' = [e_y \ e_y \ e_q \ e_b] \) and \( F(L)x_t = B_0^{-1}B(L)x_t = B_0^{-1}\varepsilon_t = A_0^{-1}\varepsilon_t = e_t \).

This identification requires assumptions about the matrix \( A_0 \). In the partially identified procedure employed here, only the row associated with structural innovations to the real exchange rate must be identified. This necessitates recovery of the coefficients from the third row of \( A_0e_t = \varepsilon_t \),

\[ a_{31}\varepsilon_{ys} + a_{32}\varepsilon_{yq} + a_{33}\varepsilon_{qt} + a_{34}\varepsilon_{b} = \varepsilon_{qt}, \]  

(6)

where \( a_{ij} \) is the \( ij^{th} \) element of \( A_0 \).

An instrumental variables approach is used to estimate \( a_{31} \) and \( a_{32} \) with the long-run stochastic trends in U.S. and foreign output used as instruments. Any nonstationarity in the real exchange
rate is driven by some linear combination of these long-run trends. Structural innovations in the real exchange rate should not affect the stochastic trends in output and thus should be uncorrelated with these long-run forecasts. This assumption is consistent with a wide range of theoretical models sharing the property that long-run output is solely a function of long-run productivity.

The stochastic trends or long-run forecasts, \( \lim_{k \to \infty} E_t X_{t+k} \), are generated by imposing the cointegrating restrictions on the long-run responses estimated from the reduced form model. The innovations in the stochastic trends in the reduced form VECM evolve according to

\[
e_\tau = -T_t F(1)^{-1} e_t
\]

(7)

Accumulating these trend innovations, \( e_\tau \), yields the stochastic trends which are used as instruments to estimate \( a_{31} \) and \( a_{32} \).

Identification of the structural innovations in the real exchange rate is completed by normalizing, \( a_{33} = 1 \), and by imposing a zero restriction, setting \( a_{34} = 0 \). This implies that contemporaneous, exogenous movements in the trade balance will not have any immediate impact on the exchange rate. These restrictions are sufficient to allow recovery of the structural innovations to the real exchange rate, \( e_\eta \).

To examine the effects of exogenous exchange rate movements on the trade balance, the moving average representation must be computed. The transformed SVAR has a moving average representation,

\[
x_t = C(L) e_t
\]

(8)

where \( C(L) = B(L)^{-1} \). Given the restrictions on the coefficients of the third row of \( A_0 \) and the estimated reduced form coefficients, \( F(L) \), the third row of \( C(L) \) can be recovered. This allows the impulse response following an exogenous exchange rate innovation to be computed.

4. Empirical Results

Impulse response functions are generated following the procedure outlined in the previous section. The variables are transformed and reduced form coefficients are estimated using a lag length of eight quarters. Using the stochastic trends as instruments, the third structural equation is identified and the moving average representation is recovered. Finally, the variables are transformed back into levels and impulse response functions are generated.

The responses of the bilateral trade balance of each country to a one standard deviation real exchange rate depreciation are found in Figure 1. One standard error bands are generated using Monte-Carlo simulations with 1000 replications. The results indicate the presence of a J-curve pattern in each of the country pairs. Following an exogenous exchange rate depreciation, the
trade balance initially worsens. The fall in the relative value of exports initially outweighs any increased export volume, causing the trade balance to deteriorate. In later periods, the volume effect dominates the value effect, leading to an improvement of the trade balance. The magnitude of the improvement is generally larger than the initial deterioration. Also, because the exogenous exchange rate innovations are very sustained, this later trade balance improvement is also very persistent.

In order to evaluate the importance of exchange rate innovations on trade balance movements, variance decompositions are performed. The results of these decompositions at different horizons are reported in Table 2. Real exchange rate shocks account for a substantial portion of the variance of the trade balance. Such innovations account for between five and thirty percent of the one-year-ahead forecast-error variance and a similar proportion at three and six year horizons.

Overall, these results indicate that there is a significant time lag between a depreciation of the real exchange rate and any improvement of the trade balance. Fiscal and monetary policies aimed at reducing trade deficits through improving the ‘competitiveness’ of a country’s exports will tend to initially worsen such deficits. Even if such policies are able to have a significant impact on the real exchange rate, any trade improvements will probably only occur after a lag of several quarters.

These results are quite different from those of Rose and Yellen (1989) and Rose (1990, 1991) who use similar data in a single-equation model. For a wide variety of instruments and lag lengths, they find no evidence of a J-curve pattern. The different conclusions that are drawn from the single-equation and system-of-equations approaches support the assertion of Demirden and Pastine (1995). It is possible that important indirect linkages are being ignored or misinterpreted in the single-equation estimations.

5. Alternate Stationarity Assumptions

There is uncertainty about the order of integration of some of the variables. Given only thirty years of data, it is difficult to distinguish between variables subject to highly persistent temporary shocks and those with unit roots. Therefore, the sensitivity of the estimation results to different stationary transformations is examined.

Although the null hypothesis of nonstationarity is not rejected at standard significance levels by the ADF tests, more sophisticated testing of the purchasing power parity hypothesis has uncovered some evidence of mean reversion in the real exchange rate which may imply stationarity. Also, many economic models use long-run purchasing power parity as a reasonable approximation of reality. Figure 2 contains the impulse responses of the trade balance to exchange rate depreciations under the specification of stationary real exchange rates. In every country, the J-curve pattern remains in place.

Because there is no stable, consistent cointegrating vector estimated across all country pairs, the case of a nonstationary real exchange rate and no cointegrating vector between output and the
real exchange rate is also examined. In each of the country pairs, impulse response functions again indicate the presence of a J-curve. The impulse response functions are reported in Figure 3. Impulse response functions for a specification where all variables are nonstationary are reported in Figure 4. This treatment is most similar to that used in Rose and Yellen (1989) where all variables including the trade balance are assumed to be nonstationary. The presence of a J-curve effect is found in five of six country pairs with the exception being the U.S.-Canada trade balance.

6. Conclusions

A partially identified structural vector error correction model is used to examine the effect of real exchange rate innovations on the trade balance. A J-curve pattern is found in the behavior of the bilateral trade data between the United States and the other G-7 countries. The initial worsening of the trade balance following a depreciation of the exchange rate lasts from one to seven quarters and is followed by a very persistent trade balance improvement. This dynamic response of the trade balance is robust to different stationarity assumptions. This robustness of the results is important due to the well-documented difficulty in distinguishing between highly persistent, stationary variables and those containing unit roots.

Endnotes

1. See Bahmani-Oskooee and Ratha (2004b) and Bahmani-Oskooee and Goswami (2003) for a discussion and examples of the benefits of using bilateral rather than aggregate trade data in J-curve estimation.

2. See, for example, Ahmed and Park (1994) or Kim (1996) for recent examples of stationary treatment of the trade balance.

3. As discussed in Dueker and Startz (1998), uncertainty over the order of the order of integration in initial tests for unit roots can be even more important than uncertainty from cointegration tests. In this paper, the main cointegration tests are performed under the assumption that the real exchange rate is difference stationary. To the extent that there is uncertainty concerning the existence of a unit root in the real exchange rate, the results of the cointegration tests could be misleading.


References


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Table 1. Cointegration Test Results

<table>
<thead>
<tr>
<th>Country Pair</th>
<th>$H_0$: $r=0$</th>
<th>$H_0$: $r=1$</th>
<th>$H_1$: $r=2$</th>
<th>Vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.-Canada</td>
<td>23.099**</td>
<td>12.853</td>
<td></td>
<td>$[1 \ -1.1190 \ 0.232]$</td>
</tr>
<tr>
<td>U.S.-France</td>
<td>54.567**</td>
<td>8.557</td>
<td></td>
<td>$[1 \ -1.4483 \ -0.026]$</td>
</tr>
<tr>
<td>U.S.-Germany</td>
<td>23.299*</td>
<td>10.563</td>
<td></td>
<td>$[1 \ -1.1807 \ -0.1497]$</td>
</tr>
<tr>
<td>U.S.-Italy</td>
<td>30.523**</td>
<td>12.522</td>
<td></td>
<td>$[1 \ -1.4368 \ -0.0924]$</td>
</tr>
<tr>
<td>U.S.-Japan</td>
<td>40.001**</td>
<td>12.339</td>
<td></td>
<td>$[1 \ -1.0684 \ -0.1228]$</td>
</tr>
<tr>
<td>U.S.-U.K.</td>
<td>39.601**</td>
<td>14.151*</td>
<td></td>
<td>$[1 \ -1.3092 \ 0.17]$</td>
</tr>
</tbody>
</table>

* Reject null hypothesis at the ten percent level.

** Reject the null hypothesis at the one percent level.

Test of number of cointegrating vectors, $r$, based on Johansen lambda-maximum test statistic estimated with four lags in the system including U.S. output, foreign output and the real exchange rate.

Table 2. Variance Decomposition: Percentage of $k$-Quarter-Ahead Forecast Error Variance Explained by Real Exchange Rate Innovations

<table>
<thead>
<tr>
<th>Country Pair</th>
<th>$k=1$</th>
<th>$k=4$</th>
<th>$k=8$</th>
<th>$k=12$</th>
<th>$k=24$</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.-Canada</td>
<td>3.68</td>
<td>9.08</td>
<td>11.44</td>
<td>14.51</td>
<td>18.82</td>
</tr>
<tr>
<td></td>
<td>(10.2)</td>
<td>(10.7)</td>
<td>(12.1)</td>
<td>(12.6)</td>
<td>(12.6)</td>
</tr>
<tr>
<td>U.S.-France</td>
<td>3.10</td>
<td>7.82</td>
<td>13.46</td>
<td>20.46</td>
<td>27.15</td>
</tr>
<tr>
<td></td>
<td>(9.1)</td>
<td>(14.1)</td>
<td>(15.6)</td>
<td>(15.8)</td>
<td>(16.2)</td>
</tr>
<tr>
<td>U.S.-Germany</td>
<td>20.89</td>
<td>29.91</td>
<td>25.36</td>
<td>20.12</td>
<td>24.04</td>
</tr>
<tr>
<td></td>
<td>(10.1)</td>
<td>(11.3)</td>
<td>(11.9)</td>
<td>(13.3)</td>
<td>(13.9)</td>
</tr>
<tr>
<td>U.S.-Italy</td>
<td>0.82</td>
<td>26.77</td>
<td>25.43</td>
<td>23.14</td>
<td>28.18</td>
</tr>
<tr>
<td></td>
<td>(10.3)</td>
<td>(14.2)</td>
<td>(15.5)</td>
<td>(16.1)</td>
<td>(16.4)</td>
</tr>
<tr>
<td>U.S.-Japan</td>
<td>0.07</td>
<td>0.64</td>
<td>6.22</td>
<td>12.22</td>
<td>16.23</td>
</tr>
<tr>
<td></td>
<td>(9.4)</td>
<td>(11.3)</td>
<td>(14.4)</td>
<td>(14.9)</td>
<td>(15.3)</td>
</tr>
<tr>
<td></td>
<td>(9.7)</td>
<td>(10.2)</td>
<td>(10.5)</td>
<td>(10.7)</td>
<td>(10.8)</td>
</tr>
</tbody>
</table>

Standard errors in parenthesis generated using Monte-Carlo simulations with 1000 replications.
Figure 1. Trade Response to a Real Exchange Rate Depreciation

Canada

France

Germany

Italy

Japan

UK
Figure 2. Response of the Trade Balance to a Real Exchange Rate Depreciation: Stationary Real Exchange Rate

Canada

France

Germany

Italy

Japan

UK
Figure 3. Response of the Trade Balance to an Exchange Rate Shock: Nonstationary Real Exchange Rate, No Cointegration
Figure 4. Trade Response to a Real Exchange Rate Depreciation:
Nonstationary Real Exchange Rate and Trade Balance

Canada

France

Germany

Italy

Japan

UK