A Vulnerability Index for Predicting Extreme Market Events in Hong Kong

W. K. Li *, Philip L. H. Yu **, K. S. Maurice Tse ***
University of Hong Kong

Abstract In this paper, we explore the possibility of developing a “vulnerability” indicator for gauging the health of the economy of Hong Kong. An important measure of “health” to be considered is the popular maximum Lyapunov exponent in the dynamical system literature which measures the sensitivity to initial conditions of a deterministic function. Other key economic and financial indicators that have impact on the Hong Kong financial market such as yield spreads & forward rates are also considered. Lyapunov exponent is often used to indicate the presence of nonlinearity and has not been used as an explanatory variable in the literature, when in fact the Lyapunov exponent also contains useful information about a dynamical system and such information can be usefully exploited. Using stepwise probit regression an indicator of the vulnerability of the financial sector is obtained which is able to indicate empirically observed crisis. An interesting feature of this indicator function is that the local maximum Lyapunov exponent plays a non-negligible role in predicting the status of the economy.

Keywords: Chaos, resilience indicator, financial crisis, vulnerability index

JEL Classifications: C53, E44, E37

Introduction

After the stock market crash on Black Monday on October 19, 1987 that wiped out $1 trillion in capital in the U.S. stock market and the Asian turmoil of 1997 that wiped off three quarters of the dollar capitalization of equities in Korea, Indonesia, Malaysia, and Thailand, it has become imminent to develop means that are able to detect whether the economy is susceptible or vulnerable to the coming of such crisis events. Needless to say finding such a means is a difficult task. Even during an economic downturn it is usually not easy to predict how vulnerable the economy could be to unfavorable events. Hence, an indicator that can help point out the vulnerability of the economy to adverse events is useful to policymakers. According to the Collins Cobuild English Language Dictionary, someone who is vulnerable is weak and without protection, that he/she is easily hurt physically and emotionally. An economy or a financial system could also be vulnerable with regard to crisis events in a similar sense. For example, the Hang Seng Index could take a big plunge upon news on the outbreak of war in the Middle East. The above interpretation of vulnerability bears very close resemblance to a concept which has been developed in the physics literature known as chaotic dynamics. Chaotic dynamics have been applied to physics and water resources research with considerable success. Chaotic dynamics are characterized by their sensitivity to initial conditions. Roughly put, in a chaotic system, slight variations in initial conditions could lead to substantially different behavior of the dynamical system over just a short period.
of time. The measure of how sensitive a dynamical system is to the initial conditions is provided by a quantity known as the Lyapunov exponent. Usually there may be several such exponents but we need only consider the largest of them.

For a one-dimensional dynamical system there is only one Lyapunov exponent $\lambda$ which measures the sensitivity to initial conditions of the system. A positive exponent $\lambda$ implies that any difference in the initial conditions is amplified at a rate of $\exp(\lambda)$ and indicates the presence of chaotic dynamics whereas a negative exponent means that the effect is unimportant and that the map is converging to a point. The potential of applying chaotic concepts and theories to the financial market has attracted much attention in the literature. Peters (1994) provides a detailed introduction to the subject. Many of the investigations, however, concentrate only on whether chaos or deterministic dynamics exists in the data and their findings are often inconclusive.

Reggiani, de Graaff and Nijkamp (2001) recently consider a resilient measure of the labour markets in West-Germany using the Lyapunov exponent. Applications of the Lyapunov exponents to analyzing financial chaos have been considered as well, for example, Dechert and Gencay (1992). Pearson and Potter (1993) summarize some recent results and findings in applying chaos theory to economic and financial analysis.

In this research, we aim at employing the Lyapunov exponent as a pure data analytic tool and not as a quantity with a theoretical construct. We do not attempt to explain why such a measure is useful by means of an economic theory. Instead we will concentrate on developing a working indicator capable of signaling the vulnerability of the financial sector of Hong Kong to adverse events.

To create a potentially useful measure that is of sufficient sensitivity to changes of the market we propose a maximum Lyapunov exponent in the next section. The maximum Lyapunov exponent will be computed for some important financial time series in Hong Kong and a potentially useful vulnerability indicator is created out of these quantities using probit regression. Our use of the Lyapunov exponent as an exploratory data analytic tool is somewhat unconventional because in most applications it is used only as an indicator of the presence of chaotic dynamics.

The paper is organized as follows. In Section 2, we discuss the Lyapunov exponent for time series data. In Section 3, we discuss the preliminary results of applying the maximum Lyapunov exponent to the key financial time series in Hong Kong. In Section 4, we detail the data set used, the development of a predictive model for the construction of the vulnerability indicator, and also the robustness of the model developed. Section 5 concludes the paper.

**Times series and the Lyapunov exponent**

A time series is a sequence of observations recorded over time. Denote this sequence by $x_1, \ldots, x_t, \ldots, x_n$, where $t$ is the time index. A popular statistical model for a time series is the so-called autoregressive model (of order one) which relates the next observation of the time series $x_{t+1}$ to its current value $x_t$ according to the following stochastic process:

$$x_{t+1} = \phi x_t + a_{t+1},$$

(1)

where $\phi$ is a parameter lying between $-1$ and $1$, and $a_t$ is a noise process. When $\phi = 1$, we have the famous random walk model. The autoregressive model (1) is a linear equation. If
we can replace $\phi x_t$ on the right-hand-side by a nonlinear relationship $f(x_t)$, then equation (1) becomes

$$x_{t+1} = f(x_t) + a_{t+1}.$$  \hspace{1cm} (2)

If $a_{t+1}$ are always zero we have a one-dimension nonlinear function

$$x_{t+1} = f(x_t).$$  \hspace{1cm} (3)

Suppose we start at time $t = 0$ with initial observation $x_0$ and iterate equation (3) for $(m-1)$ times. Then we obtain a time series of length $m$ and the Lyapunov exponent is defined as the limit of the sample average,

$$\lambda = \frac{1}{m} \sum_{t=0}^{m-1} \log f'(x_t)$$  \hspace{1cm} (4)

where $f'(x_t)$ is the derivative (assumed to exist) of $f(x)$ evaluated at $x_t$. For a higher dimensional real valued function $f(x_1, \ldots, x_d)$ there will be $d$ Lyapunov exponents. In general, as discussed in Chan and Tong (2001), it is only necessary to consider the maximum Lyapunov exponent (MLE) which will still be denoted by $\lambda$ in our following analysis and discussion. A function is considered chaotic if the MLE is positive. As mentioned earlier most of the existing literature that applies chaos theories to capital markets and finance has concentrated on whether the time series is chaotic and not on the local behavior of $x_t$ and its interpretation.

We follow Wolff (1992) and define a Lyapunov exponent at time origin $t_0$ by approximating $f(x_1, \ldots, x_d)$ with a locally linear function. The $d$-dimensional mapping is approximated around the point at $t_0$ by a $d \times d$ matrix $A^0_{t_0}$. $A^0_{t_0}$ is estimated using $L$ nearest neighbors of the observation ($d$-dimensional) at $t_0$ and their respective values under $f(x_1, \ldots, x_d)$. Similar approach has been considered by Sattin (1997) and Lai and Chen (1998). In a similar way an $A^i_{t_0}$ is obtained for $t_0 - i$ where $i = 0, \ldots, m-1$. In each case we compute the maximum eigenvalue $\lambda^i$ of the matrix $A^i_{t_0}$. The maximum Lyapunov exponent $\lambda$ is then the average of the natural logarithm of $\lambda^i$, where $i = 0, \ldots, m-1$. That is,

$$\lambda = \frac{1}{m} \sum_{i=0}^{m-1} \log (\lambda^i).$$  \hspace{1cm} (5)

In practice $d$ will be determined using techniques in dynamical systems such as the False Nearest Neighbour method (Abarbanel, 1996). In our experience with Hong Kong data $d$ is found to be 6 based on a 3-year moving window with the number of nearest neighbors of the $d$-dimensional observation at $t_0$ $L = 15$, and the length of the time series $m = 50$. In actual practice, noise is usually present in the data. A denoise programme can be applied in such situations using the method in Jayawardena, Li and Xu (2001).
Preliminary observations with the Lyapunov exponent

The maximum Lyapunov exponent (MLE) has been applied to the key financial time series, in Hong Kong, namely, the Hang Seng Index returns, the forward rates between Hong Kong dollar and US dollar, and the yield spread between 3-month U.S. Treasury yield and the 3-month Hong Kong Bond yield, for the period from December 94 through April 2002. In our analysis that follows in Section 4, the daily maximum Lyapunov exponents (MLE) were first determined using daily data so that we could calculate the monthly statistics such as the monthly average and standard deviation of MLE that are needed for the construction of predictors for monthly crisis. Figures (i)-(xii) in Appendix A1 presents the original and the denoised time series, both daily and monthly, for the Hang Seng Index returns, the forward rates, and the 3-month yield spreads for the period from December 94 through April 2002.

The denoised time series have fewer extreme observations (outliers) than the original return series. Taking the HSI for example, it can be seen from figures (xiii)-(xv) in Appendix A2 that its MLE usually fluctuates around a value of 0.2 and its monthly standard deviations may assume large values at times. For example, the standard deviation of the MLE rose sharply after August 1997, the time of the Asian financial crisis.

Recall that a positive MLE means that the mapping $f(x)$ is sensitive to initial/starting values while a negative MLE value implies that $f(x)$ is converging to a single point. An unusually large $\lambda$ therefore may suggest the market/economy is very unstable and is extremely sensitive to a large amount of news/information. In contrast, a negative $\lambda$ may suggest a very stagnant situation which is also unhealthy. Therefore, it seems reasonable to assume that in a normal situation the MLE function and its standard deviation will maintain a certain level of fluctuation. However, during an economic turmoil or in case of some very positive/negative news both the MLE and its standard deviation could behave rather differently. Like the HSI both the yield spread and forward points series rose sharply in the summer months of 1997 the time of the Asian Financial Crisis.

Therefore, it is also of interest to consider similar MLE measures for the denoised yield spread and the denoised forward points.

Data and model

To construct the “health”/“vulnerability” indicator we first need a list of past crisis events such that a statistical model could be built based on its predictive ability of the crisis events. In collaboration with the Hong Kong Monetary Authority (HKMA) a list of these crisis events were created from February 1980 to January 2002. However, since the Hong Kong bond yield data were not available until early 90s, the list of crisis events we used for this study starts from December 1994. Appendix B provides the complete list chronological list of shocks to the financial markets in Hong Kong between February 1980 and January 2002. In our model construction the period from December 1994 – May 2001 was used for fitting the model and the period from June, 2001 to May 2002 was used to test the model’s ability in forecasting crisis out of sample. The monthly target variable (score) in our model has a value of 2 for the month with extreme events defined by HKMA; a value of 1 for extreme events defined by us and a value of 0 otherwise. The HKMA score of 2 supercedes our score of 1 in case of overlapping. A time plot of the score is given in Figure 1 in Appendix C.

The potential predictors, provided in Table 1 in Appendix D, include HSI returns, 3-month yield spreads and the 3-month forward points. These variables were deemed to be most relevant in predicting or indicating the presence of crisis in the financial market of Hong
Kong. We have also considered the use of the Hong Kong Interbank Offered Rate (HIBOR) which was found to be not too useful as a predictor. Two models were constructed using three-month and one year yield spreads and forward points respectively. We considered a variety of functions of the three variables as potential predictors. The statistical model considered is a probit regression model. A stepwise variable selection method is used to select useful explanatory variables in predicting the target variable.

To develop the fitted probit model for the 3-month financial data, we first of all assume that there exists an underlying variable \( y \) such that the crisis/score variable assumes different values depending on the values of \( y \) as follows:

\[
\text{Score} = \begin{cases} 
0 & \text{if } y < \alpha_1 \\
1 & \text{if } \alpha_1 < y < \alpha_2 \\
2 & \text{if } \alpha_2 < y , 
\end{cases}
\]

\[y_i = x_i \beta + \varepsilon,\]

where \( \varepsilon \) is a standard normal variable, \( x_i \) is the vector of potential predictors and \( \beta \) is the vector of regression coefficients. The potential predictors are listed in Table 1 in Appendix D.

Under this model, the probabilities of getting a score of \( i \) (\( i = 0, 1 \) or \( 2 \)) in a month are given by

\[
P(\text{Score} = 2) = \Phi(\alpha_2 - x \beta),
\]

\[
P(\text{Score} = 1) = \Phi(\alpha_1 - x \beta) - \Phi(\alpha_2 - x \beta),
\]

\[
P(\text{Score} = 0) = 1 - \Phi(\alpha_1 - x \beta).
\]

Therefore, the probability of observing an extreme event (level 1 or 2) in a month is

\[
P(\text{Score} = 1 \text{ or } 2) = \Phi(\alpha_1 - x \beta).
\]

The modeling period is from December 1994 to May 2001 while the post-sample forecasting period is from June 2001 to May 2002. The probit regression results using stepwise variable selection are listed in Tables 2 and 3 in Appendix D.

Based on the results presented in Tables 2 and 3, the fitted model is

\[
\text{Score} = \begin{cases} 
0 & \text{if } \hat{y}_i < 2.846 , \\
1 & \text{if } 2.846 < \hat{y}_i < 5.056 , \\
2 & \text{if } \hat{y}_i > 5.056 , 
\end{cases}
\]  

(6)

and

\[
\hat{y}_i = -12.737x_1 + 325.8x_2 - 1.646x_3 - 93.268x_4
\]

where \( x_1 \) is the denoised HSI return, \( x_2 \) is the (HSI MLE – 0.2), \( x_3 \) is the yield spread, \( x_4 \) is the lag 1 of standard deviation of MLE of forward points. Note that only four variables are selected but two of these involve a MLE measure. To help us better understand the probit regression model some descriptive statistics of these four variables under the three crisis events 0, 1 and 2 are reported in Table 4 in Appendix D. From Table 4, it can be seen that the four explanatory variables have different average values under different crisis events. Perhaps not surprisingly, the denoised HSI return assumes more negative values under events 1 and 2.
on the average while values of \((\text{HSI MLE} - 0.2)^2\) during crisis are on the average twice that of their value under no crisis. The average yield spread is less negative while the average lag 1 standard deviation of the MLE of forward points has a smaller value during economic and financial crisis. What is more interesting is that all the maxima are achieved under event 0, that is, when there is no crisis. On the other hand, three of the minima, those associated with HSI return, yield spread and forward points, are associated with either event 1 or 2. This strongly suggests that the four variables contain some valuable information about the state of the financial market and the probit regression makes use to certain extent such information contained in these variables.

Based on the above fitted model, we can treat it as a classification rule and predict the score for the data in the model construction period and forecasting period. The results are summarized in Tables 5 and 6 in Appendix D.

Note that two of the score 1 events as presented in Table 6 (Appendix D) were classified as score 2 events. The mis-classification is much improved if these two types of events are combined under the same label of financial crisis. In this regard, only one score 1 event would be misclassified as score 0. The performance of the model seems to be very satisfactory from this point of view.

Construction of the Indicator

We can rewrite the model and define the indicator as \(I = \Phi(Z)\) where \(Z = y_i - \alpha_i\), and \(\Phi(\cdot)\) is the cumulative distribution function of a standard normal \(N(0,1)\) variable. According to the above classification rule, we obtain the classification rule in terms of this new indicator:

\[
\text{Score} = \begin{cases} 
0 & \text{if } \Phi(Z) < 0.5, \\
1 & \text{if } 0.5 < \Phi(Z) < 0.986, \\
2 & \text{if } 0.986 < \Phi(Z) < 1.
\end{cases}
\] (7)

In fact, \(I = \Phi(Z)\) is the predicted probability of obtaining an extreme event (1 or 2) in a month. The time plot of \(I\) is given in Figure 2 in Appendix C.

Contribution of Individual Predictor to the Indicator

It is of interest to consider the contributions of individual predictors. The results for some selected months are summarized by Figures 3a – 3v in Appendix C. In this connection, it is of interest to note the following results.

Mean of the denoised HSI Return in normal month = 0.157;
Mean of the \((\text{HSI MLE} - 0.2)^2\) in normal month = 0.003;
Mean of the yield spread in normal month = −0.326;
Mean of the lag 1 of monthly standard deviation of MLE of forward points in normal month = 0.044.

The relative strength of predictor \(i\) is defined as

\[Z - x_i \hat{\beta}_i + \text{mean of } x_i \text{ in normal month} \cdot \hat{\beta}_i,\]
The figures are self-explanatory. For example, for December-1994 (Figure 3a) both HSI denoised return and the volatility of its MLE defined above contribute about equally to the indicator. The yield spread is an important factor in August-1998 (Figure 3k) while the others are of less impact.

**Study of the Robustness of the Selected Model**

To study the robustness of the selected model over time, we divide the data (Dec 1994 – May 2002) into two sets: training data set and testing data set. The testing data set is formed by randomly selecting 12 months from the data and the rest forms the training data set. The training data set is used to estimate the parameters of the selected model and the classification rule is used to forecast the scores in the testing set. The whole process is repeated 9 more times, resulting in 10 pairs of training data sets and testing data sets. The prediction performance of the selected model is summarized in the tables 7 and 8 in Appendix D. The performance of the indicator in fact improves under this scrutiny. This suggests that the original testing period of May 2001 – May 2002 represents an unfavorable situation to the proposed methodology.

**Conclusion**

We investigate the possibility of creating a useful economic indicator for the vulnerability/health of the financial sector of Hong Kong based on the belief that even a concurrent indicator would be valuable as the determination of a crisis event is often subjective, and the existence of such an indicator would provide an objective criterion in classifying events. Based on the results of Section 4 it seems that a reasonable indicator for financial crisis of Hong Kong can be based on the indicator as defined in Section 4.2 using three-month yield spread and a volatility measure of the MLE of the three-month forward points plus the denoised HSI return and a corresponding volatility measure of its MLE. The out-sample forecast performance is very reasonable especially if both score 1 and 2 events were combined under one single category in which case there would be only one misclassification. This research seems to be able to shed some light on the information carried by the maximum Lyapunov exponent (MLE) for the time series concerned. The proposed indicator may be further tested using future financial data and events. This research should be seen as a step forward in creating a vulnerability/health indicator for the financial sector. For possible extensions to the entire economy much longer macro-economic time series will be required than as is currently available.

**Acknowledgement**

We appreciate the excellent comments of the anonymous reviewers. Their contribution to the improvement of the paper has been considerable. We, of course, are responsible for any errors that remain in the paper.

This project was supported by the Hong Kong Monetary Authority. The authors would like to thank Ms Julia Leung, Mr. Sunny Yung, Miss Ivy Yong and Mr. Kelvin Chow of the Hong
Kong Monetary Authority for their help and support in carrying out this research. The authors thank Mr. Tony Wong and Dr P. Xu for their helps in computation.

This research was partially supported by the Hong Kong Institute of Economics and Business which is funded by a grant from the University Grants Committee of the Hong Kong Special Administrative Region, China. (Project No. AoE/H-05/99)

Footnotes

* Chair Professor, Department of Statistics and Actuarial Science, University of Hong Kong
** Associate Professor, Department of Statistics and Actuarial Science, University of Hong Kong
*** Associate Professor, School of Economics and Finance, University of Hong Kong

For Correspondence: K. S. Maurice Tse, School of Economics and Finance, University of Hong Kong, Hong Kong. E-mail: kts@econ.hku.hk and Office: (852) 2857-8636.

1. The data used are time series data for Hang Seng Index returns, the forward rates between Hong Kong dollar and US dollar, and the yield spread between 3-month U.S. Treasury yield and the 3-month Hong Kong Bond yield from December 94 to April 2002.

2. The Hang Seng Index (HSI) rate of return is defined as the first order difference of the log of the time series.

3. These variables and functions are listed in Table 1 in Appendix D.

References


Appendix A1

Original and denoised time series for the Hang Seng Index returns, the forward rates between Hong Kong dollar and US dollar, the yield spread between 3-month U.S. Treasury yield and the 3-month Hong Kong Bond yield, for the period from December 94 through April 2002.

1. HSI Return, US Yield (3-Month) Rate – HK Bond Yield (3-Month) Charts
Figure (iv) Monthly Average Denoised Hang Seng Index Return

Figure (v) Daily Forward Points (3 Months)

Figure (vi) Daily Denoised Forward Points (3 Months)
Figure (x) Daily Denoised Yield Spread (3 Months)

Figure (xi) Monthly Average Yield Spread (3 Months)

Figure (xii) Monthly Average Denoised Yield Spread (3 Months)
Appendix A2: Denoised HSI Return

Figure (xiii) Monthly Average HSI MLE

Figure (xiv) (Monthly Average HSI MLE - 0.2)^2

Figure (xv) Monthly Standard Deviation of HSI MLE
Appendix B

Chronological list of shocks to the financial market in Hong Kong

Feb-Mar, 1980
Continuing the rising trend which started in the second half of 1979, the Hang Seng index passed the 900 mark on 24th January and, apart from occasional setbacks as investors realized their profits, rose almost continuously to 963 on 15th February, just before the Lunar New Year holidays. After the holidays, the index fell almost continuously by 224 points to 739 on 19th March. The principal factors behind the fall were fears of an imminent increase in interest rates following the succession of increases in US prime rates, and reports of an ICAC investigation into some local property companies.

Feb-Mar 1981
The Hang Seng Index continued the rise which had begun in the second half of 1980 and reached 1651 on 4th February, just before the Lunar New Year holidays, in response to the decline in US interest rates and to the generally favourable results announced by leading companies. After the holidays, the trend was reversed and the index fell to 1295 by 11th March with reduced turnover.

Jul-Sep, 1981
After rising to 1810 on 17th July, the Hang Seng Index fell almost continuously and particularly sharply in the latter part of September to 1218, before recovering to 1280 at the end of the month. The decline during the third quarter was 26.2%. The abrupt change in July reflected growing concern from local and overseas investors about the pressure on the exchange rate of the Hong Kong dollar, about higher levels of local interest rates, about the likely impact on investors of a series of rights issues and of new issues and about unfavourable developments in international stock markets generally.

Feb-Mar, 1982
The Hang Seng Index drifted between 1370 and 1450 in quiet trading until early February, when the market began to be affected by the continuing high levels of US interest rates, the sluggishness of overseas economies and the depressed domestic property market. Consequently, the index fell sharply and almost continuously to 1125 on 8th March.

Jul-Aug, 1982
The Hang Seng Index, after rising from 1279 at the end of June to 1313 in mid-July, fell sharply and almost continuously to reach 937 on 16th August. The fall mainly reflected increasing anxiety over the political future of Hong Kong and over the depressed state of both the domestic and overseas economies.

Oct-Dec, 1982
This was exacerbated by worries developing in the fourth quarter of the year over the political future of Hong Kong, followed by confirmation that a few property and deposit-taking companies were in financial difficulties. The index reached a low of 676 on 2 December, 54% lower than the high point of 1472 on 29 January.

May-Jun, 1983
The Hang Seng Index rose from about 760 in early January to 1067 on 14 April. The rally was reportedly brought about by overseas institutional buying and encouragement from Wall
Street’s performance. However, a large part of the gain was eroded in May and early June, with the index falling back to 863 on 9 June. The retreat was attributable in part to the depreciation of the exchange value of the Hong Kong dollar which coincided with overseas selling, and in part to the three percentage points increase in interest rates during the second quarter.

Sep, 1983
The Hang Seng index rose from 964 at the end of June to 1103 on 21 July, the highest so far this year. The market then drifted steadily downwards. Much of the fall was attributable to mixed corporate results, the renewed strength of the US dollar and concern over the future of Hong Kong. The Hang Seng index plunged 74 points to 842 on 19 September, the biggest one day fall since October 1982. It fell further to finish at 758 on 30 September.

Mar-Jul, 1984
The Hang Seng Index advanced progressively from about 870 in early January to 1170 on 19 March. However, upon renewed concern over the future of Hong Kong, which was first sparked off at the end of March by certain company announcements which were interpreted as showing a lack of confidence in the economy’s long term growth potential, the index slipped back sharply in the following months to touch 746 on 13 July.

May-Jun, 1985
The inflow of capital largely of south east Asian origin, the two reductions in local interest rates in January, the various take over bids and the favourable land sales results in February were among the other bullish factors. Whilst subsequent disclosure of poorer than expected corporate March, the successful floatation of new shares by the Furama Hotel, further reductions in interest rates, indications of a recovery in the property market and news of mortgage rate cuts by major banks again boosted sentiment and pushed the Hang Seng Index through the 1600 mark on 6 May, touching 1648 on 17 May. The index subsequently fell back by over 200 points on profit-taking and on the influence of the OTB affair.

Feb-Mar, 1986
Continuing the rising trend in 1985, the Hang Seng Index reached a high of 1827 on 8 January before falling to 1560 in mid-March. The falling US dollar, which dragged down the value of the Hong Kong dollar, and rallying securities markets in New York, Tokyo and London were believed to be the unloaded their Hong Kong stock holdings and diverted funds overseas.

Apr, 1986
The exchange rate of Hong Kong dollar against the US dollar was generally stable, and Hong Kong dollar interest rates were close to US dollar interest rates. Except for a short period in late April when the large demand for Hong Kong dollars arising from the floatation of the Cathay Pacific share drove the exchange rate to levels below HK$7.77 per US dollar, the exchange rate of the Hong Kong dollar moved within a narrow range around the linked rate of HK$7.8 per US dollar.

Aug, 1986
In August, unfounded speculation that the United States authorities might initiate currency talks with Hong Kong generated much upward pressure on the market exchange rate. The
pressure subsequently subsided as the Government reaffirmed its policy of keeping the linked rate unchanged.

**Jan, 1987**
In January 1987, speculative pressures on the linked exchange rate system led to a strengthening of the exchange rate of the Hong Kong dollar against the US dollar and a fall in the Hong Kong dollar interest rates. The differential between the Hong Kong dollar and US dollar interest rates widened in favour of the latter as a result. As these speculative pressures subsided, local money market interest rates rebounded in February and March and interest rate differential was reduced significantly. Notwithstanding these speculative pressures, the market exchange rate of Hong Kong dollar against the US dollar moved for most of the time within 0.2% of the link rate during the first quarter of 1987, with a maximum deviation of only 0.6%.

**Sep, 1987**
During most of the third quarter, the market exchange rate of Hong Kong dollar against the US dollar stayed close to the linked rate of HK$7.80 to US$1, except for a temporary strengthening in early September due to speculation that the local currency would be revalued. The speculative pressure quickly subsided after the Government reaffirmed its determination to maintain the linked rate and after local money market interest rates fell.

**Oct-Dec, 1987**
After standing at 2568 at the end of 1986, The Hang Seng Index broke the 3000 mark in early June and reached a record high of 3950 on 1 October. However, caught in a worldwide collapse of equity markets triggered off by the crash in Wall Street, the Hang Seng Index dropped by 421 points on 19 October. The Stock Exchange subsequently suspended trading for four days. When it re-opened on 26 October, the Index fell by an historic 1121 points, to 2242. Thereafter, the movements in the Hang Seng Index continued to be influenced by market uncertainties both local and overseas. The Index reached 1895, the lowest level for the year, on 7 December.

**Jan-Mar, 1988**
Under the linked exchange rate system, the market exchange rate of the Hong Kong dollar against the US dollar moved within a narrow margin close to the linked rate. Speculative pressures on a revaluation of the Hong Kong dollar, which pushed the market exchange rate above the linked rate above linked rate at the beginning of 1988, had subsided by the end of the first quarter.

**Aug-Sep, 1988**
Share prices remained relative stable in July but were generally on a downward trend since early August. The weak performance in the major stock market overseas triggered off a fall in the local stock prices, which was sustained into September. The Hang Seng Index moved between 2650 and 2770 throughout July and in early August, but fell to a low of 2423 on 20 September, before closing the month at 2441.

**May-Jun, 1989**
In Step with rallies in stock markets overseas, local stock prices were generally on an uptrend between end-1988 and mid-May 1989. The Hang Seng Index reached a post-crash high of 3310 on 15 May. However, increased uncertainties brought about by events in China led to
substantial volatility in the local stock market since then. The imposition of martial law in parts of Beijing triggered off a panic selling of stocks on 22 May, with Hang Seng Index falling by 339 points. The crackdown in Beijing led to another round of panic selling on 5 June, with the Hang Seng Index losing another 583 points, as order in Hang Seng Index closed the second quarter at 2274.

Aug-Sep, 1990
Continuing the upward trend in the first half of 1990, local share prices rose substantially during July. They then declined sharply in August, in line with the situation in major stock markets overseas following the outbreak of the Gulf crisis, and remained at a relative low level for the rest of the quarter. The extent of downward adjustment was, however, smaller than in many of the stock markets in the region. The Hang Seng Index rose from 3278 at the end of June to 3560 on 23 July, the highest level recorded since the stock market crash in October 1987, before declining to 2871 on 23 August. It subsequently recovered to 3105 on 12 September, before easing back to close the quarter at 2761.

Jul-Aug, 1992
Continuing the upward trend established during the first two quarters in 1992 and partly stimulated by the disclosure of the impressive size of Exchange Fund on 15 July, the local stock market rallied with the Hang Seng Index rising to an unprecedented high of 6163 on 16 July. However, the market began to consolidate following the plunge in Tokyo stock prices at end-July. Concern over the Section 301 negotiations between the United States and China and the relations between the two countries generally added uncertainty to the market. As a result, the Hang Seng Index dropped to a low of 5291 on 25 August.

Nov-Dec, 1992
The Hang Seng Index reached a record high of 6447 on 12 November, following satisfactory resolution of the issue between the United States and China on Section 301 of the US Trade Act. Political jitters sent the Hang Seng Index down to a low of 4978 in early December.

Jan-Mar, 1994
The Hang Seng Index, after reaching a new record high of 12201 in early January, consolidated thereafter notwithstanding the announcement of good corporate results. The threat of an immediate rise in interest rates coupled with profit-taking by investors contributed to this correction. Nevertheless, this was generally in line with the correction seen in a number of stock markets elsewhere. The index closed the first quarter at 9030, with a 24% drop during the quarter.

Nov-Dec, 1994
The Hang Seng Index touched a high of 10166 on 7 September. Renewed concern over further interest rate hikes in United States and a sharp correction in some of the major stock markets overseas, however, sparked another round of selling in the fourth quarter. The Hang Seng Index finally closed the year at 8191.

Jan, 1995
In early January 1995, following the sharp depreciation of the Mexican peso, some overseas fund managers began to trim their holdings of most foreign assets, including those in Lain America and in East Asia. While the bulk of the capital outflow was associated with portfolio fund repatriation, there were signs of speculative selling pressure on several Asian currencies,
including the Hong Kong dollar, taking advantage of bearish sentiment in the Asian financial markets. The Hong Kong Monetary Authority (HKMA) began tightening liquidity in the interbank market on 12 January. The substantial rise in the overnight interbank rate, from under 6% to as high as 12% on 13 January, imposed a high funding cost on those speculators shorting the Hong Kong dollar.

In January, the market was subject to unfavourable factors such as expectations for a further rise in US dollar interest rate, downtrend in property prices which affected the property sector’s profit outlook, and less-than-satisfactory results of the Government land auction. The Hang Seng Index dropped from 8191 at end-1994 to below the 7000 mark in late January.

Mar, 1996
As the prospect for a further cut US interest rates was reduced, the Dow Jones Industrial Average Index fell markedly on 8 March. This triggered a correction in the major stock markets world-wide, including the Hong Kong stock market. The Hang Seng Index underwent a major correction on 11 March. The growing tension between China and Taiwan at that time also contributed to the correction.

Aug, 1997
From the mid-August onwards, the market plummeted sharply, along with a sharp plunge in many of the stock markets in East Asia and amidst concern over speculative attacks on the Hong Kong dollar. Signs of correction in the New York stock market also depressed sentiment. On 1 September, the Hang Seng Index fell to a low of 13426, 3248 points below the preceding peak.

Oct, 1997
As the East Asian currency turmoil intensified, the stock market faced another spate of downslide in late October, triggered by a new bout of speculative attack on Hong Kong dollar. Market sentiment was dampened considerably by the increase in local interest rates as well as the further sharp declines in share prices in many of the regional stock markets. On 28 October, the Hang Seng Index plummeted to a low of 9060, 46% lower than the zenith attained on 7 August.

Jan, 1998
The local stock market underwent another marked correction in early January 1998, triggered by renewed instability in the regional financial markets and a further rise in local interest rates. The Hang Seng Index fell to a three-year low of 8121 on 12 January 2602 points or 24% lower than at end-1997

Apr-Jun, 1998
The local stock market then consolidated in April, and plunged more sharply again during May and June, as the regional financial markets were hit again by the social unrest in Indonesia, the protracted economic and financial problems in Japan, and weak Japanese yen. Locally, the distinct slackening in economic activity and rising unemployment also dampened market sentiment. The Hang Seng Index dropped to a three-year low of 7643 on 15 June, but recouped some of the loss along with a rebound of the yen in the latter part of the month. The Index closed the second quarter of 1998 at 8543, representing a decline of 2976 points or 26% from three months earlier.
Jul-Aug, 1998
The local stock market underwent a further setback in July, overshadowed by continuing concern over the knock-on effects of the weak Japanese yen and the severe economic and financial problems in Japan. Selling pressure on local stocks intensified in the first two weeks of August, upon the marked correction in the US stock market, another spate of currency turbulence in the region triggered mainly by the further depreciation of the Japanese yen, and heavy speculative attack on the local financial markets. This was aggravated by the build-up of very large short positions on Hang Seng Index futures. There was clear evidence of manipulation involving double-play in currency and money markets on the one hand, and the stock futures markets on the other, The Hang Seng Index plummeted to a five-year low of 6660 on 13 August. To tackle such manipulation, the Government initiated decisive counter measures with a view to preserving market integrity and stability, and buttressing the Hong Kong dollar link. As a result, the stock market tended to stabilise and the Hang Seng Index rebounded to towards the end August.

Apr-May, 2000
The local stock market was buoyant during the early part of 2000, with Hang Seng Index reaching a record high of 18302 on 28 March. The market the consolidated in April and May, amidst concern over further US interest rate hike and renewed tension across the Taiwan Strait, with the Hang Seng Index falling to a low of 13723 on 28 May.

Sep, 2000
The Hang Seng Index rallied to 17921 on 21 July, and stayed above 17000 level for most of August. Then the upsurge in international oil prices took a toll on the major stock markets worldwide, due to concern about its repercussions on inflation, interest rates and global economic growth. The local stock market was likewise hit, with the Hang Seng Index falling to 14613 on 22 September.

Feb-Mar, 2001
The local stock market bounced back at the beginning of 2001, upon an unexpected cut in US interest rates on 3 January. Expectations for further interest rate cuts pushed the Hang Seng Index to a high of 16 164 on 1 February. Yet the market lost momentum soon after, along with the slides in the US and Tokyo stock markets, the heavy plunge in the telecom and technology shares worldwide, and also the setback in financial shares. The Hang Seng Index fell back to a 17-month low of 12 586 on 23 March.

Jul-Sep, 2001
The local stock market took a sharp plunge during the third quarter of 2001, after some rebounded in the second quarter. The gloomy global economic outlook amidst wary over a protracted US economic slow-down, and the successive waves of US corporate profits warnings, pushed down share prices worldwide. The local stock market was likewise hit. Profit-taking on Mainland-related shares after and earlier upsurge in prices of such shares dampened the market more. The further interest rate cut in August did not render any noticeable stimulus to market sentiment. Then the 911 incident aroused widespread anxieties in the global financial markets, causing the Hang Seng Index to plummet to a near three-year low of 8934 on 21 September, 14% dow from the pre-911 closing level.
Jan-2002
The local stock market staged a brief rebound at the beginning of 2002, stimulated in part by a price surge in technology-related shares in the United States. The Hang Seng Index rose to a five-month high of 11 893 on 7 January. The market subsequently drifted lower, with the Hang Seng Index mostly hovering in the 10 500 – 11 000 range for the rest of January and in February in relatively quiet trading.
Appendix C

Figure 1
Time plot of the crisis indicator variable with Score defined as follows:
Score = 2: extreme events provided by HKMA (except the 911 event).
Score = 1: extreme events suggested by us.
Score = 0: otherwise.

Figure 2
Predicted probability of an extreme event (score = 1 or 2) based on models as represented by
equations (6) and (7).
Figure 3: Relative Strength Index of the individual variables

Notes:
1. Denoised HSI Return
2. \((\text{HSI MLE} - 0.2)^2\)
3. Yield spread
4. Lag 1 of monthly standard deviation of MLE of forward points
5. 

![Figure 3a](Dec-1994)

![Figure 3b](Jan-1995)

![Figure 3c](Mar-1996)

![Figure 3d](Aug-1997)

![Figure 3e](Oct-1997)

![Figure 3f](Jan-1998)
Figures 3g to 3l show the relative strength for different months. Each figure displays a bar chart with observed and predicted events, along with their respective probabilities.

- **Apr-1998**: Relative strength chart with $Pr = 0.818$, observed event = 2, predicted event = 1.
- **May-1998**: Relative strength chart with $Pr = 1.000$, observed event = 2, predicted event = 2.
- **Jun-1998**: Relative strength chart with $Pr = 1.000$, observed event = 2, predicted event = 2.
- **Jul-1998**: Relative strength chart with $Pr = 1.000$, observed event = 2, predicted event = 2.
- **Aug-1998**: Relative strength chart with $Pr = 1.000$, observed event = 2, predicted event = 2.
- **Dec-1998**: Relative strength chart with $Pr = 0.481$, observed event = 0, predicted event = 0.
Relative strength

Pr = 0.822, Observed event = 0, Predicted event = 1

Jan-1999

Figure 3m

Relative strength

Pr = 0.992, Observed event = 1, Predicted event = 2

Apr-2000

Figure 3n

Relative strength

Pr = 0.429, Observed event = 1, Predicted event = 0

May-2000

Figure 3o

Relative strength

Pr = 0.509, Observed event = 1, Predicted event = 1

Sep-2000

Figure 3p

Relative strength

Pr = 0.837, Observed event = 1, Predicted event = 1

Feb-2001

Figure 3q

Relative strength

Pr = 0.901, Observed event = 1, Predicted event = 1

Mar-2001

Figure 3r
Figure 3s

Figure 3t

Figure 3u

Figure 3v
### Appendix D

**Table 1. Original Data**

<table>
<thead>
<tr>
<th></th>
<th>HSI Return</th>
<th>Yield Spread (3 Months)</th>
<th>Forward points (3 Months)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly average</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Lag 1 of monthly average</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
</tbody>
</table>

**Denoised Data**

<table>
<thead>
<tr>
<th></th>
<th>HSI Return</th>
<th>Yield Spread (3 Months)</th>
<th>Forward points (3 Months)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Denoised monthly average</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Monthly average of MLE</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Monthly standard deviation of MLE</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>(MLE – 0.2)²</td>
<td>√</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>(MLE – 0.13)²</td>
<td>–</td>
<td>√</td>
<td>–</td>
</tr>
<tr>
<td>(MLE – 0.14)²</td>
<td>–</td>
<td>–</td>
<td>√</td>
</tr>
<tr>
<td>Range of MLE</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Lag 1 of denoised monthly average</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Lag 1 of monthly average of MLE</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Lag 1 of monthly standard deviation of MLE</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Lag 1 of (MLE – 0.2)²</td>
<td>√</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Lag 1 of (MLE – 0.13)²</td>
<td>–</td>
<td>√</td>
<td>–</td>
</tr>
<tr>
<td>Lag 1 of (MLE – 0.14)²</td>
<td>–</td>
<td>–</td>
<td>√</td>
</tr>
<tr>
<td>Lag 1 of Range of MLE</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
</tbody>
</table>

The symbol √ indicates potential predictor.
Table 2. Summary of Variable Selection

<table>
<thead>
<tr>
<th>Input Variables</th>
<th>$R^2$</th>
<th>Max-rescaled R-square</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Step 1</strong> Denoised HSI Return</td>
<td>0.432</td>
<td>0.595</td>
</tr>
<tr>
<td><strong>Step 2</strong> Yield Spread</td>
<td>0.524</td>
<td>0.721</td>
</tr>
<tr>
<td><strong>Step 3</strong> $(\text{HSI MLE} - 0.2)^2$</td>
<td>0.576</td>
<td>0.793</td>
</tr>
<tr>
<td><strong>Step 4</strong> Lag 1 of standard deviation of MLE of forward points</td>
<td>0.646</td>
<td>0.889</td>
</tr>
</tbody>
</table>

Remarks:

$$R^2 = 1 - \left( \frac{L(0)}{L(\hat{\beta})} \right)^{\frac{2}{n}}$$ and Max - rescaled R - square = \( \frac{R^2}{1 - \{L(0)\}^\frac{2}{n}} \),

where $L(0)$ is the likelihood of the intercept-only model, $L(\hat{\beta})$ is the likelihood of the specified model, and $n$ is the sample size. For both measures a higher value means a better fit. Note that the range of max-rescaled R-square covers the entire unit interval but not the range of $R^2$. 
Table 3. Maximum Likelihood Estimates of the Parameters of the Selected Model

<table>
<thead>
<tr>
<th>Variables</th>
<th>Estimate</th>
<th>Standard Error</th>
<th>P-value</th>
<th>Standardized Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept 2</td>
<td>5.056</td>
<td>1.392</td>
<td>0.0003</td>
<td></td>
</tr>
<tr>
<td>Intercept 1</td>
<td>2.846</td>
<td>1.124</td>
<td>0.0113</td>
<td></td>
</tr>
<tr>
<td>Denoised HSI Return</td>
<td>–12.737</td>
<td>2.504</td>
<td>&lt; 0.0001</td>
<td>–5.65</td>
</tr>
<tr>
<td>(HSI MLE – 0.2)^2</td>
<td>325.800</td>
<td>100.600</td>
<td>0.0012</td>
<td>1.36</td>
</tr>
<tr>
<td>Yield Spread</td>
<td>–1.646</td>
<td>0.453</td>
<td>0.0003</td>
<td>–1.94</td>
</tr>
<tr>
<td>Lag 1 of standard deviation of MLE of forward points</td>
<td>–93.268</td>
<td>30.750</td>
<td>0.0024</td>
<td>–2.23</td>
</tr>
</tbody>
</table>
Table 4. Descriptive statistics of these four variables

<table>
<thead>
<tr>
<th>Overall</th>
<th>Average</th>
<th>Std.</th>
<th>Max.</th>
<th>Min.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Denoised HSI Return</td>
<td>0.0142</td>
<td>0.4275</td>
<td>1.4079</td>
<td>-1.6347</td>
</tr>
<tr>
<td>((\text{HSI MLE} - 0.2)^2)</td>
<td>0.0032</td>
<td>0.0043</td>
<td>0.0218</td>
<td>0.0000</td>
</tr>
<tr>
<td>Yield Spread</td>
<td>-0.5071</td>
<td>1.1206</td>
<td>0.6927</td>
<td>-4.7596</td>
</tr>
<tr>
<td>Lag 1 of standard deviation of MLE of forward points</td>
<td>0.0414</td>
<td>0.0231</td>
<td>0.1398</td>
<td>0.0109</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Average</th>
<th>Event 0</th>
<th>Event 1</th>
<th>Event 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Denoised HSI Return</td>
<td>0.1570</td>
<td>-0.3996</td>
<td>-0.5908</td>
</tr>
<tr>
<td>((\text{HSI MLE} - 0.2)^2)</td>
<td>0.0026</td>
<td>0.0053</td>
<td>0.0047</td>
</tr>
<tr>
<td>Yield Spread</td>
<td>-0.3264</td>
<td>-0.0279</td>
<td>-2.4977</td>
</tr>
<tr>
<td>Lag 1 of standard deviation of MLE of forward points</td>
<td>0.0436</td>
<td>0.0315</td>
<td>0.0367</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Maximum</th>
<th>Event 0</th>
<th>Event 1</th>
<th>Event 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Denoised HSI Return</td>
<td>1.4079</td>
<td>-0.0965</td>
<td>-0.1226</td>
</tr>
<tr>
<td>((\text{HSI MLE} - 0.2)^2)</td>
<td>0.0218</td>
<td>0.0168</td>
<td>0.0148</td>
</tr>
<tr>
<td>Yield Spread</td>
<td>0.6927</td>
<td>0.1730</td>
<td>-0.4892</td>
</tr>
<tr>
<td>Lag 1 of standard deviation of MLE of forward points</td>
<td>0.1398</td>
<td>0.0504</td>
<td>0.0591</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Minimum</th>
<th>Event 0</th>
<th>Event 1</th>
<th>Event 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Denoised HSI Return</td>
<td>-0.4827</td>
<td>-0.7227</td>
<td>-1.6347</td>
</tr>
<tr>
<td>((\text{HSI MLE} - 0.2)^2)</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0001</td>
</tr>
<tr>
<td>Yield Spread</td>
<td>-3.8425</td>
<td>-0.6831</td>
<td>-4.7596</td>
</tr>
<tr>
<td>Lag 1 of standard deviation of MLE of forward points</td>
<td>0.0157</td>
<td>0.0109</td>
<td>0.0206</td>
</tr>
</tbody>
</table>
Table 5. Performance for the data in the model construction period
(December 1994 – May 2001)

<table>
<thead>
<tr>
<th>Observed events</th>
<th>Predicted events</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>61*</td>
<td>0</td>
<td>0</td>
<td>62</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>4*</td>
<td>0</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1</td>
<td>8*</td>
<td>9</td>
<td>7</td>
</tr>
<tr>
<td>Total</td>
<td>63</td>
<td>6</td>
<td>9</td>
<td>78</td>
<td></td>
</tr>
</tbody>
</table>

The asterisk * indicates correct classification. Proportion of correct classification = 73/78 = 93.59%

Table 6. Performance for the data in the forecasting period

<table>
<thead>
<tr>
<th>Observed events</th>
<th>Predicted events</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>8*</td>
<td>0</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1*</td>
<td>2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0*</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>9</td>
<td>1</td>
<td>2</td>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>

The asterisk * indicates correct classification. Proportion of correct classification = 9/12 = 75.00%

Table 7. Performance on training data sets

<table>
<thead>
<tr>
<th>Observed events</th>
<th>Predicted events</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>59.4*</td>
<td>2.0</td>
<td>0</td>
<td>61.7</td>
</tr>
<tr>
<td>1</td>
<td>2.3</td>
<td>5.6*</td>
<td>1.1</td>
<td>9</td>
<td>9.0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0.8</td>
<td>7.0*</td>
<td>7.8</td>
<td>7.8</td>
</tr>
<tr>
<td>Total</td>
<td>61.7</td>
<td>8.2</td>
<td>8.1</td>
<td>78</td>
<td></td>
</tr>
</tbody>
</table>

The asterisk * indicates correct classification. Proportion of correct classification = 72/78 = 92.31%

Table 8. Performance on testing data sets

<table>
<thead>
<tr>
<th>Observed events</th>
<th>Predicted events</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>8.7*</td>
<td>0.1</td>
<td>0</td>
<td>8.8</td>
</tr>
<tr>
<td>1</td>
<td>0.3</td>
<td>1.6*</td>
<td>0.1</td>
<td>2</td>
<td>2.0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0.3</td>
<td>0.9*</td>
<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td>Total</td>
<td>9.0</td>
<td>2.0</td>
<td>1.0</td>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>

Note to Table 8: The number in each cell represents the average frequency count on 10 training/testing data sets. The asterisk * indicates correct classification. Proportion of correct classification = 11.2/12 = 93.33%.