Implications of Parameter Estimation Uncertainty for the Central Banker Behaviour*

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Abstract
This paper studies the implications of parameter estimation uncertainty on the central banker behaviour. It first describes the optimal monetary policy rule obtained by the linear quadratic stochastic control approach. The treatment of parameter estimation uncertainty is covered by the introduction of the full variance-covariance matrix of the parameter estimates in the optimal control theory. It then examines how this rule ought to be modified according to the preferences of the monetary authorities when there is uncertainty about the estimation of the persistence and/or transmission parameters. Our application to the euro area shows that (i) the conservatism principle is found relevant only when the central banker has an inflation and output stabilization objective, (ii) introducing an interest rate smoothing objective makes the central banker more aggressive, (ii) with conventional values for the weights of the loss function, there are few differences between the parameters of the rule under certainty-equivalence and those under uncertainty.

Keywords: European monetary policy, parameter estimation uncertainty, linear quadratic stochastic control

JEL Classifications: C61, D81, E58

Introduction
Uncertainty is a characteristic of the real world that plays an important part in the decision-making process of all economic agents. As observers and experts recognise unanimously, this uncertainty applies to central banks that need to take policy decision in an environment of considerable uncertainty regarding current and future economic conditions and the functioning of the economy. Much of the uncertainty facing monetary policymakers is unavoidable. In this regard, the situation of the European Central Bank (ECB) is by no means unique, although the specific features of the euro area create additional challenges. The fact that monetary policy may impact national economies differently, the associated aggregation problem or the absence of historical data make an analysis of the euro area-wide transmission mechanism even more complicated. Since it is crucial that monetary policy itself does not become an independent source of additional uncertainty, understanding the implications for the ECB's monetary policy strategy is of very first importance. A key question confronting central banks, therefore, is how to conduct monetary policy under conditions of uncertainty.

Saying that the world is very far from the one in which market agents and policymakers operate is stating the obvious. A less obvious implication is that the structure of any monetary policy strategy must reflect the extent and the nature of the uncertainties faced by the central bank. In this study, I focus on the sources of predictable uncertainty - those of
which the degree of uncertainty can be anticipated and even calculated - and primarily on the uncertainty concerning the behaviour of economic agents. This is sometimes referred to as multiplicative or parameter uncertainty and can be intrinsic to the economy or due to econometric estimation. In contrast to uncertainty generated by additive shocks, when policymakers are uncertain on the key parameters that determine the transmission of monetary policy in the adopted structural model of the economy, aggressive policy moves are more likely to have unpredictable consequences on output and inflation, then gradual policy is optimal. This classic result, obtained by Brainard (1967), institutes the “conservatism principle”, and in some authors' view, is behind the interest rate smoothing behaviour often attributed to policymakers.

To test the veracity of the Brainard principle, a number of papers have used Bayesian methods associated with optimal control theory to determine the optimal monetary policy that minimizes the expected loss function, given a prior distribution on some uncertain parameters. This approach has recently been followed by Estrella and Mishkin (1999), Martin and Salmon (1999), Srour (1999), Svensson (1999), Rudebusch (2000) or Sack (2000) among others. These studies found Brainard's results to be empirically relevant. Two notable exceptions are Söderström (2002) and Shuey and Thompson (2000) who explained that a central banker may pursue a more aggressive monetary policy due to uncertainty about (i) inflation persistence for the first author and (ii) persistence of shocks for the second ones.

In general, it is true that uncertainty about the coefficient of a variable in the transmission mechanism will always induce the policymaker to try to minimize deviations in that variable. However, this does not necessarily translate into caution with regard to movements in the interest rate. Indeed, if there is uncertainty about all or most of the coefficients in the model, one cannot conclude a priori what this entails for the interest rate response. Whether uncertainty means moving interest rates to a greater or lesser degree depends on the relative uncertainties of the different coefficients and the structure and dynamics of the model.

This paper gives quantitative answers in a general environment and for the Euro area. First, it generalizes the Brainard model to an infinite-period horizon and a multivariate model. This allows studying the effects of uncertainty on the transmission and persistence parameters. Second, it specifies the largest preferences of the policymaker in including an interest rate smoothing objective in the loss function in addition to the traditional inflation and output stabilizations. Third, it uses all information available in the matrix of variance-covariance as a measure of uncertainty. Amman and Kendrick (2000) have found “evidence that the potential damage from ignoring the variances and covariances of the parameter estimates is substantial and that taking them into account can improve matters”. In fact, covariances may play a considerable role in the derivation of the reaction function of the monetary authorities, and since the loss function can be viewed as a quadratic approximation to the level of expected utility of a representative household, the policymaker must choose the rule that lowers this welfare criterion.

The outline of the remainder of the paper is as follows. We begin with a description of the general framework of the analysis. We then detail the extended optimal control methodology in presence of uncertainty. We then propose an empirical illustration for the Euro area, in order to measure the interactions between parameter uncertainty and the behaviour of the central banker. The simulation results and quantitative analysis of optimal policy are presented. Finally, conclusions are drawn.
The Central Bank Decision Problem

Overview

When taking monetary policy decisions, central banks face considerable uncertainty about transmission mechanism of monetary policy to the price level. In particular, the role played by monetary developments in the euro area transmission mechanism is not well understood. The problems arise from what the policymaker does not know: he must deal with the uncertainty from the base of what he knows. Especially, he must analyze whether uncertainty about the relationship between the economic variables in the economy could entail a slower, or smoother, policy response to shocks to the economy than otherwise.

This analysis, which follows a proposition first put forward by Brainard (1967), is based on the premise that uncertainty about the relationship between the interest rate and the rest of the economy creates trade-off for policymakers: the parameter estimation uncertainty may mean that movements in the interest rates themselves increase uncertainty about the future path of the economy. This could lead policymakers to use their policy instrument more cautiously in order to reduce the chance of missing the target significantly.

The central bank faces a dynamic optimal control problem whose solution describes its policy actions. These are the optimal response of monetary authorities to the evolution of the economy as captured by the relationships among the state variables. We describe such a dynamics by means of a simple closed-economy two-equation framework made up of a IS curve and a Phillips curve, which actually represent the constraints of the policymakers' optimisation problem.

Previous work on optimal control and uncertainty (especially those of Amman and Kendrick) considers the problem over finite planning horizon $T$, and provides no indication of how the length of $T$ should be determined. In that case, the resulting optimal policy as well as the final state of the system may vary substantially with $T$, depending on the controllability properties of the system. A central result of economic growth theory is the “turnpike” of capital accumulation essentially stating that for $T$ large enough, optimal growth is achieved by steering the system from the initial state to an optimal steady-state (or “turnpike”), and then leaving the turnpike towards the end of the horizon to achieve short-term unsustainable superior growth. For infinite-horizon problems a natural consequence of the turnpike property is global asymptotic convergence of the optimal trajectory, and such results have been provided under strong convexity assumptions on the Hamiltonian by Brock and Scheinkman (1976) and others. As a consequence of these results, it is natural to consider the problem of finding an optimal monetary policy rule over an infinite horizon, as this avoids unsustainable endpoint effects (for longer horizons) and variations of the optimal trajectory (for shorter horizons).

The Baseline Model

The empirical evidence from Vector AutoRegressive (VAR) studies shows that monetary policy affects the economy at different lags (Christiano, Eichenbaum and Evans, 1998 or Peersman and Smets, 2001). Furthermore, when the central bank faces an intertemporal optimization problem, forecasting the behaviour of the state variables becomes crucial to set policy as the optimal response to the developments in the economy. A typical model in this class is the one estimated by Rudebusch and Svensson (1999) who represent the model economy as follows:
\[ y_{t+1} = \alpha_1 y_t + \alpha_2 y_{t-1} - \beta (i_t - \pi_t) + \varepsilon_{y,t+1} \] (IS)  
\[ \pi_{t+1} = \delta_1 \pi_t + \delta_2 \pi_{t-1} + \delta_3 \pi_{t-2} + \delta_4 \pi_{t-3} + \gamma' + \varepsilon_{\pi,t+1} \] (Phillips)

where \( y_t \) is the output gap, \( \pi_t \) is the inflation rate, \( i_t \) is the instrument of monetary policy (here identified with the one-period nominal interest rate), and \( \varepsilon_{y,t+1} \) and \( \varepsilon_{\pi,t+1} \) are zero mean normally distributed shocks. All variables are demeaned, therefore no constants appear in the equations.

The first equation is an IS curve that explicitly models the monetary transmission mechanism by relating the output gap to its lagged values and most importantly to the difference between the short nominal interest rate and inflation, an approximate ex-post real interest rate. The second equation is a Phillips curve that captures the inflation dynamics by relating inflation to its lagged values and to current output gap. In our empirical investigation, we will not reject the hypothesis that the coefficients of the four inflation lags sum to one \( \left( \sum_{j=1}^{4} \delta_j = 1 \right) \), which implies a long-run vertical Phillips curve.

This empirical model of inflation and output, although parsimonious, embodies the minimal set of variables one may want to include for the analysis of monetary policy, and, as argued in Rudebusch and Svensson (1999), it appears to be broadly in line with the view that policymakers hold about dynamics of the economy. For the purpose of monetary policy making, which relies on forecasting method, a backward-looking model is likely to be preferred to a forward-looking one since the former over performs the latter in fitting the data (see Fuhrer, 1997). Notably, a backward-looking model does not show counterfactual dynamic responses to shocks hitting the economy, as those implied by the use of fully forward-looking models (Estrella and Fuhrer, 2002). Moreover, the main feature of this model is that the instrument of monetary policy affects output with one lag and inflation with two lags. This is roughly consistent with the empirical facts in the Euro area (see ECB, 2000 and 2002).

We can notice that this two-equation system may be easily written in a state-space form such as,

\[ x_{t+1} = Ax_t + B i_t + \varepsilon_{t+1} \] (1)

where \( x_t = (y_t, y_{t-1}, \pi_t, \pi_{t-1}, \pi_{t-2}, \pi_{t-3}, i_{t-1}) \) is the state vector, \( \varepsilon_{t+1} = (\varepsilon_{y,t+1}, \varepsilon_{\pi,t+1}) \) is the vector of shocks, and \( A \) and \( B \) are the matrices of the coefficients.

**The Optimal Interest Rate Rule**

We assume that monetary authorities operate by following a targeting rule as defined in Svensson (1999). Thus, they use all available information to bring at each point in time the target variables in line with their targets by penalizing any future deviation of the former from the latter.\(^4\) We let the central bank pursue the stabilization of the inflation around its target, the stabilization of output around its potential value and the smoothing of interest rate. The inflation target is assumed to be constant over time and it is normalised to zero because all variables are demeaned. Then, interest rates are set to minimize in each period \( t \) the following expected intertemporal loss function,
where in each period the loss function \( L(.) \) is given by

\[
L(y_t, \pi_t, \Delta i_t) = \pi_t^2 + \lambda_y y_t^2 + \lambda_i (\Delta i_t)^2
\]  

(3)

where \( \Delta i_t = i_t - i_{t-1} \).

The loss function (3) can be represented in a more compact form by defining the \( 3 \times 1 \) vector \( z_t \) of goal variables. This vector reads:

\[
z_t = Cx_t + Di_t
\]  

(4)

where the elements of (4) are given by:

\[
\begin{bmatrix}
\pi_t \\
y_t \\
\Delta i_t
\end{bmatrix}, \quad C = \begin{bmatrix}
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1
\end{bmatrix}, \quad D = \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}
\]

Accordingly, the loss function can be written as:

\[
L(y_t, \pi_t, \Delta i_t) = z_t'Qz_t
\]  

(5)

where \( 0 < \phi < 1 \) is the society's discount factor, the parameters \( \lambda_y \geq 0 \) and \( \lambda_i \geq 0 \) are the relative weights on output gap and interest rate smoothing with respect to inflation stabilization, and \( Q \) is a \( 3 \times 3 \) matrix reflecting the central bank preferences, with 1, \( \lambda_y \) and \( \lambda_i \) on the diagonal and zeros elsewhere. Specifically, it can be shown that as the discount factor approaches unity, the value of the intertemporal loss function becomes proportional to the unconditional expected value of the period loss function:

\[
E[L(y_t, \pi_t, \Delta i_t)] = \text{Var}(\pi_t) + \lambda_y \text{Var}(y_t) + \lambda_i \text{Var}(\Delta i_t) - \text{Var}(\Delta i_t)
\]  

The central bank's objective is set to a path for \( i_{t+h}, (h = 0, \ldots, \infty) \) so as to minimize function (2) subject to equations (IS) and (Phillips).

The quadratic objective function, the linear transition equation and the property \( E(\epsilon_{t+1}|x_t) = 0 \) are convenient forms for the stochastic optimal linear regulator problem. It follows that the feedback rule that solves the optimization problem is linear and independent from the errors covariance matrix as the certainty equivalence holds.

**Proposition 1** The feedback rule that solves the stochastic optimal linear regulator problem is given by:

\[
i_t =Fx_t
\]

where the vector of response coefficients is given by
\[ F = -\left[(D'QD) + \phi(B'VB)\right]^{-1}\left[(C'QD) + \phi(B'VA)\right], \]

and the matrix \( V \) is determined by the Riccati equation \( V = (C'QC) + \phi(A + BF)V(A + BF) + F'(D'QD)F + (C'QD)F + F'(C'QD) \)

**Proof.** See Ljungqvist and Sargent (2000, chap. 4).

Such a reaction function resembles a Taylor-type rule according to which monetary authorities set the interest rate in every period as the optimal response to movements in the current and lagged values of the state variables as well as lagged values of the interest rate itself.

### Dynamic Programming with Parameter Estimation Uncertainty

In the real world, monetary policy-making is undertaken in an uncertain environment. Indeed, the central banker does face a lack of information concerning the monetary transmission mechanism. Although the statistical inference upon model form and of parameter values from available data obviously entails some degree of arbitrariness and uncertainty, the previous theoretical expressions, generally used for deriving optimal policy, do not take into account these sources of approximation. In particular, no account of the effects of uncertainty in parameter estimates (which can be quite large) is commonly made.

Indeed, the introduction of parameter estimation uncertainty into the general framework will have an important effect on optimal policy. When incorporating this type of uncertainty, the certainty equivalence principle ceases to hold, and the error variances and covariances of the estimated state variables will affect the optimal rule.

The state-space formulation of the general model economy becomes

\[ x_{t+1} = A_{t+1}x_t + B_{t+1}i_t + \varepsilon_{t+1} \]  \hspace{1cm} (6)

We now suppose that we know the two first moments of the parameters and that parameter matrices \( A_t \) and \( B_t \) are then stochastic with means \( A \) and \( B \), variance matrices \( \Sigma_A \) and \( \Sigma_B \),

and covariance matrix \( \Sigma_{AB} \) (see Appendix A for the detailed forms of these matrices).\(^6\)

The policymaker faces the same control problem

\[ J(x_t) = \min_{\{z_t\}} \{z_t'Qz_t + \phi E_t J(x_{t+1})\} \]

but now, subject to

\[ x_{t+1} = A_{t+1}x_t + B_{t+1}i_t + \varepsilon_{t+1} \]

**Proposition 2** The solution is given by the following policy rule

\[ i_t = \tilde{F}x_t \]
where the vector of response coefficients depends not only on the parameter means but also on their variances and covariances, so that,

\[
\tilde{F} = - \left[ 2(D'QD) + \phi(B'\tilde{V} + \tilde{V}^rB) + 2\tilde{v}_1\Sigma_{1b} \right]^{-1} \\
\times \left[ 2(C'QD) + \phi(B'\tilde{V} + \tilde{V}^rA) + 2\tilde{v}_{11}\Sigma_{1b} + (\tilde{v}_{12} + \tilde{v}_{21})\Sigma_{2b} \right]
\]

and the matrix $\tilde{V}$ is determined by the Riccati equation\(^7\)

\[
\tilde{V} = (C'QC) + \phi(A + B\tilde{F})\tilde{V}(A + B\tilde{F}) + \tilde{F}^r(D'QD)\tilde{F} + (C'QD)\tilde{F} + \tilde{F}^r(C'QD) \\
+ \phi\tilde{v}_{11}\Sigma_{1a} + 2\Sigma_{1b} + \tilde{F}^r\Sigma_{1b} + \phi\tilde{v}_{12} + \tilde{v}_{21}\Sigma_{2b} + \phi(\tilde{v}_{12} + \tilde{v}_{21})\left(\Sigma_{1a} + \Sigma_{2b}\right)
\]

**Proof:** See Appendix B.

**Influence of Parameter Estimation Uncertainty**

To assess the influence of parameter estimation uncertainty on the central banker behavior, we apply the methodology detailed in the last section to the European central banker. Since the results obtained from a calibration exercise are obviously dependent upon the choice of parameter values, we choose to estimate the parameters and retrieve their variance-covariance matrix in order to perform simulations.

**Data**

The model is estimated on quarterly Euro area data from 1976:1 to 2002:4; graphs of the data series are shown in Figure 1. The data come from the updated Area Wide Model (AWM) database provided by Fagan, Henry and Mestre (2001). The output gap (YGA) is defined as the log deviation of real GDP to potential output where potential output is assumed to be given by a constant-returns-to-scale Cobb Douglas production function. The price series is the implicit GDP deflator (YED), seasonally adjusted, and the inflation is the quarterly percentage change in the price index, at an annual rate. The interest rate is the quarterly 3-month interest rate (STN). All variables are demeaned before estimation, so no constant appears in the regression.

**Parameter Estimation**

The dynamics of the Euro area economy is captured by applying the Maximum Likelihood (ML) method in order to recover a full covariance matrix of parameters to the IS and Phillips curves.

The hypothesis that the sum of the lag coefficients of inflation equals one had a p-value of 0.27 (obtained with the F-statistic), so this restriction was imposed in the estimation. The fit and dynamics of this model compare favourably to that of an unrestricted VAR. Indeed, the model can be interpreted as a restricted VAR, where the restrictions imposed are not at odds with the data.\(^8\)
The estimated equations are as follows, standard errors in parentheses,

\[ y_{t+1} = 0.93 y_t - 0.06 y_{t-1} - 0.05(i_t - \pi_t) + \hat{\varepsilon}_{y,t+1} \]

\[ \pi_{t+1} = 0.44 \pi_t + 0.21 \pi_{t-1} + 0.06 \pi_{t-2} + 0.29 \pi_{t-3} + 0.27 y_t + \hat{\varepsilon}_{\pi,t+1} \]

and,

\[ \sigma_{\varepsilon_y} = 0.449, \quad \sigma_{\varepsilon_\pi} = 1.031, \quad \log L = -214.603 \]

The system displays a reasonably good empirical fit with an adjusted \( R^2 = 0.866 \) (see also Figure 2). All estimates have the expected sign but the second lag of output gap in the IS equation, although it has not explanatory power. Finally, although these estimates suggest a minor initial role for monetary policy, the impact of the lagged value of the output gap in the Phillips curve is large implying that the response of output to policy rates is much greater in the long-run. Since our purpose is to measure the influence of parameter estimation uncertainty via the estimated variance-covariance matrix, we reproduce it in Table 1.

**Analysis of Parameter Estimation Uncertainty**

Based on the estimated parameters, parameter covariances and error variances, the analysis of the effect of parameter estimation uncertainty is carried out along the lines described in the past section. We begin by studying the initial response of policy and proceed by analyze the dynamic response over time. We study how the coefficients in the optimal rule are affected by parameter uncertainty, first considering uncertainty about impact and persistence parameters independently, then considering the combined effect of uncertainty about all parameters.

**The Initial Policy Response.** Table 2 provides results for several illustrative cases with different preferences over goal variables. As specified in the introduction, at first sight it seems that there are no systematic reduction or increase in the values of the coefficients. However, our framework being more general than the existing literature, it enables us to trace specific features. Indeed, as we also note it, papers that find an attenuation effect under the presence of uncertainty are limited to a loss function depending only on the variances of inflation and output gap.

On one hand, a concern not only about inflation stabilization but also about output stabilization and interest rate smoothing is more realistic for many central banks (including inflation-targeting ones). For example, the primary objective of the ECB is price stability although the Maastricht Treaty does not provide for a specific definition of this objective. However we may assimilate this objective as a flexible inflation targeting where \( \lambda_y, \lambda_i > 0 \).

On the other hand, Brainard's principle is always verified in the first three blocks of lines in which the weight of the interest rate smoothing is zero but is never in other cases. To establish it, we follow the same analysis as Söderström (2002) by dissociating uncertainty relating to the parameters of transmission and those of persistence.

Figures 3-8 show how the difference between values of coefficients under uncertainty and those under certainty equivalence evolves as the preferences parameters \( \lambda_y \) and \( \lambda_i \) vary from 0.1 to 2. This means that a negative difference would confirm the Brainard principle
whereas a positive difference would imply a central banker to be more aggressive. To save space, we only show in each figure the difference of contemporaneous output and inflation coefficients.9

First, Figures 3 and 4 show the two cases of uncertainty about the impact parameters in the transmission mechanism, $\beta$ and $\gamma$. In contrast to Söderström's result, the Brainard conservatism is not always confirmed: when there is uncertainty concerning the impact parameters, the optimal responses coefficients are not smaller than under certainty equivalence. When there is uncertainty only on $\beta$, the elasticity of output gap with respect to the real interest rate, it is true that the parameters associated to output are smaller under uncertainty. But those associated to inflation are higher for most values of the preference parameters $\lambda_y$ and $\lambda_i$, although the effect of uncertainty is weaker for high weights attached to interest rate smoothing and output stabilization. When the uncertainty relates only to $\gamma$, the parameter of transmission from output to inflation, the reverse result is found: the output coefficient is always higher under uncertainty case than the certainty equivalence case whereas the inflation coefficient is always smaller. Finally, the cumulated effects of uncertainty on the two impact parameters (Figure 5) make it difficult to draw systematic conclusions. It really depends on the preferences of the policymaker.

Second, Figures 6 and 7 show the response coefficients under uncertainty about the persistence parameters in the IS and Phillips curves. Here again, no definitive conclusions may be presented. In presence of uncertainty on persistence parameters in the IS curve $(\alpha_j, j = 1,2)$, the inflation coefficient is smaller whereas those of the output coefficient is larger. In contrast, uncertainty about persistence of inflation $(\delta_j, j = 1,...,4)$ or all persistence parameters (Figure 8), affects the optimal policy coefficients in the same direction: optimal policy is always more aggressive under uncertainty than under certainty equivalence, in contradiction to the Brainard intuition.

Figure 9 shows the case when there is uncertainty about all parameters, with estimated variances and covariances. No ambiguity is possible, whatever the configuration, the central banker behaviour is always aggressive under uncertainty with more or less strong effects according to values of the weights.

Note that we varied the preferences weights over the interval $[0.1, 2]$: if we observe what occurs in both extreme cases, when there is no weight on the output (Figure 10), the aggressive behaviour prevails, whereas when there is no weight on the interest rate smoothing (Figure 11), one clearly finds an attenuation effect. Thus the net effect of parameter uncertainty on policy depends not only on the relative variances and covariances of the parameters, but also on the weight of output stabilization and interest rate smoothing in the central bank's objective function.

The Policy Response over Time. Some features of the model are revealed by deriving the impulse response functions, which trace how each variable in the system responds to structural shocks. But these functions naturally depend on how monetary policy responds to shocks. So we must calculate how policy responds over time to shocks to output and inflation. In the first period, the economy is hit by a 1% shock either to output or to inflation. This shock is then transmitted through the economy by the state equation. The central bank responds optimally in each period according to its reaction function.

Figures 12-14 show the optimal responses from the model in the strict inflation targeting case, no interest rate smoothing case and the general case. Although the effects are typically small in all parameter configurations, it is clear that parameter uncertainty may
make the policy response more gradualist, again depending on the relative variances-covariances of the parameters and the preferences of the central bank.

As Figures 12 and 13 suggest, the model implies a very strong initial response to shocks (around 30% and 20% respectively), with large fluctuations in the central bank instrument. Consequently, this very simple model leads to substantially more interest rate volatility than what is empirically observed. But with a more realistic loss function where $\lambda_y = 0.1$ and $\lambda_i = 0.1$, the policy responses are more intuitively attractive (Figure 14), since it implies naturally more interest rate smoothing, in the sense that policy is adjusted in the same direction at least twice before returning towards zero. We note, however, that the effects of uncertainty are small in this loss function configuration. It is not too astonishing since Table 2 showed us already that with conventional values for the weights of the loss function, the difference between the parameters of the rule under certainty-equivalence and those under uncertainty is week (around 2%).

**The Implied Dynamics of the Interest Rate.** As a final experiment, we can calculate the implied optimal path of the short-term interest rate over the sample period and under parameter estimation uncertainty by applying the optimal reaction function to the actual data for the euro area economy (Figure 15).

In assuming that the weights of output and inflation smoothing are $\lambda_y = 0.1$ and $\lambda_i = 0.1$, the analysis of the preceding section showed that in this case the central banker was a little aggressive, it is thus rather natural to observe a greater volatility of the interest rates. Despite everything, we can thus come rather close mimicking the actual behaviour of the ECB by introducing parameter uncertainty into a simple optimizing model.

Moreover, it seems a necessity in this framework to take into account an interest rate smoothing objective under sorrow to obtain unrealistic interest rate path.

**Conclusion**

Monetary policymakers, especially European, face considerable uncertainty. When taking their decisions, policymakers need to be aware of this uncertainty and factor its effects into their interest rate choices.

In this paper, we examine how the traditional linear quadratic programming may be augmented to take into account the problem of parameter estimation uncertainty in which the second order moments represent a measure of this uncertainty. We illustrate the control problem by analysing the impact of uncertainty on optimal monetary policy for the Euro area. We demonstrate that the validity of the famous “conservatism principle” depends as much as the policymakers preferences than the uncertainty about parameters in a dynamic macroeconomic model. When there is no interest rate smoothing objective, the policymaker is generally more cautious but when incorporating a smoothing objective in the loss function, the policymaker has more often an aggressive behaviour (the degree of which depends on the values of the weights). Finally, with conventional values for the weights of the loss function, the average discrepancy between the parameters of the rule under certainty-equivalence and those under uncertainty is week: that wants to say that forgetting to take into account parameter estimation uncertainty will not involve deep differences for optimal monetary policy.
Footnotes

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1. See Poole (1998) or Clements and Hendry (1998) for a taxonomy of the most common types of uncertainty.

2. See the technical book by Chow (1977).


4. See Svensson (2003) for a comparison between “targeting rules” and “instrument rules”. Particularly, he shows that optimal targeting rules allow the use of judgment or extra-model information and are more robust and easier to verify (since these enter into the construction of the forecasts) than optimal instrument rules.

5. Note that $V$ is symmetric.

6. Only the uncertainty in the $B$ coefficients should matter for the conservatism result.

7. Note that $\tilde{V}$ is not necessarily symmetric in the setup with parameter uncertainty.

8. The timing assumption of our model is important. At the beginning of each period $t$, the central bank observes all state variables up to time $t$ included. On the basis of those values the central bank sets the optimal policy rate. Then nominal and real shocks hit the economy, so that at the beginning of period $t+1$ a new vector of state variables influences the central bank's decisions.

9. The variation of the other parameters is quite similar to these ones. Graphics are available upon demand.

References


**Appendix A: Form of the Variance-Covariance Matrices**

\[
\Sigma_{11}^{\text{A}} = \begin{bmatrix}
\sigma_{a_1}^2 & \sigma_{a_1a_2} & \sigma_{a_1\beta} \\
-\sigma_{a_2}^2 & \sigma_{a_2a_3} & \sigma_{a_2\beta} \\
-\sigma_{\beta}^2 & -\sigma_{\beta}^2 & \sigma_{\beta}^2
\end{bmatrix}, \quad \Sigma_{22}^{\text{A}} = \begin{bmatrix}
\sigma_{\gamma}^2 & \sigma_{\gamma}\delta_1 & \sigma_{\gamma}\delta_2 & \sigma_{\gamma}\delta_3 & \sigma_{\gamma}\delta_4 & \sigma_{\gamma}\delta_5 \\
-\sigma_{\delta_1}^2 & \sigma_{\delta_1}\delta_2 & \sigma_{\delta_1}\delta_3 & \sigma_{\delta_1}\delta_4 & \sigma_{\delta_1}\delta_5 & \sigma_{\delta_1}\delta_6 \\
-\sigma_{\delta_2}^2 & -\sigma_{\delta_2}^2 & \sigma_{\delta_2}\delta_3 & \sigma_{\delta_2}\delta_4 & \sigma_{\delta_2}\delta_5 & \sigma_{\delta_2}\delta_6 \\
-\sigma_{\delta_3}^2 & -\sigma_{\delta_3}^2 & -\sigma_{\delta_3}\delta_4 & \sigma_{\delta_3}\delta_5 & \sigma_{\delta_3}\delta_6 & \sigma_{\delta_3}\delta_7 \\
-\sigma_{\delta_4}^2 & -\sigma_{\delta_4}^2 & -\sigma_{\delta_4}\delta_5 & -\sigma_{\delta_4}\delta_6 & \sigma_{\delta_4}\delta_7 & \sigma_{\delta_4}\delta_8 \\
-\sigma_{\delta_5}^2 & -\sigma_{\delta_5}^2 & -\sigma_{\delta_5}\delta_6 & -\sigma_{\delta_5}\delta_7 & -\sigma_{\delta_5}\delta_8 & \sigma_{\delta_5}\delta_9
\end{bmatrix}
\]

\[
\Sigma_{12}^{\text{A}} = \begin{bmatrix}
\sigma_{a_1\gamma} & \sigma_{a_1\delta_1} & \sigma_{a_1\delta_2} & \sigma_{a_1\delta_3} & \sigma_{a_1\delta_4}
\\
\sigma_{a_2\gamma} & \sigma_{a_2\delta_1} & \sigma_{a_2\delta_2} & \sigma_{a_2\delta_3} & \sigma_{a_2\delta_4}
\\
\sigma_{\beta\gamma} & \sigma_{\beta\delta_1} & \sigma_{\beta\delta_2} & \sigma_{\beta\delta_3} & \sigma_{\beta\delta_4}
\end{bmatrix}, \quad \Sigma_{11}^{\text{B}} = \sigma_{\beta}^2
\]

\[
\Sigma_{11}^{\text{AB}} = \begin{bmatrix}
-\sigma_{a_1\beta} \\
-\sigma_{a_2\beta} \\
-\sigma_{\beta}^2
\end{bmatrix}, \quad \Sigma_{21}^{\text{AB}} = \begin{bmatrix}
-\sigma_{\beta\gamma} \\
-\sigma_{\beta\delta_1} \\
-\sigma_{\beta\delta_2} \\
-\sigma_{\beta\delta_3} \\
-\sigma_{\beta\delta_4}
\end{bmatrix}
\]
Appendix B: Optimal Control under Uncertainty

\[ J(x_t) = \min \{ z_t'Qz_t + \phi E_t J(x_{t+1}) \} \]

subject to

\[ x_{t+1} = A_{t+1}x_t + B_{t+1}i_t + \varepsilon_{t+1} \]

The value function is

\[ J(x_t) = x_t'\tilde{V}x_t + \tilde{d}, \]

The modified expected value taking into account the second moments is given by

\[ E_tJ(x_{t+1}) = (E_t x_{t+1})'\tilde{V}(E_t x_{t+1}) + tr(\tilde{V}\Sigma_{t+1}) + \tilde{d} \]

where the expected value of \( x_{t+1} \) is given by \( E_t x_{t+1} = Ax_t + Bi_t \) and where \( \Sigma_{t+1} \) is the covariance matrix of \( x_{t+1} \), evaluated at \( t \).

The \((i,j)\)th block of \( \Sigma_{t+1} \) is defined by

\[ \Sigma_{t+1}^{i,j} = x_t'\Sigma_A^{i,j}x_t + 2x_t'\Sigma_A^{i,j}i_t + i_t'\Sigma_B^{i,j}i_t + \Sigma_{\varepsilon}^{i,j} \]

where \( \Sigma_A^{i,j} \) is the covariance matrix of the \( i \)th block of \( A \) with the \( j \)th block of \( B \).

Since at \( t \), \( y_{t+1} \) and \( \pi_{t+1} \) are the only stochastic variables in \( x_{t+1} \), the only non zero entries of \( \Sigma_{t+1} \) are the matrices \( \Sigma_{11}^{11}, \Sigma_{22}^{22}, \Sigma_{12}^{12} \).

The only non-zero elements are then

\[ \Sigma_{t+1}^{11} = Var_t(y_{t+1}) = x_t'\Sigma_A^{11}x_t + 2x_t'\Sigma_A^{12}i_t + i_t'\Sigma_B^{12}i_t + \Sigma_{\varepsilon}^{11} \]

\[ \Sigma_{t+1}^{22} = Var_t(\pi_{t+1}) = x_t'\Sigma_A^{22}x_t + \Sigma_{\varepsilon}^{22} \]

\[ \Sigma_{t+1}^{12} = cov_t(y_{t+1}, \pi_{t+1}) = x_t'\Sigma_A^{12}x_t + x_t'\Sigma_A^{21}i_t \]

Consequently,

\[ tr(\tilde{V}\Sigma_{t+1}) = \tilde{v}_{11}(x_t'\Sigma_A^{11}x_t + 2x_t'\Sigma_A^{12}i_t + i_t'\Sigma_B^{12}i_t + \Sigma_{\varepsilon}^{11}) + \tilde{v}_{22}(x_t'\Sigma_A^{22}x_t + \Sigma_{\varepsilon}^{22}) + (\tilde{v}_{12} + \tilde{v}_{21})(x_t'\Sigma_A^{12}x_t + x_t'\Sigma_A^{21}i_t) \]
where $\tilde{v}_{ij}$ is the $(i,j)$th block of $\tilde{V}$.

The Bellman equation is

$$
x_i', \tilde{V}x_i + \tilde{d} = \min_{i'} \left\{ x_i'Qz_i + \varphi(Ax_i + B_i) + \tilde{V}(Ax_i + B_i) + \varphi tr(\tilde{V} \Sigma_{ii'}) + \varphi \tilde{d} \right\}
$$

and

$$
\frac{\partial tr(\tilde{V} \Sigma_{ii'})}{\partial t_i} = 2\tilde{v}_{11}(\Sigma_{iA}x_i + \Sigma_{ii'}i_i) + (\tilde{v}_{12} + \tilde{v}_{21})\Sigma_{iA}x_i,
$$

So the first-order condition is

$$
2(C'QD)x_i + 2(D'QD)i_i + \varphi(B'(\tilde{V} + \tilde{V}')A)x_i + \varphi(B'(\tilde{V} + \tilde{V}')B)i_i + 2\tilde{v}_{11}(\Sigma_{iA}x_i + \Sigma_{ii'}i_i) + (\tilde{v}_{12} + \tilde{v}_{21})\Sigma_{iA}x_i = 0
$$

leading to the optimal interest rate

$$
\tilde{i}_i = \tilde{F}x_i
$$

where

$$
\tilde{F} = -\frac{1}{2}
\left[
\begin{array}{c}
2(D'QD) + \varphi B'(\tilde{V} + \tilde{V}')B + 2\tilde{v}_{11}\Sigma_{ii}'
\end{array}
\right]^{-1}
\times
\left[
\begin{array}{c}
2(C'QD) + \varphi B'(\tilde{V} + \tilde{V}')A + 2\tilde{v}_{11}\Sigma_{iA}' + (\tilde{v}_{12} + \tilde{v}_{21})\Sigma_{iA}'
\end{array}
\right]
$$

Substituting back into the Bellman equation, we get

$$
x_i', \tilde{V}x_i + \tilde{d} = (Cx_i + D\tilde{F}x_i)Q(Cx_i + D\tilde{F}x_i) + \varphi(Ax_i + B\tilde{F}x_i) + \tilde{V}(Ax_i + B\tilde{F}x_i)
$$

and it can be established that $\tilde{V}$ is determined by the Riccati equation

$$
\tilde{V} = (C'QC) + \varphi(A + B\tilde{F}) + \tilde{V}(A + B\tilde{F}) + \tilde{V}'(D'QD)\tilde{F} + (C'QD)\tilde{F} + \tilde{F}'(C'QD) + \varphi \tilde{V}_{11}(\Sigma_{iA} + 2\Sigma_{iA}\tilde{F} + \tilde{F}'\Sigma_{iA}\tilde{F}) + \varphi \tilde{V}_{22}(\Sigma_{iA} + 2\Sigma_{iA}\tilde{F}) + \varphi (\tilde{v}_{12} + \tilde{v}_{21})\Sigma_{iA}' + \varphi \tilde{d}
$$
Table 1. Estimated variance-covariance matrix (coefficient *1000)

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<tr>
<th></th>
<th>α1</th>
<th>α2</th>
<th>β</th>
<th>γ</th>
<th>δ1</th>
<th>δ2</th>
<th>δ3</th>
<th>δ4</th>
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Figure 1. European data series

Figure 2. Historical and estimated series
Figure 3. Reaction function coefficients, impact parameters uncertainty (only $\beta$)

Figure 4. Reaction function coefficients, impact parameters uncertainty (only $\gamma$)
Figure 5. Reaction function coefficients, impact parameters uncertainty (all)

![Graph](image1)

Figure 6. Reaction function coefficients, persistence parameters uncertainty (IS curve)

![Graph](image2)
Figure 7. Reaction function coefficients, persistence parameters uncertainty (Phillips curve)

Output coefficient

Inflection coefficient

Figure 8. Reaction function coefficients, persistence parameters uncertainty (all)

Output coefficient

Inflection coefficient
Figure 9. Reaction function coefficients, all parameters uncertainty

Figure 10. Reaction function coefficients, all parameters uncertain ($\lambda_y = 0$)
Figure 11. Reaction function coefficients, all parameters uncertain ($\lambda_i=0$)

Figure 12. Policy response over time (with $\lambda_y=0$ and $\lambda_i=0$)

Figure 13. Policy response over time (with $\lambda_y=0.1$ and $\lambda_i=0$)
Figure 14. Policy response over time (with $\lambda y=0.1$ and $\lambda i=0.1$)

Figure 15. Historical and optimal interest rates