A New Test of Asymmetric Stationarity in the Presence of Deterministic Trends: Simulation and Empirical Evidence

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Abstract Using local-to-unity detrending, a modified momentum-threshold autoregressive test is derived to allow the unit root hypothesis to be tested against an alternative of asymmetric stationarity about a deterministic trend. Monte Carlo evidence is presented to show the increased power of the proposed test in the presence of asymmetric adjustment relative to the familiar Dickey-Fuller (1979) test and the momentum-threshold autoregressive test of Enders and Granger (1998). The empirical relevance of the test is illustrated via an application to New Zealand national output over the period 1870-2001, where in contrast to findings obtained using alternative unit root tests, the unit root hypothesis is conclusively rejected.

Keywords: Unit root tests, local-to-unity detrending, momentum-threshold autoregression

JEL Classification: C12, C15, C22

Introduction

Following the seminal research of Nelson and Plosser (1982), analysis of the trending nature of economic time series has been the focus of much attention in applied econometric research. In particular, a large literature has emerged examining whether economic series are better described as stationary about a deterministic trend (trend stationary) or instead as unit root processes (non-stationary) without any tendency to return to an underlying attractor. Given the implications of this distinction for the time series properties of economic data, the specification of econometric models and the implementation of economic policy, it is not surprising that this issue has received such widespread attention. The most commonly employed test of the unit root hypothesis in the literature is that presented by Dickey and Fuller (DF) (1979). When testing for the presence of a unit root in trending data, the appropriate specification for the DF test is as follows:

\[ y_t = \alpha + \beta t + \rho y_{t-1} + \epsilon_t \quad t = 1, \ldots, T. \]  

with the unit root null hypothesis ($H_0: \rho=1$) tested against the alternative of trend stationarity ($H_0: |\rho|=1$) via the $t$-like statistic $\tau_t$. Given the non-standard distribution of the $\tau_t$ statistic, specifically derived critical values are required to test the unit root hypothesis. However, it can be seen that (1) implicitly assumes symmetric adjustment, with reversion to an underlying attractor occurring
at a single speed determined by the value of \( \rho \). In recent research this limitation has been recognised by Enders and Granger (1998), with the DF test extended to allow adjustment to occur at differing speeds according to a defined decision rule. Drawing upon the threshold autoregressive methods of Tong (1983, 1990), Enders and Granger (EG) propose the following modification of (1):

\[
\tilde{y}_t = I_t \rho_1 \tilde{y}_{t-1} + (1 - I_t) \rho_2 \tilde{y}_{t-1} + \eta_t \quad t = 1, \ldots, T.
\]

(2)

where \( \tilde{y}_{t-1} \) is the residual obtained from the regression of \( y_t \) upon an intercept and deterministic trend, and \( I_t \) is the zero-one Heaviside indicator function given as:

\[
I_t = \begin{cases} 
1 & \text{if } \Delta \tilde{y}_{t-1} \geq 0 \\
0 & \text{if } \Delta \tilde{y}_{t-1} < 0
\end{cases}
\]

(3)

The above specification therefore allows for the possibility of asymmetric adjustment via the partitioning of \( \tilde{y}_{t-1} \) and the resulting creation of two adjustment parameters (\( \rho_1, \rho_2 \)). The unit root hypothesis is then examined using the null hypothesis \( H_0: \rho_1 = \rho_2 = 1 \) against the alternative of asymmetric stationarity. The resulting test is denoted as \( \Phi^* T \), and is referred to as momentum-threshold autoregressive as partitioning is based upon the change in, or first difference of, \( \tilde{y}_{t-1} \). It should be noted that EG also consider a further asymmetric unit root test based upon an alternative specification of the Heaviside indicator function. However, it was found that this threshold autoregressive (TAR) specification lacked power in comparison to the MTAR test.

The resulting MTAR unit root test of (2) and (3) therefore permits the unit root hypothesis to be tested against an alternative of asymmetric stationarity. This is a welcomed development for two principal reasons. First, a range of studies now exists suggesting that economic variables display asymmetric behaviour (see, inter alia, Ball and Mankiw 1994, Dixit 1992, Gale 1996, Krane 1994). Second, it is known that the DF test possesses low power in the presence of processes subject to asymmetric adjustment (see Pippenger and Goering 1993).

In this paper testing of the unit root hypothesis in the presence of asymmetric adjustment is revisited, with a new test proposed to examine the unit root null hypothesis against an alternative of asymmetric adjustment about a deterministic trend. Using local-to-unity detrending via generalised least squares (GLS), a modified MTAR unit root test is proposed. It is shown via Monte Carlo simulation that the resulting GLS-MTAR unit root test possesses greater power than both the \( \tau \) and \( \Phi^* T \) tests of DF and EG in the presence of asymmetric adjustment.

This paper will proceed as follows. In section [2] the newly proposed GLS-MTAR unit test is presented, with critical values provided for alternative sample sizes along with a simulation analysis of the power of the test relative to the \( \tau \) and \( \Phi^* T \) tests. The empirical relevance of the proposed test is illustrated in section [3] via an empirical application to long run data on New Zealand national output over the period 1870 to 2001. Section [4] provides some concluding remarks.
An Alternative MTAR Unit Root Test

In recent research, Elliott et al. (1996) have proposed local-to-unity detrending using GLS as a means of increasing the power of the DF unit root test. In the present paper, local-to-unity detrending is employed to increase the power of the MTAR test of EG. Following Elliott, et al (1996), the locally detrending version of $y_t$, denoted as $y^*_t$, is given as:

$$y^*_t = y_t - \hat{\gamma}_1 - \hat{\gamma}_2 t$$

with $\{\hat{\gamma}_i\}$ defined as the estimators derived from the regression of $\{\tilde{y}_t\}$ on $\{\tilde{z}_t\}$, where:

$$\tilde{y}_t = \{y_1, (1-\bar{\alpha}L)y_2, (1-\bar{\alpha}L)y_3, ..., (1-\bar{\alpha}L)y_T\}$$

$$\tilde{z}_t = \{z_1, (1-\bar{\alpha}L)z_2, (1-\bar{\alpha}L)z_3, ..., (1-\bar{\alpha}L)z_T\}$$

and

$$z_t = (1, t), \quad \bar{\alpha} = 1 + \bar{c}T^{-1}$$

For the specification incorporating a trend considered here, the value $\bar{c} = -13.5$ is suggested by Elliott et al. (1996) for the DF test. This value is adopted here. The detrended series $y^*_t$ is then employed in (2) and (3) above to obtain the GLS-MTAR unit root test denoted as $\Phi^*_{GLS,T}$.

To derive finite-sample critical values for the $\Phi^*_{GLS,T}$ test, the following simple unit root data generation process (DGP) is employed:

$$y_t = y_{t-1} + \xi_t \quad t = 1, \ldots, T.$$  \hspace{1cm} (4)

with $\xi_t$ generated using pseudo i.i.d. random numbers from the RNDNS procedure in the GAUSS. The initial value $y_0$ is set equal to zero with sample sizes of 100 and 250 observations considered. The resulting critical values at the 1%, 5% and 10% levels of significance derived using 50,000 replications are reported in Table One.

To investigate the power of the proposed $\Phi^*_{GLS,T}$ test relative to the $\Phi^*_{T}$ and $\tau_t$ tests when applied to asymmetric adjustment processes, the following MTAR DGP is employed:

$$\tilde{y}_t = I_t \rho_1 \tilde{y}_{t-1} + (1-I_t) \rho_2 \tilde{y}_{t-1} + v_t \quad t = 1, \ldots, T.$$  \hspace{1cm} (5)

$$I_t = \begin{cases} 1 & \text{if } \Delta \tilde{y}_{t-1} \geq 0 \\ 0 & \text{if } \Delta \tilde{y}_{t-1} < 0 \end{cases}$$  \hspace{1cm} (6)

To generate $v_t$ in (5), pseudo i.i.d. N(0,1) random numbers are derived from the RNDNS procedure in the GAUSS, with the initial value $y_0$ set equal to zero. Empirical rejection
frequencies at the 5% level of significance for the rival tests are reported in Table Two for alternative values of the asymmetric adjustment parameters ($\rho_1, \rho_2$) and two sample sizes of 100 and 250 observations respectively. From inspection of Table Two it is apparent that for the experimental designs considered, the $\Phi_{\tau}^*$ test outperforms the $\tau_t$ test in all but two experiments. The cases in which this does not hold are for the following values of the design parameters: $(\rho_1, \rho_2, T) = (0.90,0.85,100)$ and $(0.95,0.93,250)$. This finding has an intuitive explanation, as while the $\Phi_{\tau}^*$ test will be expected to possess increased power in the presence of asymmetry, this may not hold for ‘near symmetric’ cases where $\rho_1$ and $\rho_2$ are close in value and the allowance for asymmetry is outweighed by the effects of the estimation of additional parameters it entails. However, a consistent feature of the results in Table Two is the finding that the $\Phi_{\text{GLS,T}}^*$ test always exhibits the greatest power of the tests considered. As examples of this, consider the experimental results for $(\rho_1, \rho_2, T) = (0.95,0.75,100)$ and $(0.95,0.88,250)$. In these instances the increased power of the $\Phi_{\text{GLS,T}}^*$ test relative to the $\Phi_{\tau}^*$ test is substantial, with empirical rejection frequencies of $(70.49\%, 55.04\%)$ and $(90.92\%, 74.05\%)$ observed for the $\Phi_{\text{GLS,T}}^*$ and $\Phi_{\tau}^*$ tests respectively.

An Empirical Application: New Zealand GDP

To provide an empirical illustration of the above tests, the unit root hypothesis is examined for data on real, per capita GDP in New Zealand over the period 1870 to 2001. This exercise provides an interesting and important application of the newly proposed test given the economic significance of the variable considered. The natural logarithm of this series is depicted in Figure One. From inspection of this figure, it is apparent that the series possesses a clear trend. To examine whether the series is non-stationary or instead stationary about an underlying deterministic trend, the $\tau_t$ test outlined above is initially applied. However, to avoid problems with serial correlation, the test is applied in an augmented form with four lagged differenced regressors included. Using a fourth order Lagrange multiplier (LM) test of serial correlation it was found that this standard procedure of augmentation ensured an absence of serial correlation. A comparison of the resulting calculated $\tau_t$ statistic of $-2.41$ with the 5% critical value of $-3.45$ shows that the null of a unit root cannot be rejected. On the basis of this routinely employed test with its underlying assumption of symmetric adjustment, it would be concluded that New Zealand GDP is a non-stationary unit root process, with a series of resulting conclusions drawn regarding its properties and their implications for modelling and policy analysis. However, as noted above, Pippenger and Goering (1993) have shown the $\tau_t$ test to possess low power in the presence of asymmetry. To explore this issue, the $\Phi_{\tau}^*$ test of EG is employed. Applying the test in its augmented form with four lagged terms, a calculated statistic of 6.54 is obtained which marginally fails to reject the null at the 5% level of significance against a critical value of 6.78. However, application of the above $\Phi_{\text{GLS,T}}^*$ test results in a calculated statistic of 6.71. Using Monte Carlo simulation with 50,000 replications, the p-value associated with this calculated statistic for the present sample size is 0.016 or 1.6%. Therefore application of the newly proposed GLS-MTAR test convincingly rejects the null of non-stationarity against an alternative of asymmetric stationarity about a deterministic trend. With the unit root rejected, the null hypothesis of symmetry ($\rho_1 = \rho_2$) can be tested formally. It is found that the null is comfortably rejected, with a calculated p-value of 0.004 obtained for the symmetry test. Considering the values of the asymmetric adjustment parameters under the $\Phi_{\text{GLS,T}}^*$ test, the estimated values
obtained for $\rho_1$ and $\rho_2$ are $-0.042$ and $-0.055$), indicating faster adjustment following negative changes ($\Delta\tilde{y}_{t-1}$).

Conclusion

In this paper, using local-to-unity detrending of Elliott et al. (1996) has been employed to generate a modified momentum-threshold autoregressive test. In contrast to the routinely employed Dickey-Fuller test, the proposed GLS-MTAR $\Phi^*_{GLS,T}$ test allows the unit root hypothesis to be tested against an alternative of asymmetric stationarity about a deterministic trend. Using Monte Carlo simulation, the $\Phi^*_{GLS,T}$ test was found to possess greater power than the MTAR test of Enders and Granger (1998) for a range of asymmetric stationary processes. Finally, the empirical relevance of the test was illustrated via an application to New Zealand national output over the period 1870-2001 in which the $\Phi^*_{GLS,T}$ test was the only test to reject the unit root hypothesis.

Footnotes

1. I am grateful to the editor, Professor Yu Hsing, for comments which have improved the content of this paper.

2. The data were obtained from (http://www.eco.rug.nl/~Maddison/) and update figures previously reported in Maddison (1995).

References


Table 1. Finite-Sample Critical Values for the GLS-MTAR Unit Root Test*

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<thead>
<tr>
<th>Sample size</th>
<th>Significance level</th>
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<tr>
<td></td>
<td>1%</td>
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<tr>
<td>100</td>
<td>7.44</td>
</tr>
<tr>
<td>250</td>
<td>7.06</td>
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* The reported results represent critical values for the GLS-MTAR ($\Phi_{GLS,T}^*$) test for alternative sample sizes calculated via Monte Carlo experimentation using 50,000 simulations.
Table 2. Power Comparison of the DF, MTAR and GLS-MTAR Unit Root Tests*

<table>
<thead>
<tr>
<th></th>
<th>ρ₁</th>
<th>ρ₂</th>
<th>τₜ</th>
<th>Φₜ</th>
<th>Φₜ,GLS,T</th>
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<td></td>
<td></td>
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<td>0.70</td>
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<tr>
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<td>72.41</td>
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<td>18.50</td>
<td>29.08</td>
<td></td>
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</tbody>
</table>

* The reported results represent empirical rejection frequencies of the unit root hypothesis at the 5% level of significance. The results are measured in percentage terms for the alternative tests calculated using the DGP of (5) and (6) over alternative sample sizes using 50,000 simulations.
Figure 1. Real Per Capita GDP in New Zealand, 1870-2001