A Note on the Welfare Analysis on Third-degree Tariff Discrimination under International Trade Agreement

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Abstract The literature on third-degree price discrimination tariffs is skimpy at best. In the wake of globalization and international economic coordination, reduced tariffs are frequently required of each participating member country. The determination of the optimum tariff rates in the presence of differing price elasticity values needs to be examined carefully before policy implementation.

Keywords: third-degree price discrimination, optimum tariff

JEL Classification: F13, I38

1. Introduction

The major problem in formulating the optimization models is lack of solutions that are politically feasible. For instance, a prohibitive tariff is clearly not acceptable and as such we need to re-address the issue under new lens: computer simulations in absence of an analytical solution.

2. Literature Survey

The application of third-degree price discrimination is widespread even across different continents over various types of commodities (e.g., academic journal of Europe and North American according to Hamaker and Astle (1984) or demand for airline travel). However, the taxation or tariff topic has been scanty (e.g., Peng, 1993; Yang, 1993; Cheung, 1998). The first taxation paper on third-degree price discrimination is by Yang (1993). Yang’s paper primarily focuses on single taxes within the framework of profit maximization. Peng’s work (1993), however, focuses on differential taxes within third-degree price discrimination. Recently, Yang, Peng and Li (1996) prove that an optimum uniform tariff over \( n \) countries must be a convex combination of different discriminatory tariffs with a set of linear demand functions. However, their tariffs are measured in terms of dollar per unit or unit tariffs, not a commonly used practice in the literature of tariff.

I would like to mention that a different set of tariffs is the norm in international trade. For example, average tariff rates on toys were approximately 3.2%, 4.6%, 8%, 4.3%, and 41.6% for Canada, Australia, South Korea, Japan and China in 2000. The rates for food items were 18.2%, 3%, 30.2%, 16%, 40%, while those for automobiles were 4.8%, 7.3%, 7.1%, 2.1% and 25.8% respectively. Some of the industries are characterized by oligopoly which requires different sets of modeling such as Cournot competition. Brander and Spencer (1984), Hwang and Mai (1991) have proposed such theoretical models. Furthermore, Li et al. (1996) simulated a Cournot pollution tariffs using the Hwang and Mai model, which was generalized by Chou et al. (2002). Beyond that, Yang et al. (2002, 2006) proposed a Cournot spatial equilibrium model and a simulation approach on third-degree price discrimination. Note that the simulation models by Yang et al. (1996, 2006) are based on a sequential optimization model, which is known to have the “lack of solution” problem if the first stage yields rather limited solution set. In this paper, I follow the traditional one-stage optimization model to explore the much-disputed results. Most recently, Cowan (2007) derived two useful sufficient conditions to evaluate welfare position between uniform and discriminatory pricing strategy. Next section discusses the two theoretical models and computer simulations. Section IV provides a conclusion.

3. Theoretical Model

3.1. Optimum Welfare-maximizing Uniform Tariff

In this section, we first set the objective function as the sum of consumer surplus \( CS \) and profit \( \pi \) on the premise that the welfare \( W \) under regional economic integration such as APEC consists of both consumers and producers.
Max \( q_1, q_2 \) \( = CS + \pi \)

\[
= \sum_i \int_0^{q_i} p_i(q_i) dq_i - p_i q_i^* + (1-u) \sum_i p_i q_i
\]

(1)

For all \( i \in N \), where \( N \) is a set of positive integers equal to or greater than two, but not very large, and \( u \) is a uniform tariff. Clearly the first-order condition requires

\[
\frac{\partial W}{\partial q_i} = \partial \int_0^{q_i^*} p_i(q_i) dq_i - (1-u)(p_i + q_i \hat{p}_i / \partial q_i)
\]

\( = 0 \)  

(2)

For simplicity, we use \( p_i = a_i - b_i q_i \) as the demand function for nation \( i \) \((i=2)\), and \( TC = c + d \sum q_i \) as the total cost function of the producer. We now can rewrite (1) and (2) as the following,

Max \( \\begin{array}{l} W \\ q_1, q_2 \end{array} \) \( = CS + \pi \)

\[
= \frac{1}{2} b_1 q_1^2 + \frac{1}{2} b_2 q_2^2 + (1-u_1)P_1 q_1 + (1-u)P_2 q_2 - [c + d(q_1 + q_2)]
\]

\[
= \frac{1}{2} b_1 q_1^2 + \frac{1}{2} b_2 q_2^2 + (1-u)(a_1 q_1 - b_1 q_1^2 + a_2 q_2 - b_2 q_2^2) - c - dq_1 - dq_2
\]

(3)

\[
\frac{\partial W}{\partial q_1} = b_1 q_1 + (1-u)(a_1 - 2b_1 q_1) - d = 0
\]

(4)

\[
\frac{\partial W}{\partial q_2} = b_2 q_2 + (1-u)(a_1 - 2b_2 q_2) - d = 0
\]

(5)

The second-order condition can be shown in terms of the bordered Hessian \( H \)

\[
|H| = \begin{vmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{vmatrix} = \begin{vmatrix} b_1 (2u-1) & 0 \\ 0 & b_2 (2u-1) \end{vmatrix} = b_1 b_2 (2u-1)(2u-1)
\]

(6)

If \( b_1 (2u-1) < 0 \), \( b_2 (2u-1) < 0 \) and \( b_1 b_2 (2u-1)(2u-1) > 0 \), then \( |H_1| < 0 \), and \( |H_2| > 0 \).

In this case, we are assured to have a true maximum.
From (4) and (5), optimum quantities and prices can be derived or

\[
q_1^* = \frac{a_1(u - 1) + d}{b_1(2u - 1)} \quad (7)
\]

\[
q_2^* = \frac{a_2(u - 1) + d}{b_2(2u - 1)} \quad (8)
\]

\[
p_1^* = \frac{a_1u - d}{2u - 1} \quad (9)
\]

\[
p_2^* = \frac{a_2u - d}{2u - 1} \quad (10)
\]

In the second stage, as is often the case, a policy may mandate target tariff revenue \( R_0 \) for an importing country. Import tariff is viewed as a supplementary revenue source, which is more amenable to political impasse than is income or sales tax. The objective of the second stage optimization is to seek the optimum unit tariff \( u \) that maximized \( \text{CS} \) and \( \pi \) while guarantees a tariff revenue at \( R_0 \) or

\[
\text{Max}_{u_1, u_2} \quad \text{CS} + \pi
\]

\[
= \frac{1}{2}b_1q_1^{*2} + \frac{1}{2}b_2q_2^{*2} + (1 - u)(P_1^*q_1^* + P_2^*q_2^*) - [c + d(q_1^* + q_2^*)]
\]

s.t. \( u(P_1^*q_1^* + P_2^*q_2^*) = R_0 \) \hspace{1cm} (12)

We first solve for \( q_1^* \) from (12)

\[
q_1^* = \frac{1}{uP_1^*}(R_0 - uP_2^*q_2^*) \quad (13)
\]

Substituting (13) into (11) immediately leads to
\[
\begin{align*}
\text{Max}_{u_1, u_2} & \quad CS + \pi \\
& = \frac{1}{2} b_1 \left[ \frac{1}{uP_1^1} (R_0 - uP_2^* q_2^*) \right]^2 + \frac{1}{2} b_2 q_2^* \\
& \quad + (1 - u) \left\{ P_1^* \left[ \frac{1}{uP_1^1} (R_0 - uP_2^* q_2^*) + P_2^* q_2^* \right] \right\} \\
& \quad - \left\{ c + d \left[ \frac{1}{uP_1^1} (R_0 - uP_2^* q_2^*) + q_2^* \right] \right\}
\end{align*}
\]

Again, replacing \( p_1, p_2, q_1, q_2 \) with (7) (8) (9) (10) and (12), and taking derivate with respect to \( u \), we obtain a complicated form of solution, via the following

\[
\frac{\partial (CS + \pi)}{\partial u} = \frac{\partial (CS + \pi)}{\partial q_2} \frac{\partial q_2}{\partial u} = 0
\]

The resulting relation between \( u \) and the welfare is rather involved and can be best illustrated by simulations as shown in Table 1.

A glimpse on Table 1 indicates that the welfare-maximizing tariff is around \( u = 0.1667 \) within all the feasible rates \( 0 \leq \mu \leq 1 \) and at \( R_0 = 500,000 \) units. Note that 16.67% may well be politically acceptable within the framework of APEC especially for agriculture produce. Of course, different target tariff amount \( R_0 \) would lead to a different set of optimum tariffs. This being the case, as simulation is necessary to accommodate different nonlinear demand functions and \( R_0 \).

### 3.2 Optimum Welfare-maximizing Discriminatory Tariffs

The essence of a third-degree tariff discrimination model lies in charging different tariffs based on different demand price elasticity among different economic regions. As was done in the previous section, a two-stage optimization process is carried out to solve for the best tariff rates \( u_1 \) and \( u_2 \) for two regions (countries).
\[\begin{align*}
Max & \quad W \\
& = CS + \pi \\
& = \frac{1}{2}b_1q_1^2 + \frac{1}{2}b_2q_2^2 + (1 - u_1)P_1q_1 + (1 - u_2)P_2q_2 - [c + d(q_1 + q_2)] \\
& = \frac{1}{2}b_1q_1^2 + \frac{1}{2}b_2q_2^2 + (1 - u_1)(a_1q_1 - b_1q_1^2) \\
& + (1 - u_2)(a_2q_2 - b_2q_2^2) - c - dq_1 - dq_2
\end{align*}\]

The resulting first-order conditions are listed below:

\[\begin{align*}
\frac{\partial W}{\partial q_1} &= b_1q_1 + (1 - u_1)(a_1 - 2b_1q_1) - d = 0 \\
\frac{\partial W}{\partial q_2} &= b_2q_2 + (1 - u_2)(a_2 - 2b_2q_2) - d = 0
\end{align*}\]

Which gives rise to

\[\begin{align*}
q_1^* &= \frac{a_1(u_1 - 1) + d}{b_1(2u_1 - 1)} \\
q_2^* &= \frac{a_2(u_2 - 1) + d}{b_2(2u_2 - 1)}
\end{align*}\]

The second-order condition is satisfied by the following bordered Hessian

\[|H| = \begin{vmatrix} W_{11} & W_{12} \\
W_{21} & W_{22} \end{vmatrix} = \begin{vmatrix} b_1(2u_1 - 1) & 0 \\
0 & b_2(2u_2 - 1) \end{vmatrix} = b_1b_2(2u_1 - 1)(2u_2 - 1)\]

If \(b_1(2u_1 - 1) < 0\), \(b_2(2u_2 - 1) < 0\) and \(b_1b_2(2u_1 - 1)(2u_2 - 1) > 0\), it implies that \(|H_1| < 0\), and \(|H_2| > 0\). In this case, \(W\) again has reached a true maximum.

Following the previous formulation, we pursue the optimum \(u_1\) and \(u_2\) that maximize \(CS\) and \(\pi\) while maintaining a fixed amount of tariff revenue \(R_0\).
Max $CS + \pi$

s.t. $u_1P_1q_1 + u_2P_2q_2 = R_0$ (21)

The Lagrangian equation and its derivative can be shown as

\[
L = \frac{1}{2}b_1q_1^2 + \frac{1}{2}b_2q_2^2 + (1-u_1)P_1q_1 + (1-u_2)P_2q_2 - \left[c + d(q_1 + q_2)\right] + \lambda\left[R_0 - (u_1P_1q_1 + u_2P_2q_2)\right]
\] (22)

\[
\frac{\partial L}{\partial u_1} = b_1q_1 + \frac{\partial (P_1q_1)}{\partial u_1} - P_1q_1 - u_1\frac{\partial (P_1q_1)}{\partial u_1} - dq_1\frac{\partial q_1}{\partial u_1} - \lambda
\] (23)

\[
\frac{\partial L}{\partial u_2} = b_2q_2 + \frac{\partial (P_2q_2)}{\partial u_2} - P_2q_2 - u_2\frac{\partial (P_2q_2)}{\partial u_2} - dq_2\frac{\partial q_2}{\partial u_2} - \lambda
\] (24)

\[
\frac{\partial L}{\partial \lambda} = R_0 - u_1P_1q_1 - u_2P_2q_2 = 0
\] (25)

Substituting (18), (19), into (23), (24) and (25) gives rise to a complicated mathematic form in terms of $W$, $u_1$ and $u_2$, which can best be demonstrated via simulations. Table 2 reports 25 sets of relations among $W$, $u_1$ and $u_2$.

4. Summary and Conclusions

A glimpse at Table 2 immediately suggests there are three sets of equally optimal solutions: $u_1=0.1$ and $u_2=0.2$; $u_1=0.2$ and $u_2=0.15$; and $u_1=0.3$ and $u_2=0.1$. Given that the objective of WTO is to reduce tariff for its members, we opt for the first set of solution: $u_1=0.1$ and $u_2=0.2$. If a uniform tariff is required of each member, we opt for $u_1 = u_2 = 0.15$ or $u_1 = u_2 = 0$ as suboptimal solutions especially in the same economic community where differential tariffs are not allowed. As is well known in the literature, a uniform tariff is only a special case of third-degree tariff discrimination. It is to be noted that while the suboptimal solutions are suggested (e.g., $u_1 = 0.1$ and $u_2 = 0.2$ or $u_1 = u_2 = 0.15$), a true optimum tariff may escape the simulations because the grits we used are rather rough tariff = 0.05 via LINGO software package. A more refined grid can be easily used to produce slightly better solutions. This would be substantially less expensive than the case of a misguided policy.
Since a linear demand or $q(p)$ is used in our simulation, it automatically satisfies log concavity condition or $qq''/(q')^2 \leq 1$ due to $q''=0$. (Cowan, 2007, p.423) In a world without tariff, the Cowan result suggests total welfare with discrimination is lower than with uniform pricing. It is in agreement with the well-known result that linear demand gives rise to higher profit but less welfare for third-degree price discrimination. The focus of this paper is on optimum uniform and discriminatory tariffs while Cowan’s paper presents two useful sufficient conditions on welfare and output effects without tariffs. It comes as no surprise that welfare position even using Cowan’s conditions is by and large indeterminate. Tariff introduces complexity into already-complicated third-degree price discrimination model and as such simulations are needed to ferret out the best tariff rates.

**Endnote**

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**References**


Table 1. Optimum Uniform Tariff Which Maximizes the Welfare ($CS + \pi$)

<table>
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<tr>
<th>$u$</th>
<th>0.09</th>
<th>0.1</th>
<th>0.12</th>
<th>0.15</th>
<th>0.166667</th>
<th>0.18</th>
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<td>7,414,230</td>
<td>7,492,415</td>
<td>7,498,000</td>
<td>7,495,768</td>
<td>7,487,227</td>
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<table>
<thead>
<tr>
<th>$u$</th>
<th>0.21</th>
<th>0.24</th>
<th>0.27</th>
<th>0.3</th>
<th>0.33</th>
<th>0.36</th>
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<td>7,464,092</td>
<td>7,446,580</td>
<td>7,430,437</td>
<td>7,415,919</td>
<td>7,402,970</td>
<td>7,387,868</td>
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Table 2. Optimum Discriminatory Tariff Under Third Degree Price Discrimination

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<thead>
<tr>
<th>$u_2 \backslash u_1$</th>
<th>0.1</th>
<th>0.15</th>
<th>0.2</th>
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