On the Risk Management of South American Financial Markets: Do Integration and Contagion Matter? 

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Abstract: We add to the discussion about risk management by testing different specifications of Value at Risk (VaR) measures for the main stock market indices in South America: MERVAL (Argentina), IBOVESPA (Brazil), IPSA (Chile), IGBC (Colombia), and IGBVL (Peru). As a benchmark VAR, we rely on the hypotheses of unconditional moments of a Gaussian probability distribution. We relax these assumptions by allowing for time-varying specifications for the moments of a well-specified probability distribution in terms of fitting. Finally, we take into account the evidence reported in Matos, Siqueira and Trompieri (2014) by measuring the effects of contagion and integration on risk management, based on the conditional time-varying moments of the best-fitting distribution, which are extracted from a multivariate GARCH approach. We compare all these specifications based on relevant backtesting models.

Keywords: Value at Risk; Time-varying moments; Laplace probability distribution function.

JEL Classification: G15, C32.

1. Introduction

Kose et al. (2009) provide evidence that emerging countries have received growing capital flows during the past decade. Regions such as Latin America and Eastern Europe have experienced financial liberalization from the 1990s onwards, and have moved towards an open, market-based development model in place of one that relied on the state and was closed regarding capital flows and foreign trade, as emphasized by Stallings (2006).

However, over the last decades, the financial markets have been characterized by the incessant search for better returns and lower risks, and this has led to a continuing race for the international markets. In this context, one should also mention that the main stock markets of the emerging economies are able to offer higher levels of expected return, even if these are accompanied by higher levels of risk.

Another relevant issue is that the emerging economies are more likely to experience periods of turmoil due to economic crises, not only local ones but also global crises or crises occurring in other economies. According to Fidrmuc and Korhonend (2010) and Ozkan and Unsal (2012), one possible reason is the greater intensity of contagion effects among emerging economies, for which there is robust empirical evidence. Within this context, we address risk management modelling that considers the cross-effects for the main South American stock market indices, because of their relevance for domestic and foreign investors.

More specifically, South American economies have commonly come to the attention of researchers, investors and policy makers, although for quite distinct reasons. One can highlight the solid Chilean financial market, the internal demand of the Brazilian population as a whole, the
antidemocratic convergence in Argentina, the Colombian process of internal peace-making, and even the elevated growth rates of the Peruvian economy. To summarize, one can see great differences between these countries, with their similarities most commonly identified under an historical prism.

Methodologically, we deal with risk management by following the literature about modern risk management theory, commonly associated with the concept established by J.P. Morgan in 1994 called Value at Risk (VaR). This metric seems to be able to capture an extreme total risk, taking into account only the moments of the asset’s own probability distribution functions. In its most commonly used versions, VaR depends on a parametric statistical framework that is based on some unreliable assumptions. Unconditional Gaussian VaR – used by the Basel Committee as the legal framework for its signatory countries – depends on the premise that one should not reject the null hypothesis that the net return on financial assets follows a normal probability distribution function (pdf), with moments fixed over time.

A promising route of related research has discussed the suitability of an unconditional normal distribution rather than a more appropriate distribution function. Another step relates to deal with the issue of conditional moments and, specifically, the mean and standard deviation used to measure VaR, by extracting both as time-varying series.

Our main contribution is to provide an empirical exercise that can be used to answer the question of whether these current risk measures are able to capture cross-effects and thus to provide good predictions. In other words, we aim to discover whether or not the cross-effects are second order effects. In South American financial markets, this issue is relevant, since there is robust evidence of contagion and integration effects, as recently reported by Matos, Siqueira and Trompieri (2014). Aligned to our main issue, we also mention the research of Hegerty (2014), which uses multivariate conditional volatility models to verify the contagion effect between South American countries.

Here, we aim to address these effects on risk management for the main South American stock market indices, by employing an innovative VaR suggested by Matos, Fonseca and de Jesus Filho (2016), which depends on the time-varying moments of a best-fitting distribution that is derived to capture the cross-effects associated with common risk drivers.

We apply three different specifications of VaR (Basel VaR, and conditional best-fitting VaR in its univariate and multivariate versions) to daily series of nominal returns on the main stock market indices of Argentina, Brazil, Chile, Colombia and Peru during the period from January 07, 1998 to December 31, 2013.

This paper is structured so that in section 2 we offer a brief review of related literature. In section 3 we detail the methodology proposed, while in Section 4 we present the empirical exercise and discuss the results. The final considerations are in the fifth section.

2. Related Literature

The literature identifies several contributions to risk measures. More specifically, it is necessary to progress methodologically and to analyze the statistical frameworks aligned to the evaluation of ratings and the Basel agreements. When J.P. Morgan proposed RiskMetrics by defining VaR as a simple and unique metric for the risk, assuming normality in the returns and modeling the volatility as the standard deviation using an exponential smoothing, this science has evolved. In
practice, even financial market procedures have changed to a point at which this metric had been widely used, so that the Basel Committee established it as a standard for risk calculations, making it part of the legal framework in its signatory countries.\textsuperscript{iv}

Nonetheless, inherent to the evolution of this science is the necessity to achieve a better accommodation of the basic violations that are characteristic of the series of net returns of financial assets. First, as can be seen in several samples of returns over the course of time, one can evidence heteroscedasticity and leptokurtosis, making it impossible to use the unconditional Gaussian VaR. Danielsson and de Vries (2000) showed the importance of incorporating this point in VaR.

A second step in this literature was taken by Lee and Lee (2011) and Jánský and Rippel (2011) that suggested innovation through using Autoregressive Moving Average (ARMA) modeling for return on the asset in association with the use of Generalized Autoregressive Heteroscedasticity (GARCH) for volatility and thus created the VaR ARMA-GARCH family. On the other hand, Berkowitz and O’Brien (2002) analyzed the VaR models of large North American banks and compared them with the VaRs measured by the ARMA-GARCH models of conditional volatility, concluding that the VaRs of the analyzed banks do not capture the changes.

Within this context, West and Cho (1995) demonstrated that, in the short run, models using the GARCH family framework, which was initially developed by Engle (1982) and then generalized by Bollerslev (1986), are more precise and better at predicting volatility than models with the constant standard deviation alone or even other frameworks of conditional volatility.

In Cappielo, Engle, and Sheppard (2006), this model was improved by the inclusion of the crossed effects of one asset into another by the use of the GARCH multivariate model, obtaining multivariate volatilities. These extensions, although very robust, still suffer from the same problem: the probability distribution function is not adequate. Actually, another research stream aims to deal with this by incorporating the statistical benefits of modeling the fitting correctly through non-normal distributions such as the hyperbolic secant, in Vaughan (2002) and Klein and Fischer (2003).

In articles closer to this study, it is necessary to list the contribution of Matos et al. (2015), who analyzed the returns series for BRIC countries, and Hegerty (2014), who used a multivariate GARCH to analyze product volatility in a set of Latin American countries. Here we aim to build on the literature, not only by applying the techniques used in these empirical works, but also by suggesting innovation, using the empirical evidence for the Latin American economies obtained in Matos Siqueira, and Trompieri (2014), who analyzed integration and financial contagion in South America. We also follow Mejías-Reyes (2000), who showed that Latin American business cycles are modeled from macroeconomic variables, through the frameworks of Markovian chains and using real GDP per capita, for a sample containing data from Argentina, Bolivia, Brazil, Chile, Colombia, Mexico, Peru, and Venezuela between 1950 and 1995.

Given this literature overview, our main intention is to measure the conditional risk of the main South American financial markets by considering the crossed effects between the conditional volatility series of these economies through the GARCH multivariate framework. Another related contribution is that by Silva et al. (2010), who applied the VaR metric for the stock exchange indices of Latin American countries using the volatility forecasting models EWMA and EQMA.
3. Methodology

3.1. VaR

Taking the Gaussian distribution as an example, the relation of the unconditional VaR to a determined level of confidence, c(%) (which is usually 95% or 99%), is simply given by:

\[
VaR_{G,I}(c\%) = \mu - \alpha_{c(\%)} \sigma,
\]  

(1)

where \( \mu \) is the population mean parameter, \( \sigma \) is the parameter that measures the population standard deviation, and \( \alpha_{c(\%)} \) is the characteristic critical alpha in a standard normal distribution, assuming the value 2.32630 for a 1% accumulated probability and 1.64485 for a 5% accumulated probability, for example.

This relation is merely the quantile function of the normal – that is, the inverse of the cumulative distribution function associated with a one-tailed probability, 5% or 1%, which is related to the confidence level according to the relation given by \( 1 - c(\%) \). In this relation, note that the insertion of the time-varying moments is trivial, since the mean and standard deviation, which are both no longer obtained from the sampling but are obtained by means of the ARMA-GARCH modeling and are given, respectively, by \( \mu_t \) and \( \sigma_t \), will replace the constant population parameters, producing the following relation for the VaR ARMA-GARCH univariate Gaussian:

\[
VaR_{G,C}(c\%) = \mu_t - \alpha_{c(\%)} \sigma_t
\]  

(2)

The extension to the multivariate framework consists only in using, in relation (2), the mean and standard deviation of the time series now obtained via the ARMA-GARCH multivariate model of the AGDCC (Asymmetric Generalized Dynamic Conditional Correlation) type, whose specification meets the best Schwarz criterion and does not have the unwanted violations of this type of model.

The question then becomes is how to obtain the analogous relation to this conditional Gaussian VaR, incorporating the information that the most appropriate distribution in terms of fitting is not the normal distribution. In this case, the search for the distribution needs to impose a certain limitation on the range of continuous distribution families, for the only distributions that can be used are those in which the standard deviation and the mean are given by univariate bijective functions (functions that have as their argument only one of the respective distribution parameters). In short, it is necessary to identify exactly which parameter is time-varying so that the mean can also be time-varying; the same applies for the standard deviation. Otherwise, the evidence that the mean and the deviation are both time-varying does not have an exact counterpart, on the assumption that the distribution parameters will also be exactly time-varying. A better way of putting this, assuming that the mean is conditional but depends on two or more parameters of the distribution in question, is to consider how to carry out the necessary bijection so that the parameter can be substituted by the mean in the quantile function formula.
For example, in the Dagun function (4p), which is well suited to banks’ returns in Brazil, the standard deviation function has, as its arguments, all four parameters of the distribution; these parameters also appear in the quantile function, making it impossible to establish a relation between the inverse of the accumulated function and the standard deviation, which is replaced by the conditional standard deviation.

This happens because in most probability distributions the parameters are not exactly given by the mean and the deviation in the same way as they are given in the normal. Therefore, it is necessary to obtain a bijection such that the parameters become a function of the mean and the deviation, respectively, and allow the inverse cumulative function to be expressed by the mean and the standard deviation and, finally, to insert them as time-varying.

The ultimate goal is therefore to obtain two relations at the same level of confidence \( c \%, \) both based on a probability distribution with a proper fitting. Consider the parametric quantile, the VaR best-fitting unconditional univariate, \( VaR^{BF,J}(c\%) \), given by:

\[
VaR^{BF,J}(c\%) = F_{BF}^{-1}(1 - c|\Theta),
\]  

(3)

The ARMA-GARCH univariate conditional best-fitting VaR will be given by a quantile function, but it is no longer a function of the parameter vector itself; it has as arguments the time-varying mean and standard deviation, according to the following relation:

\[
VaR^{BF,C}(c\%) = F_{BF}^{-1}(1 - c|\mu_t, \sigma_t)
\]  

(4)

Using the Laplace probability distribution function as the best-fitting distribution, whose parameters are \( \mu \) and \( \lambda \), in which the standard deviation is given by \( \sigma = \sqrt{2}/\lambda \), and incorporating the extracted conditional moments of an ARMA-GARCH multivariate, \( VaR^{MLapC} \) can be generated and is given by:

\[
VaR^{MLapC}(c\%) = \mu_t + \sigma_t \frac{\ln(2(1-c\%))}{\sqrt{2}}
\]  

(5)

### 3.2. Validating backtesting framework

Finally, for each VaR specification used herein, predictions are made one step forward in the sample for each return series, so they can be compared. The methods of Basel backtesting and those set out in Lopez (1999), Kupiec (1995), and Christoffersen (1998) are used to compare the series, each of these having certain features that will help in verifying whether the effects of financial interconnection incorporated into the VaR influence the risk calculation and with what magnitude.

More specifically, as Jorion (2006) says, when VaR models cannot predict the risk precisely, they lose their usefulness. Adhesion tests (backtesting) are employed to check the performance of these models in order to examine whether the losses foreseen by VaR are consistent with the reality of
the data series. Backtesting is important from two perspectives, risk management and statistics, according to Campbell (2006). Campbell discusses various types of testing methodologies and explains how all of them have weaknesses and that more than one should therefore be applied to find a diagnosis closer to reality.

4. Empirical Exercise
4.1. Database

In an attempt to analyze the risk of the major financial indices of Latin America, it is necessary to use a long and consistent time series of returns of these indices. For this reason, the financial markets suggest that, because of their transaction volume and composition, the major indices are: i) IBOVESPA (Index of the São Paulo Stock Exchange, Brazil), ii) IGBC (Colombia Stock Exchange General Index, Colombia), iii) MERVAL (Buenos Aires Stock Exchange Merval Index, Argentina), iv) IGBVL (Lima Stock Exchange General Index, Peru), and v) IPSA (Santiago Stock Exchange IPSA Index, Chile). In terms of time series, 4,118 observations of nominal daily net returns on these indices from January 07, 1998 to December 31, 2013 were extracted from the CMA Trade®.

The use of this set of indices is aligned with the research of Matos, Siqueira and Trompieri (2014), where the presence of integration and financial contagion was detected. The indices differ in their maturity. The most traditional is the IBOVESPA, which began in 1968 and underwent a change in its calculation methodology in 2013; on the other hand, the IGBC’s calculation methodology only became official in 2001, but there are data available at Economática from 1991. The Argentinian index is composed from the stock price and thus differs from the others, which have a methodology in common that includes a rebalancing on the basis of composition weighted by the capitalization of the stock markets.

4.2. Summary statistics

From the visual analysis shown in graph 1, there is similar behavior for periods of growth and decline, which can especially be seen for the Brazilian index from June 1999 to June 2002; for almost this entire period, this was above the other indices as a consequence of the currency depreciation policy that favored the growth of the industrial sector, according to Cardoso (2007).

The Peruvian index shows a much higher growth trend than the others between the second half of 2006 and the second half of 2007; this growth reflects foreign capital inflows for investment in the mining sector as a result of a new policy in relation to international trade agreements.

The Argentinian index had a very similar performance to the Chilean index, until December 2012; however, from January 2013, an opposite shift to the others can be observed, particularly in the second half of 2013, when it started to show greater gains, reaching a cumulative return of 676.79%, while the others showed a downward trend. During 2008, there was, overall, a declining trend in the South American indices, especially in the second half of that year. This period of decline had a greater effect on the Peruvian stock exchange, where there was a drop of approximately 74.62% due to strong economic dependence on foreign capital, whereas the maximum loss for the IBOVESPA was 61.3%.
After this period of a general fall in the South American indices, it is clear that the Peruvian index recovered more strongly, achieving an average daily gain, from the beginning of January 2009 until the end of the sample period, of around 0.073%; this is second only to the Argentinian index, which recovered to show an average daily gain of 0.141% as a result of the almost exponential growth seen in 2013. The presence of clusters of volatility in the daily return series would be expected, for observations of returns show several non-linear patterns, according to Gourieroux and Jasiak (2001).

Table 1 shows the main descriptive statistics for the indices. Throughout the sample period, the Argentinian index accumulated a net gain of 676.79%, while the IPSA showed a gain of only 261.28%. The MERVAL had the highest standard deviation and the highest average daily return among the analyzed indices, but its performance, according to the Sharpe ratio, was similar to that of the Chilean index.

The performance of the Colombian and Peruvian indices was very similar, with the Peruvian index having the largest metric associated with a gain, while the Colombian index, compared with the Peruvian, showed better measures of risk, which resulted in the highest Sharpe ratio among the indices observed. In its turn, the IBOVESPA had the worst Sharpe ratio and the second lowest downside risk, while the Chilean index had the lowest values for both the standard deviation and the downside risk.

All the indices, except for the IGBVL, showed asymmetry to the right, with this being more pronounced for the IBOVESPA and less pronounced for the MERVAL.

All the indices feature leptokurtosis, since they show a kurtosis greater than the normal distribution, which is 3, and the magnitude is greatest for the Brazilian index and lowest for the Chilean one. This evidence suggests a priori the non-normality of the index return series. In this sense, with the aim of verifying the Gaussian nature more properly, the Jarque-Bera test was used; the result of this test indicates the rejection of the null hypothesis of normality for all the series at a 1% level of significance. According to Mahadeva and Robinson (2004), the biggest problem of using regression models when there are non-stationary variables is that the standard error obtained is biased. Non-stationary series are not suitable if the final purpose is to make predictions, as they have little practical value because the behavior of the series is conditional on time. To examine whether the series are stationary, the Dickey and Fuller (1979) unit root test, in its augmented version, also known as the ADF test, was performed, and also the Phillips and Perron (1988) test. According to Mahadeva and Robinson (2004), the Phillips and Perron test (1988) is used as an alternative to the ADF test because, being a non-parametric test, it has advantages in various applications.

Thus, it was verified that for all the series, at a 1% significance level in both tests, the null hypothesis of the presence of a unit root is rejected; this is expected in returns series. Also, in order to determine whether or not there is heteroscedasticity to be modeled in the residues, Engle’s ARCHLM test was performed, and the results are shown in Table 1. It can be seen that for all series, at a 99% confidence level, the null hypothesis of the homoscedasticity of the residues is rejected. According to Engle (2001), in the presence of heteroscedasticity the regression coefficients estimated by the ordinary least squares method remain unbiased, but they give a false sense of precision. Engle (2001) argues that the autoregressive conditional heteroskedasticity
ARCH and Generalized Autoregressive Heteroskedasticity (GARCH) models treat heteroscedasticity as a variance to be modeled, instead of considering it as a problem to be fixed.

Figure 2 shows the behavior of the returns series for the indices. From a visual analysis of these graphics, it is possible to highlight the presence of volatility clusters (volatility clustering), and large swings, which can particularly be seen at the end of 2008 and between 1998 and 1999, when there were common fluctuations: in 2008, as a result of the subprime crisis peak and from 1998 to 1999 as a result of the 1999 currency crisis.

It can also be seen that the Colombian index shows turbulence even before the crisis, with the largest negative peaks in June 2006. The Argentinian index has its most turbulent period in 2002.

Finally, Table 2 reports the initial ranking positions in terms of the fitting of a wide range of distribution functions for the probability present, using the EasyFit software. It is not exactly surprising that the normal distribution does not perform very well in the fitting rankings reported in this table. It can be observed that, in the aggregate ranking that considers the more than 50 continuous distributions that make up the Easy database, the normal distribution occupies positions like 10th and 17th, with the best distributions being identified as the Laplace for the Brazilian, Chilean and Argentinian indices. The best distribution for the Peruvian and Colombian indices is the Johnson SU.

Thus, for only these two indices, among the subset of distributions that can establish the bijection necessary for the quantile function to have a time-varying mean and standard deviation as arguments, it is noted that the Laplace function, which occupies second place in the overall ranking, presents the most appropriate fitting. Other distributions that satisfy this condition and have adequate fitting for the sector indices and for shares in the Brazilian capital markets are the logistic distribution and the secant hyperbolic distribution, among others.

4.3. Results: ARMA-GARCH

Based on Table 3, the series have different specifications, both in terms of the ARMA modeling, and in terms of the GARCH modeling, with the IGBC showing the simplest linear framework.

The parameters of the models estimated for the IBOVESPA, IGBC and IPSA indices are individually significant, even at the 1% level, both in the ARMA specification and in the GARCH framework. The IGBVL index obtained some parameters that were individually insignificant even at the 10% significance level in the ARMA specification, but the estimated GARCH parameters showed no problem with individual significance, even at the 1% level. However, the estimated model for the MERVAL presented a problem of individual significance in some parameters, for both the ARMA specification and the GARCH framework.

Still considering Table 3, the p-value of the F statistic for the estimated ARMA-GARCH models is reported. The results demonstrate, in all the models estimated, that the null hypothesis that the slope coefficients of the estimated equations are jointly statistically insignificant is rejected at a 99% confidence level. Thus, the F-test confirms that the estimated models can be used to represent the return series of the South American indices for both the models estimated for the IGBVL and MERVAL indices, which showed some individual insignificant parameters, as well as for the other indices. The GARCH models obtained, except for the Argentinian index, are aligned with
the results of Ferreira (2013), who states that “financial series are often better adjusted to low order GARCH models, with GARCH (1.1) being a very popular choice.”

4.4. Results: Joint estimation

As a result of the contagion and financial integration for the main market indices in South America highlighted in Matos, Siqueira, and Trompieri (2014), it is intuitively clear that there can be benefits in risk modeling when considering the cross-effects in estimating the ARMA-GARCH frameworks. This happens through the joint estimation of this framework, that is, by using a multivariate GARCH of the AGDCC type (Asymmetric Generalized Dynamic Conditional Correlation). Theoretically, this framework accommodates all the major criticism of the unconditional Gaussian VaR used in most parametric approaches to risk management.

In practice, the estimation results of the conditional risk series show, as can be seen in Figure 3, a pattern of conditional volatility very similar to the ARMA-GARCH estimations in a univariate and isolated way. The peaks occur on the same days, with rare exceptions, and the orders of magnitude are also similar. However, this apparently negligible difference does not translate into exactly identical VaRs, since the return series fitted change greatly within the univariate and multivariate frameworks. In short, it is possible to observe a pattern of an inferior envelope being provided by the Laplace Multivariate Conditional VaR, with more extremes, that is, with an excess of conservatism that apparently provides a model with fewer exceptions.

4.5. Results: validation of models through backtesting

Aiming to analyzing cross-effects due to contagion and integration, in Figure 4 we plot the time evolution of the VaR series generated by following the multivariate metric, with a confidence level of 99% and a horizon of one day, as well as the daily return series for each banking index. Visual analysis allows us to suggest that, for all series, the maximum expected losses predicted by the multivariate VaR are closer to the realized losses than those predicted by the base VaR.

According to Figure 4, there are three moments for all the South American markets when the highest values of VaR are seen, and these match the times of greatest volatility: i) the first half of 2009, still reflecting the subprime crisis in the United States; ii) the first half of 2010, as a result of a time of instability in the euro zone due to the first signs of the sovereign debt crisis; and iii) the second half of 2011, with the emergence of the same crisis signaling the possibility of some government defaults. Our purpose is to draw inferences about the relevance of contagion and financial integration effects between the South American stock market indices, and therefore multivariate conditional best-fitting VaR, which incorporates these cross-effects, is compared with the corresponding univariate version.

These models are similar in all aspects, except for the conditional moments incorporated into the distribution with the better fitting, which, in the latter VaR, are estimated from a univariate ARMA-GARCH, instead of a multivariate framework. To continue this comparison we must use the backtesting methods defined in the previous section. We report the results of the proposed backtesting in Table 4.
Using Basel backtesting, which takes into account violations in the absolute or relative amounts over the 4,118 daily observations, we reject Basel VaR for all economies, while univariate VaR is rejected for Argentina. Multivariate VaR fails for Colombia, Argentina, and Peru. For all the economies, the number of violations is higher for Basel VaR than for univariate or multivariate VaR. Using backtesting that takes into account the frequency and conditionality of losses exceeding the VaR (the tests proposed by Kupiec (1995) or Christoffersen (1998) or the joint test proposed by these authors), while Basel VaR is rejected for all five economies, we fail to reject univariate conditional best-fitting VaR for all markets, while the multivariate version is rejected only for Peru and Chile. Since for most of the indices there are no successive violations when we use the univariate or multivariate VaR measures, we cannot measure a value for the statistical test proposed by Christoffersen (1998) or for the joint test.

Table 4 also reports useful partial metrics to measure the average violation and the excessive conservatism. For the IBOVESPA in Brazil and the MERVAL in Argentina, multivariate VaR shows the best performance regarding both partial statistics, supporting a conclusion that contagion and integration effects are relevant for these two financial markets, which are the largest in the continent.

5. Conclusion

Risk management, whether on the local scene for a particular company, or in a broader and more complex international context that involves, for example, a country’s index, requires a statistical framework that can accommodate the characteristics of the time series in question. What can be seen from the commonly used parametric frameworks is the adoption of assumptions that prove invalid, such as the Gaussian nature of the returns and homoscedasticity. In this context, this study follows Matos et al. (2015) in order to accommodate these two violations in a VaR framework that is described as univariate best-fitting. Subsequently, this study follows Matos, Fonseca, and de Jesus Filho (2016) by adding the possible cross-effects associated with contagion and the existing financial integration of this group of countries.

The results, in summary, suggest that the statistical refinement associated with the use of the Laplace probability distribution generates a risk management model with the lowest number of violations and with the average violation having a lower order of magnitude, but yet with the absence or small incidence of violation clusters, so avoiding excessive conservatism.

The insertion of the cross-effects does not generally translate into improvements if the model is validated through backtesting, except in relation to the IBOVESPA.

References


Figure 1. Cumulative daily return on South American stock market indices \(^a,b\)

\(\text{Figure 1. Cumulative daily return on South American stock market indices } \quad a,b\)

\textbf{Figure 1. Cumulative daily return on South American stock market indices} \(^a,b\)

\begin{center}
\includegraphics[width=\textwidth]{figure1.png}
\end{center}

\(^a\) Source: CMA Trade. \(^b\) Cumulative daily return on South American major market indices, during the period from January 07, 1998 to December 31, 2013, 4118 observations.
Table 1. Summary statistics and violation tests applied to returns on South American stock market indices

<table>
<thead>
<tr>
<th>Statistics/Index</th>
<th>MERVAL</th>
<th>IPSA</th>
<th>IGBVL</th>
<th>IGBC</th>
<th>IBOV</th>
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<td></td>
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<td>0.063%</td>
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<td>781.886%</td>
<td>776.445%</td>
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<tr>
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<td>1.109%</td>
<td>1.423%</td>
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<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
</tr>
<tr>
<td><strong>ARCH LM test</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F Statistic $^d$</td>
<td>217.318</td>
<td>195.754</td>
<td>908.633</td>
<td>289.053</td>
<td>95.987</td>
</tr>
<tr>
<td>P-value</td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
</tr>
</tbody>
</table>

$^a$ Panel containing daily time series of nominal net returns on major stock exchanges in Latin America countries from 1998 to 2013. (4118 observations); $^b$ Jarque-Bera test for series normality, the test statistics measure the difference between skewness and kurtosis of the series with a normal distribution under the null hypothesis that the series follows a normal distribution; $^c$ Phillips-Perron unit root test, at a level with constant and trend, spectral estimation method Default (Bartlett Kernel); $^d$ Engle’s ARCH LM Test, the "Lagrange multiplier" type, for the hypothesis of residuals from ARMA models returns having an ARCH structure, under the null hypothesis that there is no ARCH with a lag. Null Hypothesis: There is no ARCH effect.
Figure 2. Nominal net returns on South American major market indices

a. IBOVESPA  

b. MERVAL  

c. IPSA  

d. IGBVL  

e. IGBC  

* Daily series of nominal net return obtained from the time series closing price (end-of-day) of the indices in question during the period from January 07, 1998 to December 31, 2013, 4,118 observations - Source: CMA Trade.
Table 2. Identification of the best fitting probability distribution function $^a$

<table>
<thead>
<tr>
<th>Index</th>
<th>Country</th>
<th>Best Fitting probability distribution function(pdf) $^a$</th>
<th>Anderson-Darlin statistic test</th>
<th>Parameters</th>
<th>Best fitting pdf critical value(1%)</th>
<th>Global ranking(best fitting pdf) $^b$</th>
<th>Global ranking (normal distribution)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IBOVESPA</td>
<td>Brazil</td>
<td>Laplace</td>
<td>3.002</td>
<td>$l= 68.033; \mu=6.00 E-4$</td>
<td>-5.690%</td>
<td>1$^a$</td>
<td>17$^b$</td>
</tr>
<tr>
<td>IGBC</td>
<td>Colombia</td>
<td>Laplace</td>
<td>13.789</td>
<td>$l= 107.35; \mu=6.1360 E-4$</td>
<td>-3.583%</td>
<td>2$^a$</td>
<td>15$^b$</td>
</tr>
<tr>
<td>IGBVL</td>
<td>Peru</td>
<td>Laplace</td>
<td>13.306</td>
<td>$l= 99.367; \mu=6.3003 E-4$</td>
<td>-3.874%</td>
<td>2$^a$</td>
<td>10$^b$</td>
</tr>
<tr>
<td>IPSA</td>
<td>Chile</td>
<td>Laplace</td>
<td>2.040</td>
<td>$l= 127.49; \mu=3.73 E-4$</td>
<td>-3.031%</td>
<td>1$^a$</td>
<td>16$^b$</td>
</tr>
<tr>
<td>MERVAL</td>
<td>Argentina</td>
<td>Laplace</td>
<td>4.085</td>
<td>$l= 65.991; \mu=7.27 E-4$</td>
<td>-5.855%</td>
<td>1$^a$</td>
<td>12$^b$</td>
</tr>
</tbody>
</table>

$^a$ The best-fitting pdf is identified and the ranking is done based on Anderson and Darling (1952) statistic. Our search for this specific and idiosyncratic distribution needs to impose a limitation on the range of continuous distribution families, because we can only use pdf's in which the standard deviation and the mean are given by univariate bijection, i.e., each moment depends on only one pdf parameter. $^b$ This is an unrestricted ranking, considering all continuous timing distributions.
Table 3. Estimation ARMA-GARCH model. \(^{a,b}\)

<table>
<thead>
<tr>
<th>Index</th>
<th>ARMA-GARCH</th>
<th>Estimation</th>
<th>Log verossim.</th>
<th>Akaike</th>
<th>Schwarz</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Period: from 1998:01 to 2013:12 (4120 observations)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IBOVESPA</td>
<td>ARMA(2,1)-GARCH(1,1)</td>
<td>(y_t = 0.000905 + 1.525010 y_{t-1} - 0.723076 y_{t-2} - 1.529007 \epsilon_{t-1} + 0.712479 \epsilon_{t-2} )</td>
<td>( \sigma_t^2 = 0.000007 + 0.087503 \epsilon_{t-1}^2 + 0.894567 \sigma_{t-1}^2 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>IGBC</td>
<td>AR(1)-GARCH(1,1)</td>
<td>(y_t = 0.000705 + 0.180992 y_{t-2} )</td>
<td>( \sigma_t^2 = 0.000000 + 0.002787 \epsilon_{t-1}^2 + 0.732474 \sigma_{t-1}^2 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td></td>
</tr>
<tr>
<td>IKVRL</td>
<td>ARMA(2,1)-GARCH(1,1)</td>
<td>(y_t = 0.000949 + 0.957639 y_{t-1} - 0.030871 y_{t-2} - 0.760778 \epsilon_{t-1} - 0.120363 \epsilon_{t-2} )</td>
<td>( \sigma_t^2 = 0.00000615 + 0.202787 \epsilon_{t-1}^2 + 0.732474 \sigma_{t-1}^2 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>MERVAL</td>
<td>ARMA(1,2)-GARCH(2,2)</td>
<td>(y_t = 0.001109 + 0.730897 y_{t-1} - 0.677681 \epsilon_{t-1} - 0.021408 \epsilon_{t-2} )</td>
<td>( \sigma_t^2 = 0.000002 + 0.059806 \epsilon_{t-1}^2 + 0.106012 \epsilon_{t-2}^2 + 0.195203 \sigma_{t-1}^2 + 0.593848 \sigma_{t-2}^2 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>IPSA</td>
<td>ARMA(2,1)-GARCH(1,1)</td>
<td>(y_t = 0.000719 + 0.908219 y_{t-1} - 0.143530 y_{t-2} - 0.721234 \epsilon_{t-1} )</td>
<td>( \sigma_t^2 = 0.0000025 + 0.130792 \epsilon_{t-1}^2 + 0.850493 \sigma_{t-1}^2 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td></td>
</tr>
</tbody>
</table>

\(^{a}\) ARMA models estimated via OLS using the Newey-West coefficient for heteroscedasticity. \(^{b}\) ARMA-GARCH models estimated via ARCH, with normal errors distribution (Gaussian), using the Bollerslev-Wooldridge covariance coefficient for heteroscedasticity.
Figure 3. Conditional volatility of returns on South American stock market indices \(^a,b\)

\(^a\) Daily series of nominal net return obtained from the time series closing price (end-of-day) of the indices in question during the period from January 07, 1998 to December 31, 2013, 4,118 observations - Source: CMA Trade. \(^b\) One-step-ahead prediction performed using the ARMA-GARCH models estimated jointly.
Figure 4. Laplace-based absolute VaR (99%, 01 day) of South American stock market indices

- **a. IPSA (Chile)**
- **b. IBOVESPA (Brazil)**
- **c. IGBC (Colombia)**
Figure 4. Laplace-based absolute VaR (99%, 01 day) of South American stock market indices

-20%  -15%  -10%  -5%   0%   5%   10%   15%   20%
jan-98 jan-99 jan-00 jan-01 jan-02 jan-03 jan-04 jan-05 jan-06 jan-07 jan-08 jan-09 jan-10 jan-11 jan-12 jan-13

--- Laplace-based multivariate conditional VaR
  
  d. MERVAL (Argentina)

--- Daily return

-25%  -20%  -15%  -10%  -5%   0%   5%   10%   15%   20%
jan-98 jan-99 jan-00 jan-01 jan-02 jan-03 jan-04 jan-05 jan-06 jan-07 jan-08 jan-09 jan-10 jan-11 jan-12 jan-13

--- Laplace-based multivariate conditional VaR
  
  e. IGBVL (Peru)

--- Daily return
Table 4. Backtesting methods applied to VaR of returns on South American stock market indices

<table>
<thead>
<tr>
<th>Index (country)</th>
<th>VaR specification</th>
<th>Basel test (4118 observations)</th>
<th>Partial statistics</th>
<th>Kupiec Test (^c) (crit. value (X^2(1) = 6.63))</th>
<th>Christoffersen Test (^d) (crit. value (X^2(1) = 6.63))</th>
<th>Kupiec-Christoffersen Test (^d) (crit. value (X^2(1) = 9.21))</th>
</tr>
</thead>
<tbody>
<tr>
<td>IBOVESPA (Brazil)</td>
<td>Gaussian-based unconditional</td>
<td>57</td>
<td>1.384% reject</td>
<td>0.331% 5.253%</td>
<td>5.48 no reject</td>
<td>14.93 reject</td>
</tr>
<tr>
<td></td>
<td>Laplace-based univariate conditional</td>
<td>31</td>
<td>0.753% no reject</td>
<td>0.147% 6.076%</td>
<td>2.78 no reject</td>
<td>- x - no applic</td>
</tr>
<tr>
<td></td>
<td>Laplace-based multivariate conditional</td>
<td>29</td>
<td>0.704% no reject</td>
<td>0.146% 6.075%</td>
<td>4.06 no reject</td>
<td>- x - no applic</td>
</tr>
<tr>
<td>IGBC (Colombia)</td>
<td>Gaussian-based unconditional</td>
<td>74</td>
<td>1.797% reject</td>
<td>0.287% 3.320%</td>
<td>21.37 reject</td>
<td>39.94 reject</td>
</tr>
<tr>
<td></td>
<td>Laplace-based univariate conditional</td>
<td>38</td>
<td>0.923% no reject</td>
<td>0.083% 3.835%</td>
<td>0.25 no reject</td>
<td>- x - no applic</td>
</tr>
<tr>
<td></td>
<td>Laplace-based multivariate conditional</td>
<td>48</td>
<td>1.166% no reject</td>
<td>0.092% 3.845%</td>
<td>1.08 no reject</td>
<td>5.45 no reject</td>
</tr>
<tr>
<td>MERVAL (Argentina)</td>
<td>Gaussian-based unconditional</td>
<td>74</td>
<td>1.797% reject</td>
<td>0.383% 5.414%</td>
<td>21.37 reject</td>
<td>6.28 no reject</td>
</tr>
<tr>
<td></td>
<td>Laplace-based univariate conditional</td>
<td>46</td>
<td>1.117% reject</td>
<td>0.170% 6.276%</td>
<td>0.55 no reject</td>
<td>0.37 no reject</td>
</tr>
<tr>
<td></td>
<td>Laplace-based multivariate conditional</td>
<td>44</td>
<td>1.068% reject</td>
<td>0.183% 6.271%</td>
<td>0.19 no reject</td>
<td>0.46 no reject</td>
</tr>
<tr>
<td>IGBVL (Peru)</td>
<td>Gaussian-based unconditional</td>
<td>78</td>
<td>1.894% reject</td>
<td>0.348% 3.587%</td>
<td>26.34 reject</td>
<td>47.62 reject</td>
</tr>
<tr>
<td></td>
<td>Laplace-based univariate conditional</td>
<td>38</td>
<td>0.923% no reject</td>
<td>0.104% 4.134%</td>
<td>0.25 no reject</td>
<td>- x - no applic</td>
</tr>
<tr>
<td></td>
<td>Laplace-based multivariate conditional</td>
<td>45</td>
<td>1.093% reject</td>
<td>0.131% 4.139%</td>
<td>0.35 no reject</td>
<td>10.32 reject</td>
</tr>
<tr>
<td>IPSA (Chile)</td>
<td>Gaussian-based unconditional</td>
<td>68</td>
<td>1.651% reject</td>
<td>0.199% 2.802%</td>
<td>14.75 reject</td>
<td>38.88 reject</td>
</tr>
<tr>
<td></td>
<td>Laplace-based univariate conditional</td>
<td>35</td>
<td>0.850% no reject</td>
<td>0.073% 3.219%</td>
<td>0.99 no reject</td>
<td>- x - no applic</td>
</tr>
<tr>
<td></td>
<td>Laplace-based multivariate conditional</td>
<td>40</td>
<td>0.971% no reject</td>
<td>0.084% 3.228%</td>
<td>0.03 no reject</td>
<td>12.11 reject</td>
</tr>
</tbody>
</table>

\(^a\) We performed the backtestsings reported here on the daily series of absolute measures of VaR, with 99% confidence level one day ahead, during the period from January 07, 1998 to December 31, 2013, 4,118 observations. We may reject the VaR specification where the test statistic is higher than the critical value. \(^b\) Unconditional coverage test proposed by Kupiec (1995), with confidence around 99% defined by a ratio of log-likelihood having chi-square asymptotic distribution with one degree of freedom under the null hypothesis that the level of VaR confidence is the real likelihood of losses. \(^c\) Unconditional coverage test proposed by Christoffersen (1998), with the region of confidence of approximately 99%, defined by a ratio of log-likelihood that has asymptotic chi-square with one degree of freedom under the null hypothesis that the exceptions are independent serially. \(^d\) Joint test of conditional and unconditional coverage, with confidence of approximately 99%, defined by a ratio of log-likelihood having chi-square asymptotic distribution with two degrees of freedom under the null hypothesis that the confidence level of the VaR is the real likelihood of losses and the exceptions are serially independent.
First draft July 2016. Financial support from CNPQ-Brazil is gratefully acknowledged. We are thankful to anonymous referees and the editor for their insightful comments. We thank seminar participants at Federal University of Ceará for their comments and suggestions. The usual disclaimer applies.

The value-at-risk model can also be used to optimize portfolios, according to Andersson at all (2001), who applied this technique to optimize credit risk; additionally, Tokpavi and Vaucher (2012) suggested the use of conditional value-at-risk to find portfolios whose goal is to minimize risk.

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