NOTE: This is probably one of the most important skills required for any calculus class. Your success in the class would partly depend on this material.

**MAIN IDEA:**

Rational expressions in algebra are nothing but algebraic fractions. Similar to the arithmetic fractions that you learned in elementary grades, recall the following facts:

To add or subtract any two rational expressions, first you need to find the **least common denominator** (LCD) of the fractions involved.

However, to multiply or divide rational expressions, you do not need an LCD.

**Examples:**

1. Simplify and write as a single fraction: \( \frac{1}{x} + \frac{3}{2x} \)

   The LCD of the denominators \( x \) and \( 2x \) is clearly \( 2x \). We need to convert each denominator into \( 2x \).

   \[
   \frac{1}{x} + \frac{3}{2x} = \frac{1}{x} \cdot \frac{2}{2} + \frac{3}{2x} = \frac{2}{2x} + \frac{3}{2x} = \frac{2 + 3}{2x} = \frac{5}{2x}
   \]

2. Simplify and write as a single fraction: \( \frac{2}{x} - \frac{3}{x^2} \)

   The LCD of the denominators \( x \) and \( x^2 \) is clearly \( x^2 \). Therefore, we need to convert both denominators into \( x^2 \).

   \[
   \frac{2}{x} - \frac{3}{x^2} = \frac{2}{x} \cdot \frac{x}{x} - \frac{3}{x^2} = \frac{2x}{x^2} - \frac{3}{x^2} = \frac{2x - 3}{x^2}
   \]

3. Simplify and write as a single fraction: \( \frac{3}{4x} - \frac{5}{6x^2} + \frac{7}{2x} \)

   The LCD of the three denominators \( 4x, 6x^2, \) and \( 2x \) is \( 12x^2 \).

   \[
   \frac{3}{4x} - \frac{5}{6x^2} + \frac{7}{2x} = \frac{3}{4x} \cdot \frac{3x}{3x} - \frac{5}{6x^2} \cdot \frac{2}{2} + \frac{7}{2x} \cdot \frac{6x}{6x} = \frac{9x}{12x^2} - \frac{10}{12x^2} + \frac{42x}{12x^2} = \frac{9x - 10 + 42x}{12x^2} = \frac{51x - 10}{12x^2}
   \]
4. **Good!** Simplify and write as a single fraction: \( x - \frac{1}{2x} \)

\[
x - \frac{1}{2x} = \frac{x}{1} - \frac{1}{2x} = \frac{x}{1} \cdot \frac{2x}{2x} - \frac{1}{2x} = \frac{2x^2}{2x} - \frac{1}{2x} = \frac{2x^2 - 1}{2x}
\]

5. **Very Good!** Simplify and write as a single fraction: \( \sqrt{x} + \frac{3}{2\sqrt{x}} \)

\[
\sqrt{x} + \frac{3}{2\sqrt{x}} = \frac{\sqrt{x}}{1} + \frac{3}{2\sqrt{x}} = \frac{\sqrt{x}}{1} \cdot \frac{2\sqrt{x}}{2\sqrt{x}} + \frac{3}{2\sqrt{x}} = \frac{2x}{2\sqrt{x}} + \frac{3}{2\sqrt{x}} = \frac{2x + 3}{2\sqrt{x}}
\]

6. **EXCELLENT!** Simplify the complex fraction: \( \frac{1}{x} - \frac{1}{y} \)

First rewrite:

\[
\frac{1}{x} - \frac{1}{y} = \frac{1}{x} - \frac{1}{y}
\]

Now look at the four denominators of the smaller fractions. They are \( x \), \( y \), \( 1 \), and \( 1 \). The combined LCD of these four denominators is simply \( xy \). One method of simplifying this complex fraction is to multiply the numerator and the denominator of the main fraction by this LCD \( xy \), as shown below. There is also a second method, but it is not described here.

\[
\frac{1}{x} - \frac{1}{y} = \frac{1}{x} \cdot \frac{y}{y} - \frac{1}{y} \cdot \frac{x}{x} = \frac{y - x}{xy(x - y)} = \frac{-1}{xy}
\]

7. **EXCELLENT!** Simplify the complex fraction: \( \frac{1}{x+2} - \frac{1}{x} \)

\[
\frac{1}{x+2} - \frac{1}{x} = \frac{1}{x+2} - \frac{1}{x} = \frac{1}{x+2} \cdot \frac{x(x+2)}{x} = \frac{1}{x+2} \cdot \frac{x(x+2)}{x} - \frac{1}{x} \cdot \frac{x(x+2)}{x} = \frac{x(x+2)}{2(x+2)} - \frac{1}{x} \cdot \frac{x(x+2)}{x+2} = \frac{x(x+2)}{2x(x+2)} - \frac{1}{x+2} = \frac{2x - x - 2}{2x(x+2)} = \frac{-2}{2x(x+2)} = \frac{-1}{x(x+2)}
\]
8. FANTASTIC! Simplify the expression in to a single fraction: \( (x + 1) \cdot \frac{1}{2 \sqrt{x}} + 1 \cdot (\sqrt{x} + 2) \)

Make every attempt to do problems like this, as it is a must in this class:

\[
\begin{align*}
(x + 1) \cdot \frac{1}{2 \sqrt{x}} + 1 \cdot (\sqrt{x} + 2) &= \frac{(x + 1)}{2 \sqrt{x}} + \frac{(\sqrt{x} + 2)}{1} \\
&= \frac{(x + 1)}{2 \sqrt{x}} + \frac{(\sqrt{x} + 2)}{2 \sqrt{x}} \\
&= \frac{(x + 1)}{2 \sqrt{x}} + \frac{(\sqrt{x} + 2) \cdot 2 \sqrt{x}}{2 \sqrt{x}} \\
&= \frac{(x + 1)}{2 \sqrt{x}} + \frac{2x + 4 \sqrt{x}}{2 \sqrt{x}} \\
&= \frac{x + 1 + 2x + 4 \sqrt{x}}{2 \sqrt{x}} \\
&= \frac{3x + 4 \sqrt{x} + 1}{2 \sqrt{x}}
\end{align*}
\]

9. Simplify the expression into a single fraction, and leave the answer in a factored form:

\[
\frac{(4t^2 - 1) \cdot 3(2t + 3)^2 \cdot 2 - (2t + 3)^3 \cdot 8t}{(4t^2 - 1)^2}
\]

\[
\begin{align*}
\frac{(4t^2 - 1) \cdot 3(2t + 3)^2 \cdot 2 - (2t + 3)^3 \cdot 8t}{(4t^2 - 1)^2} &= \frac{6(4t^2 - 1)(2t + 3)^2 - 8t(2t + 3)^3}{(4t^2 - 1)^2} \\
&= \frac{2(2t + 3)^2[3(4t^2 - 1) - 4t(2t + 3)]}{(4t^2 - 1)^2} \\
&= \frac{2(2t + 3)^2[12t^2 - 3 - 8t^2 - 12t]}{(4t^2 - 1)^2} \\
&= \frac{2(2t + 3)^2(4t^2 - 12t - 3)}{(4t^2 - 1)^2}
\end{align*}
\]

10. Simplify the expression into a single fraction, and write without any fractional exponents:

\[
x \cdot \frac{1}{2} \cdot (x^2 + 1)^{-\frac{1}{2}} \cdot 2x + 1 \cdot (x^2 + 1)^\frac{1}{2}
\]
\[ x \cdot \frac{1}{2} \cdot (x^2 + 1)^{-\frac{1}{2}} \cdot 2x + 1 \cdot (x^2 + 1)^{\frac{1}{2}} = x^2 \cdot (x^2 + 1)^{-\frac{1}{2}} + (x^2 + 1)^{\frac{1}{2}} \]

\[ = \frac{x^2}{(x^2 + 1)^{\frac{1}{2}}} + \frac{(x^2 + 1)^{\frac{1}{2}}}{1} = \frac{x^2}{(x^2 + 1)^{\frac{1}{2}}} + \frac{(x^2 + 1)^{\frac{1}{2}}}{(x^2 + 1)^{\frac{1}{2}}} = \frac{x^2}{(x^2 + 1)^{\frac{1}{2}}} + \frac{x^2 + 1}{(x^2 + 1)^{\frac{1}{2}}} \]

\[ = \frac{x^2 + x^2 + 1}{(x^2 + 1)^{\frac{1}{2}}} = \frac{2x^2 + 1}{(x^2 + 1)^{\frac{1}{2}}} = \frac{2x^2 + 1}{\sqrt{x^2 + 1}} \]