1. \(ABC\) is a right triangle with the right angle at \(C\). Suppose \(CD\) is the altitude of this triangle drawn from the angle \(C\) to the opposite side \(AB\). Given that \(AD = 2 \text{ cm}\) and \(BD = 18 \text{ cm}\), find \(CD\).

Answer:

**METHOD I (Use Pythagorean Theorem):**

Let \(AC = p\) and \(BC = q\). We will use the Pythagorean Theorem three times for the right triangles \(ACD, BCD,\) and \(ABC:\)

From the right triangle \(ACD:\)
\[
p^2 = x^2 + 2^2 \tag{1}
\]

From the right triangle \(CD:\)
\[
q^2 = x^2 + 18^2 \tag{2}
\]

From the right triangle \(ABC:\)
\[
20^2 = p^2 + q^2 \tag{3}
\]

We now three equations, and three unknowns \(p, q,\) and \(x\). However, we only want to find the variable \(x\). The best way is to eliminate \(p\) and \(q\) from equations (1) – (3): First, add the equations (1) and (2) to obtain:
\[
p^2 + q^2 = 2x^2 + 4 + 324 \tag{4}
\]

However, by equation (3) we know that \(p^2 + q^2 = 400\). Substitute this in equation (4) to obtain:
\[
400 = 2x^2 + 328 \tag{5}
\]

Therefore \(2x^2 = 72\). This yields \(x^2 = 36\). This means that \(x = \pm 6\). However, since \(x\) represents the length of a side it cannot be negative. Thus, \(x = 6\). In other words, \(CD = 6 \text{ cm}\).
**METHOD II (Use Law of Similar Triangles):**

Note that $ACD$ and $BCD$ are similar triangles (not congruent triangles). What this means is that, their corresponding angles are equal, meaning that $AD\angle C = BD\angle C$, $DA\angle C = B\angle D$, and $AC\angle D = C\angle BD$. Now, according to the Law of Similar Triangles, for any two similar triangles, their corresponding sides are proportional. We want to use this principle for the triangles $ACD$ and $BCD$:

First note that the side $CD$ of the triangle $ACD$ corresponds to the side $BD$ of the triangle $BCD$. Likewise, the side $AD$ of the triangle $ACD$ corresponds to the side $CD$ of the triangle $BCD$. Therefore, using the Law of similar triangles, we will get:

\[
\frac{CD}{BD} = \frac{AD}{CD} \tag{6}
\]

The above equation (6) implies

\[
\frac{x}{18} = \frac{2}{x} \tag{7}
\]

Using the cross multiplication principle, the equation (7) reduces to, $x^2 = 36$. Taking the square root of both sides gives $x = \pm 6$. This finally yields $AD = 6$ cm.

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2. **TRY THIS ONE:**

$ABC$ is a right triangle with the right angle at $C$. Suppose $CD$ is the altitude of this triangle drawn from the angle $C$ to the opposite side $AB$. Given that $AD = 3$ cm and $BC = 2$ cm, find the exact value of $CD$.

(Answer: $CD = \sqrt{3}$ cm)

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3. **TRY THIS ONE:**

The perimeter of a rectangle is $14/3$ inches, and the length of one of its diagonals is equal to $5/3$ inches. Find the exact dimensions of the rectangle.

(Answer: 4/3 inches and 1 inch)

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4. **TRY THIS ONE:**

The volume of a cube is equal to 64 cubic inches. Find the length of its main diagonal and the total surface area.

(Answers: main diagonal is $4\sqrt{3}$ in. and the total surface area is 96 square in.)