1. \[ \sum_{n=2}^{\infty} \frac{n}{n^3+1} = \sum_{n=2}^{\infty} \frac{n}{n^3} = \sum_{n=2}^{\infty} \frac{1}{n^2} \]

So, let \( a_n = \frac{n}{n^3+1} \) & \( b_n = \frac{1}{n^2} \) \( \forall n > 2 \) \( \Rightarrow a_n, b_n > 0 \) \( \forall n > 2 \)

\[ \lim_{n \to \infty} \frac{a_n}{b_n} = \lim_{n \to \infty} \left( \frac{n}{n^3+1} \cdot \frac{n^2}{1} \right) = \lim_{n \to \infty} \left( \frac{n^3}{n^3+1} \right) = \lim_{n \to \infty} \left( \frac{1}{1+\frac{1}{n^3}} \right) = 1 \]

But \( \sum b_n = \sum \frac{1}{n^2} \) is cgt by p-serial test with \( p = 2 > 1 \)

\therefore \text{By LCT, } \sum a_n = \sum_{n=2}^{\infty} \frac{n}{n^3+1} \text{ is cgt}

2. \[ \sum_{n=1}^{\infty} \frac{2^n}{(2n+1)!} \]

Let \( a_n = \frac{2^n}{(2n+1)!} \) \( \forall n > 1 \)

\[ \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \left| \frac{2^{n+1}}{(2n+3)!} \cdot \frac{(2n+1)!}{2^n} \right| = \lim_{n \to \infty} \frac{2^{n+1}}{(2n+3)(2n+2)(2n+1)} = 0 < 1 \]

\therefore \text{By Ratio Test, } \sum_{n=1}^{\infty} \frac{2^n}{(2n+1)!} \text{ is cgt}

3. \[ \sum_{n=1}^{\infty} \left( \frac{2n+3}{n+4} \right)^n \]

Let \( a_n = \left( \frac{2n+3}{n+4} \right)^n \) \( \forall n > 1 \)

\[ \lim_{n \to \infty} |a_n| = \lim_{n \to \infty} \left( \frac{2n+3}{n+4} \right)^n = \lim_{n \to \infty} \left( \frac{2+\frac{3}{n}}{1+\frac{4}{n}} \right)^n = 2 \]

\therefore \text{By Root Test, } \sum_{n=1}^{\infty} \left( \frac{2n+3}{n+4} \right)^n \text{ is cgt}

4. \[ \sum_{n=2}^{\infty} \frac{1}{n^2+n} = \sum_{n=2}^{\infty} \frac{1}{n(n+1)} \]

\[ \frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1} \]

\( 1 = A(n+1) + Bn \)

\( n=0 \Rightarrow 1 = A \)

\( n=-1 \Rightarrow 1 = B(-1) \Rightarrow B = -1 \)

\[ \therefore a_n = \frac{1}{n^2+n} = \frac{1}{n} - \frac{1}{n+1} \quad n=2,3,4,\ldots \]
Contd. 4

\[ a_n = \frac{1}{n} - \frac{1}{n+1} \]

\[ a_2 = \frac{1}{2} - \frac{1}{3} \]
\[ a_3 = \frac{1}{3} - \frac{1}{4} \]
\[ a_4 = \frac{1}{4} - \frac{1}{5} \]

\[ a_{n-2} = \frac{1}{n-2} - \frac{1}{n-1} \]
\[ a_{n-1} = \frac{1}{n-1} - \frac{1}{n} \]
\[ a_n = \frac{1}{n} - \frac{1}{n+1} \]

\[ \therefore \quad a_n = \frac{1}{2} - \frac{1}{n+1} \]

\[ \lim_{n \to \infty} a_n = \lim_{n \to \infty} \left[ \frac{1}{2} - \frac{1}{n+1} \right] = \frac{1}{2} \]

The given series is cgt and converges to \( \frac{1}{2} \).

---

\[ \sum_{n=2}^{\infty} \frac{1}{n \cdot \ln n} \]

Use Integral Test. Let \( a_n = \frac{1}{n \cdot \ln n} \), \( n > 2 \)

\[ \text{let } f(x) = \frac{1}{x \cdot \ln x}, x > 2 \]

(i) \( f(x) > 0 \) for \( x > 2 \)

(ii) \( f \) is cts on \([2, \infty)\)

(iii) \( f \) is decreasing on \([2, \infty)\)

So conditions are satisfied to start the Int. Test.

Consider \( \int_{2}^{\infty} \frac{1}{x \cdot \ln x} \, dx = \lim_{b \to \infty} \int_{2}^{b} \frac{1}{x \cdot \ln x} \, dx = \lim_{b \to \infty} \left[ \ln |\ln x| \right]_{2}^{b} \)

\[ = \lim_{b \to \infty} \left[ \ln |\ln b| - \ln (\ln 2) \right] = \infty - \ln (\ln 2) = \infty \]

\[ \therefore \int_{2}^{\infty} \frac{1}{x \cdot \ln x} \, dx \text{ is dgt} \]

\[ \therefore \text{ By Int. Test, } \sum_{n=2}^{\infty} \frac{1}{n \cdot \ln n} \text{ is dgt. } / / \]
\( 7.8121212 \ldots = 7.8 + 0.012 + 0.00012 + 0.0000012 + \ldots \)
\[= 7.8 + \left( \frac{12}{10^3} + \frac{12}{10^5} + \frac{12}{10^7} + \ldots \right) \]
This part is a Geometric Series with \( a = \frac{12}{10^3} \) and \( C.R = r = \frac{1}{10^2} \). It is q.g.t. since \( |r| < 1 \).
\[= \frac{78}{10} + \frac{12}{10^3 - 10} = \frac{78}{10} + \frac{12}{990} = \frac{7899}{165} \]

\( f(x) = \cos 2x \)
\[P_4(x) = f(0) + \frac{f'(0)}{1!} x + \frac{f''(0)}{2!} x^2 + \frac{f'''(0)}{3!} x^3 + \frac{f^{IV}(0)}{4!} x^4 \]
\[f(x) = \cos 2x \quad \Rightarrow f(0) = 1 \]
\[f'(x) = -2 \sin 2x \quad \Rightarrow f'(0) = 0 \]
\[f''(x) = -2^2 \cos 2x \quad \Rightarrow f''(0) = -2^2 \]
\[f'''(x) = 2^3 \sin 2x \quad \Rightarrow f'''(0) = 0 \]
\[f^{IV}(x) = 2^4 \cos 2x \quad \Rightarrow f^{IV}(0) = 2^4 \]
\[\therefore P_4(x) = 1 - \frac{2^2}{2!} x^2 + \frac{2^4}{4!} x^4 = \sum_{i=0}^{2} (-1)^i \frac{2^i}{(2i)!} x^{2i} \]

\[\sum_{n=1}^{\infty} \frac{1}{n} \sin \left[ \frac{(2n-1)\pi}{2} \right] = \sum_{n=1}^{\infty} \frac{1}{n} (-1)^{n+1} = \sum_{n=1}^{\infty} \frac{1}{n} \left( \sum_{n=1}^{\infty} \frac{1}{n} \right) \]
Let \( a_n = \frac{1}{n} \), \( \forall n \)
\( \therefore a_n > 0 \quad \forall n \)
\( i) \{a_n \} \) is a m.d seqn
\( \therefore \) by AST, \( \sum_{n=1}^{\infty} \frac{1}{n} \sin \left[ \frac{(2n-1)\pi}{2} \right] \) is c.q.t.
\( \therefore \lim_{n \to \infty} a_n = \lim_{n \to \infty} \frac{1}{n} = 0 \)
\( \therefore \)
But \( \sum_{n=1}^{\infty} \left| \frac{1}{n} \sin \left[ \frac{(2n-1)\pi}{2} \right] \right| = \sum_{n=1}^{\infty} \frac{1}{n} \) is c.q.t by p-series Test.
\( \therefore \) The given series is conditionally convergent.
1. Is the series \( \sum_{n=2}^{\infty} \frac{n}{n^2 + 1} \) convergent or divergent? Justify your answer.

2. Is the series \( \sum_{n=1}^{\infty} \frac{2^n}{(2n+1)!} \) convergent or divergent? Justify your answer.

3. Is the series \( \sum_{n=1}^{\infty} \left( \frac{2n+3}{n+4} \right)^n \) convergent or divergent? If convergent, find the sum. Justify your answer.

4. Is the series \( \sum_{n=2}^{\infty} \frac{1}{n^2 + n} \) convergent or divergent? If convergent, find the sum. Justify your answer.

5. Is the series \( \sum_{n=2}^{\infty} \frac{1}{n \ln n} \) convergent or divergent? Justify your answer.

6. Write the repeating decimal \( 7.812121212... = 7.8\overline{12} \) as a geometric series. Use this geometric series to express the repeating decimal as a quotient of two integers.

7. Find the fourth Maclaurin Polynomial for \( f(x) = \cos(2x) \). Express your answer in the summation notation.

8. Is the series \( \sum_{n=1}^{\infty} \frac{1}{n} \sin \left( \frac{(2n-1)\pi}{2} \right) \) absolutely convergent, conditionally convergent or divergent? Justify your answer.

9. Is the series \( \sum_{n=1}^{\infty} \frac{2^n + 3^{n-1}}{6^n} \) convergent or divergent? If convergent, find the exact sum. Justify your answer.

10. Find the interval of convergence and the radius of convergence of the power series \( \sum_{n=1}^{\infty} \frac{(x-2)^{n+1}}{(n+1)4^n} \)
SHOW ALL WORK

1. Is the series $\sum_{n=2}^{\infty} \frac{n}{n^3 + 1}$ convergent or divergent? Justify your answer.

2. Is the series $\sum_{n=1}^{\infty} \frac{2^n}{(2n+1)!}$ convergent or divergent? Justify your answer.

3. Is the series $\sum_{n=1}^{\infty} \left( \frac{2n + 3}{n + 4} \right)^n$ convergent or divergent? If convergent, find the sum. Justify your answer.

4. Is the series $\sum_{n=2}^{\infty} \frac{1}{n^2 + n}$ convergent or divergent? If convergent, find the sum. Justify your answer.

5. Is the series $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ convergent or divergent? Justify your answer.

6. Write the repeating decimal $7.812121212... = 7.\overline{812}$ as a geometric series. Use this geometric series to express the repeating decimal as a quotient of two integers.

7. Find the fourth Maclaurin Polynomial for $f(x) = Cos \ 2x$. Express your answer in the summation notation.

8. Is the series $\sum_{n=1}^{\infty} \frac{\sin \frac{(2n-1)\pi}{2}}{n}$ absolutely convergent, conditionally convergent or divergent? Justify your answer.

9. Is the series $\sum_{n=1}^{\infty} \frac{2^n + 3^{n-1}}{6^{2n}}$ convergent or divergent? If convergent, find the exact sum. Justify your answer.

10. Find the interval of convergence and the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x - 2)^n}{(n+1)4^{n+1}}$. 