1. Find the derivatives of the following functions. DO NOT simplify the answers:
(a) \( y = \frac{4}{3\sqrt{x}} \)
(b) \( f(x) = x^2 \tan(3x) \)

2. Find the derivatives of the following functions. DO NOT simplify the answers:
(a) \( f(x) = \frac{x}{\sqrt{1-x^2}} \)
(b) \( y = \cos^3(3x^2) \)

3. Find the equation of the tangent line to the graph of \( f(x) = (x^2 - x - 1)^4 \) at \( x = -1 \). Provide the answer in the slope-intercept form.

4. Find the slope of the tangent line to the graph of \( f(x) = \frac{1 - \tan x}{1 + \tan x} \) at \( x = \frac{2\pi}{3} \). Provide the exact and simplified answer, with a rational denominator.

5. Given \( y = x + \sqrt{x^2 + 1} \), find \( \frac{d^2y}{dx^2} \). Simplify your answer completely.

6. Find the x-coordinates of the points on the graph of \( f(x) = 2\sin^2x + \sin 2x \), \( 0 \leq x \leq 2\pi \), at which there is a horizontal tangent line. Show your work by hand.

7. Find the equations to the tangent lines to the graph of \( y^2 = 4x + 1 \) that pass through the point \((5, -5)\).

8. Find the equation of the tangent line to the graph of \( x^2 + y^2 - 3\sec^2(xy) = 2y + \frac{\pi^2}{16} - \frac{\pi}{2} - 5 \) at \( \left(1, \frac{\pi}{4}\right) \). Simplify your answer. Give the final answer in the slope-intercept form.

9. At a sand and gravel plant, sand is falling off a conveyor onto a conical pile at a rate of at 10 cubic feet per minute. The diameter of the base of the cone is approximately three times the altitude. At what rate is the height of the pile changing when the pile is 15 feet high?

[Identical to your homework problem]
1. (a) \( y = \frac{4}{3\sqrt{x}} = \frac{4}{3} x^{-\frac{1}{2}} \)

\[ \frac{dy}{dx} = \frac{4}{3} (-\frac{1}{2}) x^{-\frac{3}{2}} \]

(b) \( f(x) = x^2 \tan(3x) \)

\[ f'(x) = 2x \cdot \sec^2(3x) \cdot 3 + 2x \cdot \tan(3x) \]

2. (a) \( f(x) = \frac{x}{\sqrt{1-x^2}} = \frac{x}{(1-x^2)^{\frac{1}{2}}} \)

\[ f'(x) = \frac{(1-x^2)^{\frac{1}{2}} \cdot 1 - x \cdot \frac{1}{2} (1-x^2)^{-\frac{1}{2}} (-2x)}{(1-x^2)} \]

(b) \( y = \left[ \csc(3x^2) \right]^3 \)

\[ \frac{dy}{dx} = 3 \left[ \csc(3x^2) \right]^2 \cdot [-\csc(3x^2) \cdot \cot(3x^2)] \cdot 6x \]

3. \( f(x) = (x^2 - x - 1)^4 \) at \( x = -1 \)

\( f(-1) = (1 + 1)^4 = 1 \)

\[ f'(x) = 4 (x^2 - x - 1)^3 (2x - 1) \]

\( f'(0) = 4 (1 + 1)^3 (-2 - 1) = -42 \text{ } m \)

Eqn. of tgt line at \( x = -1 \) is \( y - y_1 = m (x - x_1) \)

\[ y - 1 = -12 (x + 1) \]

\[ y = -12x - 11 \]

4. \( f(x) = \frac{1 - \tan x}{1 + \tan x} \) at \( x = \frac{2\pi}{3} \)

\[ f'(x) = \frac{(1 + \tan x)(-\sec^2 x) - (1 - \tan x)(\sec^2 x)}{(1 + \tan x)^2} \]

\[ = -\sec^2 x \frac{(1 + \tan x) + (1 - \tan x)}{(1 + \tan x)^2} \]

\[ = -2 \sec^2 x \]

\( m = f'(\frac{2\pi}{3}) = -2 (-2)^2 = -\frac{8}{(1 - \sqrt{3})^2} = -\frac{8 (1 + \sqrt{3})}{4} = -8 (1 + 2\sqrt{3}) = -4 (2 + \sqrt{3}) \)
\[ y = x + \sqrt{x^2 + 1} \]
\[
\frac{dy}{dx} = 1 + \frac{1}{2} (x^2 + 1)^{-\frac{1}{2}} (2x) = 1 + \frac{x}{(x^2 + 1)^{\frac{3}{2}}} 
\]
\[
\frac{d^2y}{dx^2} = 0 + (x^2 + 1)^{-\frac{3}{2}} (1 - x^2) \cdot \frac{1}{2} (x^2 + 1)^{-\frac{1}{2}} \cdot 2x = \frac{(x^2 + 1) - x^2}{(x^2 + 1)(x^2 + 1)^{\frac{3}{2}}} = \frac{1}{(x^2 + 1)^{\frac{3}{2}}}
\]

\( f(x) = 2 \sin^2 x + \sin(2x) \quad 0 \leq x \leq 2\pi \)
\[
f'(x) = 2 \left[ 2 \sin x \cos x + \cos 2x \right] = 2 \sin 2x + 2 \cos 2x
\]
\[
\therefore f'(x) = 2 \left[ \sin 2x + \cos 2x \right]
\]

Set \( f'(x) = 0 \) and solve for \( x \):
\[
2 \left( \sin 2x + \cos 2x \right) = 0
\]
\[
\sin 2x + \cos 2x = 0 \quad \Rightarrow \quad \sin 2x = -\cos 2x \quad \Rightarrow \quad \tan(2x) = -1.
\]
\[
2x = \frac{3\pi}{4}, \quad \frac{7\pi}{4}, \quad \frac{11\pi}{4}, \quad \frac{15\pi}{4}
\]
\[
\therefore x = \frac{3\pi}{8}, \quad \frac{7\pi}{8}, \quad \frac{11\pi}{8}, \quad \text{or} \quad \frac{15\pi}{8}
\]

\[ y^2 = 4x + 1; \quad \text{through (5, 5)} \]
\[
2y \frac{dy}{dx} = 4
\]
\[
\therefore \frac{dy}{dx} = \frac{2}{2y} = \frac{2}{y}
\]
\[
\left. \frac{dy}{dx} \right|_P = \frac{2}{y} = \frac{2}{t} \quad \therefore \text{slope of } PA = \frac{2}{t} \quad \text{EQUAL!}
\]
\[
\text{Also, slope of } PA = \frac{t + 5}{(t^2 - 1) - 5} = \frac{4(t + 5)}{t^2 - 21}
\]
\[
\therefore \frac{2}{t} = \frac{4(t + 5)}{t^2 - 21}
\]
\[
\therefore (t + 3)(t + 7) = 0 
\]
\[
\therefore t = -3 \quad \text{or} \quad t = -7
\]
\[
t = -3: \quad P_1 = (3, -3), \quad m_1 = \frac{2}{-3} = -\frac{2}{3} \quad \text{eqn. of } t \parallel y = -\frac{2x - 3}{3}
\]
\[
t = -7: \quad P_1 = (12, -7), \quad m_1 = -\frac{2}{7} \quad \text{eqn. of } t \parallel y = -\frac{2}{7}(x - 12)
\]
\( x^2 + y^2 - 3 \sec^2(xy) = 2y + \frac{x^2}{16} - \frac{\pi}{2} - 5 \) at \( (1, \frac{\pi}{4}) \)

\[
2x + 2y \frac{dy}{dx} - 3x \sec(xy) \cdot \sec(xy) \tan(xy) \left( x \frac{dy}{dx} + y \right) = 2 \frac{dy}{dx}
\]

\[
2x + 2y \frac{dy}{dx} - 6x \sec(xy) \tan(xy) \frac{dy}{dx} - 6y \sec^2(xy) \tan(xy) = 2 \frac{dy}{dx}
\]

\[
\therefore \frac{dy}{dx} \left( 2y - 6x \sec(xy) \tan(xy) - 2 \right) = -2x + 6y \sec^2(xy) \tan(xy)
\]

\[
\therefore \frac{dy}{dx} = \frac{2x + 6y \sec^2(xy) \tan(xy)}{y - 3x \sec(xy) \tan(xy) - 2}
\]

\[
\frac{dy}{dx} \bigg|_{(1, \frac{\pi}{4})} = \left. \frac{-1 + 3 \cdot \frac{\pi}{4}}{\frac{\pi}{4} - 3 \times 2 \times 1 - 1} \right. = \frac{\frac{3\pi}{2} - 1}{\frac{\pi}{4} - 7} = \frac{6\pi - 4}{\pi - 28} = m.
\]

eqn. of tgt: \( y - \frac{\pi}{4} = \frac{6\pi - 4}{\pi - 28} (x - 1) \)

\[
y = \frac{(6\pi - 4) x}{(\pi - 28)} - \frac{(6\pi - 4)}{(\pi - 28)} + \frac{\pi}{4}
\]

\[
y = \frac{(6\pi - 4) x}{(\pi - 28)} + \frac{\pi^2 - 52\pi + 16}{4(\pi - 28)}
\]

\[\text{Given:} \quad \frac{dv}{dt} = 10 \text{ ft}^3/\text{min} \]

\[\text{Know:} \quad V = \frac{1}{3} \pi r^2 h \quad \text{and} \quad 2r = 3h \]

\[r = \frac{3h}{2}, \quad h = h, \quad V = \text{vol.}\]

Find: \[\frac{dh}{dt} \bigg|_{h=15} \text{ ft/min} \]

Now: \[\frac{dv}{dt} = \left( \frac{dv}{dh} \right) \cdot \frac{dh}{dt} \]

\[10 = \frac{9\pi h^3}{4} \cdot \frac{dh}{dt} \]

\[\therefore \frac{dh}{dt} = \frac{40}{9\pi h^2} \]

\[\therefore \frac{dh}{dt} \bigg|_{h=15} = \frac{40}{9\pi (15)^2} = \frac{40 \cdot 8}{9\pi \times 225} = \frac{8}{405 \pi} \text{ ft/min} \]