1. \( f(x) = 2x^3 - 3x^2 + 1 \)
   
   \[ f'(x) = 6x^2 - 6x = 6x(x-1) \]
   
   \[ \text{Zero: } x = 0, \ x = 1 \]
   
   \[ \text{DNE: } \text{Never} \]
   
   Only critical points are \( x = 0, 1 \).

   Sign of \( f'' \):
   
   \[
   \begin{array}{c|c|c|c}
   & + & - & + \\
   \hline
   0 & - & + & - \\
   \end{array}
   \]

   \( \therefore f \) is increasing on \((-\infty, 0) \cup [1, \infty)\)

   \( f \) is decreasing on \([0, 1] \).

2. \( f(x) = \frac{x^3}{48} - \frac{1}{x} \)
   
   \[ f'(x) = \frac{3x^2}{48} + \frac{1}{x^2} = \frac{x^3 + 1}{16x^2} \]

   \[ f''(x) = \frac{2x}{16x^3} = \frac{x}{8x^3} = \frac{x^2 - 4}{8x^3} = \frac{(x-2)(x^2+4)}{8x^3} \]

   \[ \text{Zero: } x = -2, \ x = 2 \]

   \[ \text{DNE: } x = 0 \text{ (not in domain)} \]

   The only hypercritical points at \( x = -2, x = 2 \).

   Sign of \( f'' \):
   
   \[
   \begin{array}{c|c|c|c|c}
   & + & - & + & - \\
   \hline
   -2 & + & - & + & - \\
   \end{array}
   \]

   \( \therefore f \) is concave down of \((-\infty, -2) \cup (-2, 2) \)

   \( f \) is concave up on \([-2, 0) \cup (0, \infty) \)

   Points of inf. at \((-2, \frac{1}{3})\) and \((2, -\frac{1}{3})\).

3. **Mean Value Theorem**: Suppose \( f \) is a function defined on the closed interval \([a, b] \). Suppose further \( f \) is continuous on \([a, b] \) and differentiable on \((a, b)\). Then there is a real number \( c \) in the open interval \((a, b)\) such that \( f'(c) = \frac{f(b) - f(a)}{b - a} \).
Given \( f(x) = \frac{x+1}{x} = 1 + \frac{1}{x} \) on \( \left[ \frac{1}{2}, 2 \right] \).

\( f \) is cont on \( \left[ \frac{1}{2}, 2 \right] \) (\( f \) is a rational function, and its only discont at \( x = 0 \)).

\( f \) is diff. on \( \left( \frac{1}{2}, 2 \right) \).

Also \( \frac{f(b) - f(a)}{b-a} = \frac{\frac{3}{2} - \frac{3}{2}}{2 - \frac{1}{2}} = -1 \)

\( \therefore \) By M.V.T, there exists \( c \in \left( \frac{1}{2}, 2 \right) \) s.t. \( f'(c) = -1 \).

Now: \( f'(x) = -\frac{1}{x^2} \) SET \( f'(x) = -1 \) and solve for \( x \).

\(-\frac{1}{x^2} = -1 \)

\( x^2 = 1 \Rightarrow x = \pm 1 \) not in \( \left( \frac{1}{2}, 2 \right) \)

\( \therefore x = 1 \) i.e. \( c = 1 \) only.

\[ \int \frac{2x+1}{\sqrt{x}} \, dx = \left( \int \frac{2x}{x^{1/2}} + \frac{1}{x^{1/2}} \right) \, dx = \left( \int 2x^{1/2} + x^{-1/2} \right) \, dx = \frac{4}{3}x^{3/2} + 2x^{1/2} + C \]

\[ \int \frac{2 + \sec^2 t}{\tan^2 t} \, dt = \left( \int 2\csc^2 t + \csc t \cot t \, dt \right) = \left( \int [2(\csc t - 1) + \csc t \cot t] \, dt \right) = \left( \int (2\csc^2 t - 2 + \csc t \cot t) \, dt \right) = -2\csc t - 2t - \csc t + C \]
6. \[ x''(t) = -32 \]
   \[ x'(t) = \int -32 \, dt \]
   \[ x'(t) = -32 \, t + C_1 \]
   \[ u(t) = -32 \, t + C_1 \]

   \( t = 0 \):
   \[ 60 = -32(0) + C_1 \Rightarrow C_1 = 60 \]
   \[ x'(t) = -32 \, t + 60 \]
   \[ x(t) = \int (-32 \, t + 60) \, dt \]
   \[ x(t) = -16 \, t^2 + 60 \, t + C_2 \]

   \( t = 0 \):
   \[ 6 = -16(0)^2 + 60(0) + C_2 \Rightarrow C_2 = 6 \]
   \[ x(t) = -16 \, t^2 + 60 \, t + 6 \]

At the maximum height, the velocity is zero, so use 1 and set \( u(t) = 0 \):
\[ 0 = -32 \, t + 60 \Rightarrow t = \frac{60}{32} = \frac{15}{8} \text{ sec} \] (how long it will take the ball to reach the max. h.t.)

To find the max. ht., plug in \( t = \frac{15}{8} \) on eqn 2:
\[ x\left(\frac{15}{8}\right) = -16 \left(\frac{15}{8}\right)^2 + 60 \left(\frac{15}{8}\right) + 6 = \frac{243}{4} \text{ ft} \]

7. \( f(x) = x \sqrt{4-x^2} \)

- **Domain:** \( -2 \leq x \leq 2 \)

- **Critical points:**
  \[ f'(x) = \frac{x \frac{d}{dx} \sqrt{4-x^2} + \sqrt{4-x^2}}{\sqrt{4-x^2}} = \frac{-x^2 + 4 - x^2}{\sqrt{4-x^2}} = \frac{2(2-x^2)}{\sqrt{4-x^2}} \]

  \[ \text{The critical points are } x = \pm \sqrt{2} \]

- **Increasing/decreasing intervals**

  \[ f \text{ is dec. on } [-\sqrt{2}, 2] \cup [\sqrt{2}, 2] \; \text{ and conc. on } [-\sqrt{2}, \sqrt{2}] \]

- **Local Extrema:**
  - Local minimum at \( x = -\sqrt{2} \): \((-\sqrt{2}, -2)\)
  - Local maximum at \( x = \sqrt{2} \): \((\sqrt{2}, 2)\)
(iv) Concavity & pts of inflection: 
\[ f'(x) = \frac{2(2-x^2)}{4-x^2} \]

\[ f''(x) = 2 \left[ \frac{\sqrt{4-x^2} (-2x) - (2-x^2) \cdot \frac{\sqrt{4-x^2} \cdot x}{\sqrt{4-x^2}}}{4-x^2} \right] = 2 x \left[ -2(4-x^2) + (2-x^2) \right] \frac{1}{4-x^2} \]

\[ f''(x) = 2 x \left( x^2 - 6 \right) \frac{1}{(4-x^2)^{3/2}} \]

Zero: \( x = \pm \sqrt{6} \); not in domain. 
DNE: \( x = \pm 2 \)

: Hypercritical Points
are \( x = \pm 2 \)

Sign of \( f'' \):

\[
\begin{array}{c|c|c}
-2 & 0 & 2 \\
+ & - & - \\
\end{array}
\]

\( f \) is concave up on \( [-\sqrt{6}, \sqrt{6}] \); concave down on \( [0, 2] \); Inf pt \((0, 0)\).

(v) Horizontal and Vertical Asymptotes:

There are no horizontal asymptotes either.

Also note that \( \lim_{x \to \infty} f(x) \) and \( \lim_{x \to -\infty} f(x) \) both do not exist.

(vi) Intercepts: 
X-int: Set \( y = 0 \Rightarrow (0, 0), (-2, 0) \) and \( (2, 0) \) are the X-int.
Y-int. Set \( x = 0 \Rightarrow (0, 0) \) is the Y-int.

GRAPH:

\[
\begin{array}{c}
\text{Graph}
\end{array}
\]

\[
\begin{array}{c}
\text{Let's use upper sums } S(n).
\end{array}
\]

\[
\begin{array}{c}
\Delta x = \frac{4-1}{n} = \frac{3}{n}
\end{array}
\]

\[
\begin{array}{c}
S(n) = \sum_{l=1}^{n} f(x_l) \Delta x = \sum_{l=1}^{n} f(1 + \frac{3l}{n}) \frac{3}{n} = \frac{3}{n} \sum_{l=1}^{n} \left[ (1 + \frac{3l}{n}) + 1 \right] \frac{3}{n}
\end{array}
\]

\[
\begin{array}{c}
S(n) = \frac{3}{n} \left[ \sum_{l=1}^{n} \left( 1 + \frac{3l}{n} + \frac{9l^2}{n^2} \right) + 1 \right] \frac{3}{n} = \frac{3}{n} \left[ \frac{4 + \frac{18}{n} + \frac{27}{2n^2} \cdot \sum_{l=1}^{n} l^2}{n} \right] \frac{3}{n}
\end{array}
\]

\[
\begin{array}{c}
S(n) = \frac{3}{n} \left[ 4 + \frac{18}{n} + \frac{27}{2n^2} \cdot \sum_{l=1}^{n} l^2 \right] = \frac{3}{n} \left[ 4 + \frac{18}{n} \cdot \frac{n(n+1)}{2} + \frac{27}{2n^2} \cdot \frac{n(n+1)(2n+1)}{6} \right]
\end{array}
\]

\[
\begin{array}{c}
S(n) = \frac{3}{n} \left[ 4 + 9(1+\frac{1}{n}) + \frac{9}{2} (1+\frac{1}{n})(2+\frac{1}{n}) \right] = \frac{3}{n} \left[ 4 + 9 + \frac{9}{2} \cdot \frac{3}{n} \right]
\end{array}
\]

\[
\begin{array}{c}
\text{Required Area} = \lim_{n \to \infty} S(n) = 3 \left[ 4 + 9 + 9 \right] = 66 \text{ units}^2
\end{array}
\]
ONLY answer ANY 7 questions. DO NOT attempt all 8 problems. You might run out of time if you do so.

1. Find the intervals on which the function \( f(x) = 2x^3 - 3x^2 + 1 \) is increasing or decreasing.

2. Find the intervals on which the function \( f(x) = \frac{x^3}{48} - \frac{1}{x} \) is concave up/down. Also find the \( x \) and \( y \)-coordinates of the points of inflection. Provide the exact answers.

3. State the Mean Value Theorem in precise terms. Draw a BIG CLEAR graph to illustrate the idea behind the theorem.

Consider the function \( f(x) = \frac{x + 1}{x} \) defined on the closed interval \([1/2, 2]\). Explain precisely why does the Mean Value Theorem guarantee that there is a real number \( c \) in the interval \((1/2, 2)\) such that \( f'(c) = -1 \). Also find \( c \).

4. Calculate the integral \( \int \frac{2x + 1}{\sqrt{x}} \, dx \)

5. Calculate the integral \( \int \frac{2 + \text{Sect}}{\text{Tan}^2 t} \, dt \)

6. A ball is thrown vertically upward from a height of 6 ft with an initial velocity of 60 ft/sec. How high the ball will go? Use that the acceleration due to gravity is \(-32 \text{ ft/sec}^2\). Do this problem without assuming any equations from physics.

7. Do all the usual steps and graph the function \( f(x) = x\sqrt{4 - x^2} \). Provide the exact answers.

8. Use either the lower sums or the upper sums to find the area of the region bounded by the graphs of \( f(x) = 3x^2 + 1, x = 1, x = 4 \), and the \( x \)-axis. Provide the exact answer.