1. Find the derivatives of the following functions. **Do not simplify** your answers.
   (a) \( y = \sqrt{1 - x^2} \)
   (b) \( f(x) = x \sin^2 x \)

2. Find the equation of the tangent line to the graph of \( f(x) = -x^4 + 3x^2 - 4 \) at \( x = -1 \). Provide the answer in the slope-intercept form.

3. Find the \( x \)-coordinates of the points on the graph of \( f(x) = 3(3x + 4)^3 (1 - 2x)^2 \) at which there is a horizontal tangent line. Provide the exact answers.

4. Find the slope of the tangent line to the graph of \( y = 3 \tan^2 (2x) \) at \( x = \frac{\pi}{3} \). Provide the exact and simplified answer in slope-intercept form.

5. Given \( f(x) = \frac{3}{(x^2 + 1)^2} \), find and completely simplify \( f'(\frac{1}{2}) \). Show your work carefully by hand, and provide the exact answer.

6. Given \( f(x) = \sqrt{3x^2 + 4} \) find and completely simplify \( f''(x) \).

7. A ball is thrown vertically from the ground-level at an initial velocity of 100 ft/sec. Its height \( h \) (in feet) from the ground-level, at time \( t \) (in seconds) is given by the formula \( h(t) = -16t^2 + 100t \). Find the velocity and the acceleration of the ball after 4 seconds. Do this problem using principles of calculus, without using any equations from physics.

8. Find the \( x \)-coordinates of the points on the graph of \( f(x) = \sin x (2 + \cos x) \) where \(-3\pi < x \leq 2\pi\), at which the tangent line is parallel to the line \( x - 2y + 1 = 0 \). Make sure to do this problem by hand. Provide the exact answers.

9. Find the equations of the tangent lines to the graph of \( f(x) = \frac{2}{1 + x} \) which pass through the point \((1, -3)\). Provide the exact answers.
1. (a) \( y = \sqrt{1-x^2} = (1-x^2)^{\frac{1}{2}} \)

\[ \frac{dy}{dx} = \frac{1}{2} (1-x^2)^{-\frac{1}{2}} \cdot (-2x) \]

(b) \( f(x) = x \sin^2 x = x (\sin x)^2 \)

\[ f'(x) = x \cdot 2 \sin x \cdot \cos x + 1 \cdot (\sin x)^2 \]

2. \( f(x) = -x^4 + 3x^2 - 4 \)

\[ f'(x) = -4x^3 + 6x = -4x^3 + 6x \]

\[ m = \text{Slope of tgt} = f'(-1) = -4(-1)^3 + 6(-1) = 4 - 6 = -2 \]

at \( x = -1 \)

\[ x_1 = -1 \quad \text{and} \quad y_1 = f(-1) = -(-1)^4 + 3(-1)^2 - 4 = -1 + 3 - 4 = -2 \]

pt. on the curve = \((x_0, y_1) = (-1, -2)\)

eqn. of tgt : \( y - y_1 = m(x - x_1) \)

\[ y + 2 = -2(x + 1) \]

\[ y + 2 = -2x - 2 \]

\[ y = -2x - 4 \]

3. \( f(x) = 3(3x+4)^2(1-2x)^2 \)

\[ f'(x) = 3 [ (3x+4)^2 \cdot 2 (1-2x)(-2) + 3(3x+4)^2 \cdot 3 \cdot (1-2x)^2] \]

\[ f'(x) = 3 \left[ -4(3x+4)^2(1-2x) + 9(3x+4)^2(1-2x)^2 \right] \]

\[ f''(x) = 3(3x+4)^2 (1-2x) \left[ -4(3x+4) + 9(1-2x) \right] \]

\[ f'(x) = 3(3x+4)^2 (1-2x) \left( -12x + 16 + 9 - 18 \right) \]

\[ f'(x) = 3(3x+4)^2 (1-2x) \left( -30x + 7 \right) \]

Now set \( f'(x) = 0 \) and solve for \( x \).

\[ x = -\frac{4}{3}, \frac{1}{2}, -\frac{7}{30} \]

4. \( y = 3 \tan^2(2x) = 3 \left[ \tan(2x) \right]^2 \)

\[ \frac{dy}{dx} = 3 (2) \left[ \tan(2x) \right] \cdot \sec^2(2x) \cdot 2 = 12 \tan(2x) \sec^2(2x) \]

\[ m = \left. \frac{dy}{dx} \right|_{x = \frac{\pi}{6}} = 12 \tan \left( \frac{2\pi}{3} \right) \cdot \sec^2 \left( \frac{2\pi}{3} \right) = 12(-\sqrt{3}) \cdot 4 = -48 \sqrt{3} \]
\( f(x) = \frac{3}{(x^2+1)^2} = 3(x^2+1)^{-2} \)

\[
\therefore f'(x) = 3(-2)(x^2+1)^{-3} \cdot 2x
\]

\[
f'(x) = \frac{-12x}{(x^2+1)^3}
\]

\[
\therefore f'(\frac{1}{2}) = \frac{-12\left(\frac{1}{2}\right)}{(\frac{1}{4}+1)^3} = \frac{-6}{(\frac{5}{4})^3} = \frac{-6}{\left(\frac{125}{64}\right)} = \frac{-384}{125}
\]

\[
\therefore f'(\frac{1}{2}) = \frac{-384}{125}
\]

\( f(x) = \sqrt{3x^2+4} = (3x^2+4)^{\frac{1}{2}} \)

\[
\therefore f'(x) = \frac{1}{2} (3x^2+4)^{-\frac{1}{2}} \cdot 6x = \frac{3x}{(3x^2+4)^{\frac{1}{2}}}
\]

\[
\therefore f''(x) = 3 \left[\frac{(3x^2+4)^{\frac{1}{2}} \cdot 1 - x \cdot \frac{1}{2} (3x^2+4)^{-\frac{1}{2}} \cdot 6x}{(3x^2+4)^{\frac{1}{2}} \cdot 3x^2+4}\right]
\]

\[
\therefore f''(x) = 3 \left[\frac{\sqrt{3x^2+4} - \frac{3x^2}{\sqrt{3x^2+4}}}{(3x^2+4)^{\frac{1}{2}} \cdot \sqrt{3x^2+4}}\right]
\]

\[
\therefore f''(x) = 3 \left[\frac{(3x^2+4) - 3x^2}{(3x^2+4)^{\frac{3}{2}}}ight]
\]

\[
\therefore f''(x) = 3 \left[\frac{4}{(3x^2+4)^{\frac{3}{2}}}ight] = \frac{12}{(3x^2+4)^{\frac{3}{2}}}
\]

\( h(t) = -16t^2 + 100t \)

\( v(t) = h'(t) = -32t + 100 \)

\( h(4) = v(4) = -32(4) + 100 = -28 \text{ ft/sec} \) is the velocity after 4 seconds

\( a(t) = h''(t) = -32 \text{ ft/sec}^2 \) is the acceleration at any time.
\[ f(x) = \sin x \ (2 + \cos x) \quad -3\pi \leq x \leq 2\pi \]

\[ f'(x) = \sin x \ (-\sin x) + \cos x \ (2 + \cos x) \]

\[ f''(x) = -\sin^2 x + 2 \cos x + \cos^2 x = \text{slope of tgt at any } x\text{-value} \]

Now, find the slope of \[ x - 2y + 1 = 0 \]

\[ 2y = x + 1 \]

\[ y = \frac{1}{2} x + \frac{1}{2} \]

\[ \therefore \text{ slope of line } = \frac{1}{2} \]

\[ \therefore \text{ Set } f'(x) = \frac{1}{2} \text{ and Solve for } x \]

\[ -\sin^2 x + 2 \cos x + \cos^2 x = \frac{1}{2} \]

\[ (\cos^2 x - 1) + 2 \cos x + \cos^2 x = \frac{1}{2} \]

\[ 2 \cos^2 x + 2 \cos x - 1 = \frac{1}{2} \]

\[ 4 \cos^2 x + 4 \cos x - 3 = 0 \]

\[ (2 \cos x - 1)(2 \cos x + 3) = 0 \]

\[ \therefore \cos x = \frac{1}{2} \text{ or } \cos x = -\frac{3}{2} \]

\[ \text{This cannot happen since } -1 \leq \cos x \leq 1 \forall x \in \mathbb{R}. \]

\[ \therefore \cos x = \frac{1}{2} \]

\[ \text{Ref 4 = } \frac{\pi}{3} \]

\[ \therefore x = -\frac{7\pi}{3}, -\frac{5\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{3} \]

There are 5 answers.
\(f(x) = \frac{2}{1+x}\)

\[\therefore f'(x) = -2(1+x)^{-2}\cdot 1\]

\[\therefore f'(x) = \frac{-2}{(1+x)^2}\]

So, slope of \(PA = f'(t) = \frac{-2}{(1+t)^2}\)

Also, slope of \(PQ = \frac{2}{1+t} + 3 = \frac{2+3(1+t)}{(1+t)(1+t)} = \frac{5+3t}{(t-1)(1+t)}\)

\[\therefore \text{By 1 and 2, } \frac{-2}{(1+t)^2} = \frac{(5+3t)}{(t-1)(1+t)}\]

\[\therefore -2(t-1) = (5+3t)(1+t)\]

\[-2t^2 + 2 = 5 + 8t + 3t^2\]

\[0 = 3t^2 + 10t + 3\]

\[0 = (3t + 1)(t + 3)\]

\[\therefore t = -\frac{1}{3} \text{ or } t = -3\]

With \(t = -\frac{1}{3}\):

\[Q = (t, \frac{2}{1+t}) = (-\frac{1}{3}, 3)\]

and \(m = \frac{-2}{(1+t)^2} = -\frac{9}{2}\)

eqn: \(y - y_1 = m(x - x_1)\)

\[y - 3 = -\frac{9}{2}(x + \frac{1}{3})\]

\[y = -\frac{9}{2}x - \frac{3}{2} + 3\]

\[\therefore y = -\frac{9}{2}x + \frac{3}{2}\]

With \(t = -3\):

\[Q = (t, \frac{2}{1+t}) = (-3, -1)\]

and \(m = \frac{-2}{(1+t)^2} = -\frac{1}{2}\)

eqn: \(y - y_1 = m(x - x_1)\)

\[y - 1 = -\frac{1}{2}(x + 3)\]

\[y = -\frac{1}{2}x - \frac{3}{2} - 1\]

\[\therefore y = -\frac{1}{2}x - \frac{5}{2}\]

\[\therefore \text{The eqns of the 2 tangents are } y = -\frac{9}{2}x + \frac{3}{2} \text{ and } y = -\frac{1}{2}x - \frac{5}{2}\]