SHOW ALL WORK

1. Find the exact value of \( \lim_{x \to 2} \frac{2x^2 + x - 10}{2x^2 - 8} \). Show your work by hand.

2. Find the exact value of \( \lim_{x \to 1} \frac{1 - x^3}{\sqrt{x + 3} - 2} \). Show your work by hand.

3. Given \( f(x) = -x^2 + 2x - 6 \), calculate \( \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \) by hand. Simplify your answer. Show your work carefully.

4. Draw a BIG, CLEAR graph of \( f(x) = \begin{cases} 
-3x - 1 & \text{if } x < 3 \\
-2x^2 + 8 & \text{if } x \geq 3 
\end{cases} \)

Is \( f \) continuous at \( x = 3 \)? Give the precise mathematical reason using the definition of continuity (without relating to the graph). If discontinuous, name the type of discontinuity.

5. Find the equation of the tangent line to the graph of \( f(x) = -\sqrt{1-x} \) at \( x = -3 \). Simplify your answer into the slope-intercept form.

6. Write the definition of a vertical asymptote in precise mathematical terms.

Find all the vertical asymptotes of \( f(x) = Cosec(5x) \) for \( 1/2 \leq x \leq 1 \). Provide the exact answers. As a part of your answer, make sure to show the necessary limit calculations. In other words, find the limit of the function \( f(x) \) as \( x \) approaches the asymptote \( x \)-value either from left or right.

7. State the Intermediate Value Theorem (IVT) in precise terms.

Draw a BIG and CLEAR diagram to illustrate the IVT, labeling all the necessary features.

8. Recall that for any real number \( x \), \([x]\) denotes the greatest integer less than or equal to \( x \).

Draw a BIG and CLEAR graph of \( f(x) = [x+1] \) for \( -3 \leq x \leq 3 \).

Also find \( \lim_{x \to -20^-} [x + 1] \).

9. Find the exact value of the \( \lim_{x \to \pi/4} \frac{\sin x - \cos x}{4x - \pi} \). Show your work by hand.
1. \[
\lim_\limits{x \to 2} \frac{2x^2 + x - 10}{2x^2 - 8} = \lim_\limits{x \to 2} \frac{(2x + 5)(x - 2)}{2(x - 2)(x + 2)} = \lim_\limits{x \to 2} \frac{2x + 5}{2(x + 2)} = \frac{9}{8}
\]

2. \[
\lim_\limits{x \to 1} \frac{1 - x^3}{\sqrt{x + 3} - 2} = \lim_\limits{x \to 1} \frac{(1 - x)(1 + x + x^2)}{(\sqrt{x + 3} - 2 + (\sqrt{x + 3} + 2)} = \lim_\limits{x \to 1} \frac{(1 - x)(1 + x + x^2)}{(x + 3) - 4} = \lim_\limits{x \to 1} \frac{-(x - 1)(1 + x + x^2)(\sqrt{x + 3} + 2)}{(x - 1)} = -(3)(2 + 2) = -12
\]

3. Given: \( f(x) = -x^2 + 2x - 6 \)
   \[
   \lim_\limits{h \to 0} \frac{f(x + h) - f(x)}{h} = \lim_\limits{h \to 0} \frac{[-(x + h)^2 + 2(x + h) - 6] - [-x^2 + 2x - 6]}{h} = \lim_\limits{h \to 0} \frac{-2x - h}{h} = -2x - 2
   \]

4. \( f(x) = \begin{cases} 
3x - 1 & \text{if } x < 3 \\
-2x^2 + 8 & \text{if } x \geq 3 
\end{cases} \)
   Yes, \( f \) is cts at \( x = 3 \)
   Mathematical Reason:
   (a) \( f(3) \) exists and \( f(3) = -2(3)^2 + 8 = -10 \)
   (b) \( \lim_\limits{x \to 3} f(x) \) exists and is equal to \(-10\), because, both \( \lim_\limits{x \to 3^-} f(x) = -10 = \lim_\limits{x \to 3^+} f(x) \)
   (c) Also, \( f(3) = \lim_\limits{x \to 3} f(x) \)

5. Given \( f(x) = -\sqrt{1-x} \), at \( x = -3 \)
   \[
   f'(x) = \lim_\limits{h \to 0} \frac{f(x + h) - f(x)}{h} = \lim_\limits{h \to 0} \frac{-\sqrt{1-x-h} + \sqrt{1-x}}{h} = \lim_\limits{h \to 0} \frac{\sqrt{1-x} - \sqrt{1-x-h}}{h} (h = \sqrt{1-x} + \sqrt{1-x-h})
   \]
   \[
   = \lim_\limits{h \to 0} \frac{h}{h} \frac{(1-x)(1-x-h)}{(1-x) + (1-x-h)} = -\frac{1}{2}
   \]
\[ f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = \lim_{h \to 0} \frac{1}{\sqrt{1-x} + \sqrt{1-x-h}} = \frac{1}{2\sqrt{1-x}} \]

.: \[ f'(x) = \frac{1}{2\sqrt{1-x}} \]

eqn. of tgt line: \( y - y_1 = m(x - x_1) \); \( y + 2 = \frac{1}{4}(x+3) \) ie \( y = \frac{1}{4}x - \frac{5}{4} \).

6. See class notes for the defn of vertical asymptotes.

Find V.A's of \( f(x) = \text{Cosec}(5x) \) for \( \frac{1}{2} \leq x \leq 1 \).

.: \( f(x) = \frac{1}{\sin(5x)} \)

So candidates for V.A's are obtained by solving \( \sin(5x) = 0 \)

.: \( 5x = \ldots -\pi, 0, \pi, 2\pi, 3\pi \ldots \) etc

.: \( x = \ldots -\frac{\pi}{5}, 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5} \ldots \) etc.

.: only 1 Candidate for V.A's: \( x = \frac{\pi}{5} \) only

Check \( x = \frac{\pi}{5} \): \( \lim_{x \to \frac{\pi}{5}} \frac{1}{\sin(5x)} = \frac{1}{0^+} = +\infty \) (an inf. limit)

.: \( x = \frac{\pi}{5} \) is a V.A.

7. See class notes for the statement of IVT.

Diagram illustrating the IVT: (Make sure you have all the necessary comp's)
8) Draw $f(x) = \lfloor x + 1 \rfloor$

$$\lim_{x \to -20^-} \lfloor x + 1 \rfloor = -20$$

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9) Find

$$\lim_{x \to \frac{\pi}{4}} \frac{\sin x - \cos x}{4x - \pi}$$

Form $\frac{0}{0}$ Indet. Form

$$= \lim_{x \to \frac{\pi}{4}} \frac{(\sin x - \cos x)(\sin x + \cos x)}{(4x - \pi)(\sin x + \cos x)}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\sin^2 x - \cos^2 x}{(4x - \pi)(\sin x + \cos x)}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{-\sin (\frac{\pi}{2} - 2x)}{2 \left(2x - \frac{\pi}{2}\right)}$$

$$= \lim_{x \to \frac{\pi}{4}} \left[ \frac{\sin (\frac{\pi}{2} - 2x)}{(\frac{\pi}{2} - 2x)} \cdot \frac{1}{2 \left(\sin x + \cos x\right)} \right]$$

$$= 1 \cdot \frac{1}{2 \left(\frac{\pi}{4} + \frac{1}{2}\right)} = \frac{1}{2\sqrt{2}}$$

$$\therefore \lim_{x \to \frac{\pi}{4}} \frac{\sin x - \cos x}{4x - \pi} = \frac{1}{2\sqrt{2}}$$

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~ END OF TEST 1 ~ ~