1. Find the local extrema of \( f(x) = 2x^3 + 3x^2 - 12x + 1 \).

2. Find the intervals where the function \( f(x) = 2x^2(1 - 2x)^5 \) is increasing or decreasing. Provide the exact answers.

3. Find the intervals where the function \( f(x) = 8x^2 + \frac{1}{x} \) is concave up or concave down. Provide the exact answers.

4. Solve the differential equation \( f''(x) = 3 - 5\sqrt{x} \), given that \( f'(1) = -2 \), and \( f(0) = 1 \).

5. Find the area bounded by the graphs of \( y = 2\sin x - 1 \), \( y = 0 \), \( x = \pi / 3 \), and \( x = \pi \). Provide the exact answer.

6. Evaluate \( \int_0^2 (x^2 + 2\sqrt{12 - 3x^2}) \, dx \). Provide the exact answer.

7. A ball is thrown vertically upward from a height of 6 ft at an initial velocity of 100 ft/sec.
   (a) Find the velocity of the ball after 4 seconds (exact answer).
   (b) Find how long the ball will stay more than 142 feet above the ground level (exact answer).
   Assume that the acceleration due to gravity is \(-32 \text{ ft/sec}^2\). DO NOT assume any equations from Physics to do this problem.

8. Calculate \( \int \frac{\cos^2 x + 1}{\cos^2 x - 1} \, dx \)

9. Use either the lower sums or the upper sums to find the area of the region bounded by the graphs of \( f(x) = 3x^2 + 2x \), \( x = 2 \), \( x = 5 \), and the x-axis. Provide the exact answer.
1. \( f(x) = 2x^3 + 3x^2 - 12x + 1 \)

   (a) Domain = \( \mathbb{R} \)

   (i) Critical points:
   \[ f'(x) = 6x^2 + 6x - 12 = 6(x^2 + x - 2) = 6(x+2)(x-1) \]
   Critical points are \( x = -2, \ 1 \)

   (ii) Local Extrema
   
<table>
<thead>
<tr>
<th>Sign of ( f'(x) )</th>
<th>(-2)</th>
<th>(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local maximum at ( x = -2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Local minimum at ( x = 1 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. \( f(x) = 2x^2 (1 - 2x)^5 \)

   (a) Domain = \( \mathbb{R} \)

   (i) Critical points:
   \[ f'(x) = 4x(1-2x)^4 \left[ -8x + (1-2x) \right] = 4x(1-2x)^4(1-7x) \]
   Critical points: \( x = 0, \ x = \frac{1}{2}, \ x = \frac{1}{7} \)

   (ii) Increasing/Decreasing intervals
   
<table>
<thead>
<tr>
<th>Sign of ( f'(x) )</th>
<th>(-)</th>
<th>(+)</th>
<th>(-)</th>
<th>(-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Increasing on ([0, \frac{1}{7}]) and Decreasing on ((-\infty, 0) \cup \left[\frac{1}{7}, \frac{1}{2}\right] \cup \left[\frac{1}{2}, \infty\right))</td>
<td></td>
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</tbody>
</table>

3. \( f(x) = 8x^2 + \frac{1}{x} \)

   (a) Domain = \( (-\infty, 0) \cup (0, \infty) \)

   (i) Concavity
   \[ f'(x) = 16x - \frac{1}{x^2} \]
   \[ f''(x) = 16 + \frac{2}{x^3} = \frac{16x^3 + 2}{x^3} = \frac{2(8x^3 + 1)}{x^3} \]

<table>
<thead>
<tr>
<th>Sign of ( f''(x) )</th>
<th>(-)</th>
<th>(+)</th>
<th>(+)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concave up on ((-\infty, -\frac{1}{2}) \cup (0, \infty))</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concave down on ([-\frac{1}{2}, 0))</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4. \[ f''(x) = 3 - 5\sqrt{x} \] with \[ f'(1) = -2 \] and \[ f(0) = 1 \]

\[
\int f''(x) \, dx = \int (3 - 5\sqrt{x}) \, dx
\]

\[
f'(x) = 3x - 5\left(\frac{2}{3}\right)x^{\frac{1}{2}} + C_1.
\]

\[
f'(1) = -2 = 3(1) - \left(\frac{10}{3}\right)(1) + C_1
\]

\[
\therefore C_1 = -2 - 3 + \frac{10}{3} = -\frac{5}{3}
\]

\[
f'(x) = 3x - \frac{10}{3}x^{\frac{3}{2}} - \frac{5}{3}
\]

So now \[ \int f'(x) \, dx = \int \left[ 3x - \frac{10}{3}x^{\frac{3}{2}} - \frac{5}{3} \right] \, dx \]

\[
f(x) = \frac{3x^2}{2} - \frac{10}{3}\left(\frac{2}{5}\right)x^{\frac{5}{2}} - \frac{5}{3}x + C_2.
\]

So \[ f(0) = 1 = 0 + C_2 \]

\[ \therefore C_2 = 1 \]

\[ \therefore \text{The solution of the diff. eqn is: } f(x) = \frac{3x^2}{2} - \frac{4}{3}x^{\frac{5}{2}} - \frac{5}{3}x + 1. \]

5. \[ A = \text{Total area} = A_1 + A_2 = \int_{\frac{\pi}{3}}^{\pi} (2\sin x - 1) \, dx + (-1) \int_{\frac{\pi}{3}}^{\frac{5\pi}{6}} (2\sin x - 1) \, dx \]

\[
A = \left[-2\cos x - x\right]_{\frac{\pi}{3}}^{\pi} - \left[-2\cos x - x\right]_{\frac{5\pi}{6}}^{\pi}
\]

\[
A = \left\{ \left(\sqrt{3} - \frac{5\pi}{6}\right) - \left(1 - \frac{\pi}{3}\right) \right\} - \left\{ \left(2 - \pi\right) - \left(\sqrt{3} - \frac{5\pi}{6}\right) \right\}
\]

\[
A = \left(\sqrt{3} + 1 - \frac{\pi}{2}\right) - \left(-\sqrt{3} + 2 - \frac{\pi}{6}\right) = \sqrt{3} + 1 - \frac{\pi}{2} + \sqrt{3} - 2 + \frac{\pi}{6}
\]

\[ \therefore \text{Area} = 2\sqrt{3} - 1 - \frac{\pi}{3} \text{ Sq. units.} \]

6. \[ \int_{0}^{\frac{\pi}{2}} \left(x^2 + 2\sqrt{12-3x^2}\right) \, dx \]

\[ = \frac{1}{2} \int_{0}^{\frac{\pi}{2}} x^2 \, dx + 2 \int_{0}^{\frac{\pi}{2}} \sqrt{12-3x^2} \, dx \]

\[ = \left[\frac{x^3}{3}\right]_0^{\frac{\pi}{2}} + 2\sqrt{3} \int_{0}^{\frac{\pi}{2}} \sqrt{4-x^2} \, dx \]

\[ = \frac{8}{3} + 2\sqrt{3} \pi^4 \]

\[ \text{Had done in class using area concept.} \]

\[ Y = \sqrt{4-x^2} \]

\[ \int_{0}^{\frac{\pi}{2}} \sqrt{4-x^2} \, dx = \text{Shaded area} \]

\[ = \frac{\pi}{4} (2)^2 = \pi \]
Let \( t = \text{time} \)
\( x(t) = \text{position at time } t, \text{ from the ground level} \)

**Given:** \( x''(t) = -32 \)

\[ \therefore \int x''(t) \, dt = \int -32 \, dt \]

\( v(t) = x'(t) = -32t + c_1 \)

\[ \therefore v(0) = 100 = -32(0) + c_1 \]

\[ \therefore c_1 = 100 \]

\[ \therefore v(t) = x'(t) = -32t + 100 \quad (1) \]

\[ \int x'(t) \, dt = \int (-32t + 100) \, dt \]

\( x(t) = -16t^2 + 100t + c_2 \)

\( x(0) = 6 = -16(0)^2 + 100(0) + c_2 \)

\[ \therefore c_2 = 6 \]

\[ \therefore x(t) = -16t^2 + 100t + 6 \quad (2) \]

Using (1) and (2), you can answer all the questions:

(a) \( v(4) = -32(4) + 100 = -128 + 100 = -28 \text{ ft/sec} \) is the velocity after 4 seconds

(b) Set \( x(t) = 142 \) on (2) and solve for \( t \).

\[ 142 = -16t^2 + 100t + 6 \]

\[ 16t^2 - 100t + 136 = 0 \]

\[ 4t^2 - 25t + 34 = 0 \]

\[ (4t - 17)(t + 2) = 0 \]

\[ \therefore t = \frac{17}{4} \text{ sec and } t = 2 \text{ secs} \]

\[ \therefore \text{The ball will stay more than } 142 \text{ ft above the ground for } \left(\frac{17}{4} - 2\right) \text{ secs. } \Rightarrow \frac{3}{4} \text{ seconds} \]
\[
\int \frac{\cos^2 x + 1}{\cos^2 x - 1} \, dx = \int \frac{\cos^2 x}{-\sin^2 x} \, dx + \int \frac{1}{-\sin^2 x} \, dx = \int (\cot^2 x - \csc^2 x) \, dx = \int (\csc^2 x + 1 - \csc^2 x) \, dx = \int (-2\csc^2 x + 1) \, dx = -2\cot x + x + C.
\]

9. \( f(x) = 3x^2 + 2x \) from \( x = 2 \) to \( x = 5 \).

Let's use upper sums:
\[
\Delta x = \frac{b-a}{n} = \frac{5-2}{n} = \frac{3}{n}
\]
\[
x_i = 2 + i\Delta x = 2 + \frac{3i}{n}, \quad i = 0, 1, 2, \ldots, n
\]

\[
S(n) = \sum_{i=1}^{n} f(x_i) \Delta x
\]

\[
S(n) = \sum_{i=1}^{n} f\left(2 + \frac{3i}{n}\right) \cdot \frac{3}{n}
\]

\[
\therefore \quad S(n) = \frac{3}{n} \sum_{i=1}^{n} \left[ 3\left(2 + \frac{3i}{n}\right)^2 + 2\left(2 + \frac{3i}{n}\right) \right]
\]

\[
= \frac{3}{n} \sum_{i=1}^{n} \left( 12 + \frac{36i}{n} + \frac{27i^2}{n^2} + 4 + \frac{6i}{n} \right)
\]

\[
= \frac{3}{n} \sum_{i=1}^{n} \left( 16 + \frac{42i}{n} + \frac{27i^2}{n^2} \right)
\]

\[
\therefore \quad S(n) = \frac{3}{n} \left\{ \sum_{i=1}^{n} \frac{16}{n} + \frac{42}{n} \sum_{i=1}^{n} i + \frac{27}{n^2} \sum_{i=1}^{n} i^2 \right\}
\]

\[
S(n) = \frac{3}{n} \left\{ \frac{16n + \frac{42}{n} \cdot n(n+1) + \frac{27}{n^2} \cdot n(n+1)(2n+1)}{2} \right\}
\]

\[
S(n) = \frac{3}{n} \left\{ 16 + 21 \left(1 + \frac{1}{n}\right) + \frac{9}{2} \left(1 + \frac{1}{n}\right) \left(2 + \frac{1}{n}\right) \right\}
\]

\[
\therefore \text{Area} = \lim_{n \to \infty} S(n) = \lim_{n \to \infty} 3 \left\{ 16 + 21 \left(1 + \frac{1}{n}\right) + \frac{9}{2} \left(1 + \frac{1}{n}\right) \left(2 + \frac{1}{n}\right) \right\} = 3(16 + 21 + 9) = 138 \text{ units}^2
\]