1. Find \( \lim_{x \to 2} \frac{2x - 4}{4 - x^2} \)

\[
\begin{align*}
\lim_{x \to 2} \frac{2(x - 2)}{(2 + x)(2 - x)} &= \lim_{x \to 2} \frac{2(x - 2)}{-2(x - 2)} \\
&= \frac{2}{2} = 1
\end{align*}
\]

2. Find \( \lim_{x \to 1} \frac{x - 2}{\sqrt{x + 3} - 2} \)

\[
\begin{align*}
\lim_{x \to 1} \frac{(x - 1)(\sqrt{x + 3} + 2)}{(\sqrt{x + 3} - 2)(\sqrt{x + 3} + 2)} &= \lim_{x \to 1} \frac{(x - 1)(\sqrt{x + 3} + 2)}{(x + 3) - 4} \\
&= \frac{4}{1} = 4
\end{align*}
\]

3. Find \( \lim_{x \to 0} \frac{\frac{1}{2 + x} - \frac{1}{2}}{x} \)

\[
\begin{align*}
\lim_{x \to 0} \frac{\frac{1}{2 + x} - \frac{1}{2}}{x} &= \lim_{x \to 0} \frac{2 - 2(x + x)}{2x(2 + x)} \\
&= \lim_{x \to 0} \frac{-2}{2(2 + x)} = \frac{-1}{4}
\end{align*}
\]

4. (a) Find \( \lim_{x \to 0} \frac{\sin(8x)}{4x} \)

\[
\begin{align*}
\lim_{x \to 0} \left[ \frac{\sin(8x)}{8x} \cdot \frac{8}{4} \right] &= \lim_{x \to 0} \left[ \frac{8}{4} \right] = 2
\end{align*}
\]

(b) Find \( \lim_{x \to \pi} \frac{1 - \tan x}{\sin x - \cos x} \)

\[
\begin{align*}
\lim_{x \to \pi} \frac{(1 - \sin x)}{\cos x} &= \lim_{x \to \pi} \frac{(\sin x - \cos x) \cdot \cos x}{(\sin x - \cos x) \cdot \cos x} \\
&= \lim_{x \to \pi} \frac{(\cos x - \sin x)}{4}
\end{align*}
\]
3. Find \( \lim_{x \to 0} \frac{\frac{1}{2+x} - \frac{1}{2}}{x} \)

\[ = \lim_{x \to 0} \frac{\left( \frac{1}{2+x} - \frac{1}{2} \right) 2(2+x)}{x \cdot 2(2+x)} = \lim_{x \to 0} \frac{2 - (2+x)}{2 \cdot 2(2+x)} = \lim_{x \to 0} \frac{-x}{2(2+x)} = \frac{-1}{2(2+0)} = \frac{-1}{4} \]

4. (a) Find \( \lim_{x \to 0} \frac{\sin 8x}{4x} \)

\[ = \lim_{x \to 0} \left[ \frac{\sin 8x}{8x} \cdot \frac{8}{4} \right] = (1) \left( \frac{8}{4} \right) = 2 \]

(b) \( \lim_{x \to \frac{\pi}{4}} \frac{1 - \tan x}{\sin x - \cos x} \)

\[ = \lim_{x \to \frac{\pi}{4}} \frac{\left( 1 - \frac{\sin x}{\cos x} \right) \cdot \cos x}{\left( \sin x - \cos x \right) \cdot \cos x} = \lim_{x \to \frac{\pi}{4}} \frac{\cos x - \sin x}{\sin x - \cos x} = \lim_{x \to \frac{\pi}{4}} \frac{\cos x - \sin x}{\sin x - \cos x} \cos x = \frac{-1}{\cos \left( \frac{\pi}{4} \right)} = \frac{-1}{\left( \frac{\sqrt{2}}{2} \right)} = -\sqrt{2} \]
5. The graph of a function \( y = f(x) \) is given below. Use the graph to answer the following questions:

(a) Find \( \lim_{{x \to 1^+}} f(x) = 2 \)

(b) Find \( \lim_{{x \to 1}} f(x) \) does not exist.

(c) Is \( f \) continuous at \( x = -2 \)?
   
   No: \( (f(-2) \) does not exist \)

(d) Find all the \( x \)-values at which the function has a removable discontinuity.
   
   Only at \( x = -2 \)

6. Consider the function \( f(x) = \begin{cases} 
  x^3 - 1, & x < 1 \\
  x + 1, & x \geq 1 
\end{cases} \)

(a) Draw a clear graph of the function \( f \).

(b) State the mathematical definition of the continuity of a function at a given point. Use this definition to check whether the above function is continuous at \( x = 1 \). Show complete work.

**Defn:** A fn \( f \) is continuous at a point \( x = a \) means:

(i) \( \lim_{{x \to a}} f(x) \) exists

(ii) \( f(a) \) exists

(iii) \( \lim_{{x \to a}} f(x) = f(a) \).

For the given function in the question, the condition (i) breaks, i.e. \( \lim_{{x \to 1^+}} f(x) \) does not exist. This is because \( \lim_{{x \to 1^-}} f(x) = -1 \) while \( \lim_{{x \to 1^+}} f(x) = 2 \). \( \therefore \lim_{{x \to 1^-}} f(x) \neq \lim_{{x \to 1^+}} f(x) \).

Thus, \( f \) is not continuous at \( x = 1 \).
Consider the function \( f(x) = \begin{cases} x^3 - 1, & x < 1 \\ x + 1, & x \geq 1 \end{cases} \)

(a) Draw a clear graph of the function \( f \).

(b) State the mathematical definition of the continuity of a function at a given point. Use this definition to check whether the above function is continuous at \( x = 1 \). Show complete work.

**Defn:** A fn \( f \) is continuous at a pt \( x = a \) means:

- \( \lim_{x \to a} f(x) \) exists
- \( f(a) \) exists
- \( \lim_{x \to a} f(x) = f(a) \)

For the given function in the question, the condition (i) breaks, i.e., \( \lim_{x \to 1} f(x) \) does not exist. This is because \( \lim_{x \to 1} f(x) = 0 \), while \( \lim_{x \to 1} f(x) = 2 \). \( \therefore \lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x) \).

Thus, \( f \) is not continuous at \( x = 1 \).

7) Use the difference quotient to find the derivative of \( f(x) = -2x - 3x^2 \).

\[
\begin{align*}
f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{-2(x+h) - 3(x+h)^2 + 2x + 3x^2}{h} \\
&= \lim_{h \to 0} \frac{-2x - 2h - 3x^2 - 6xh - 3h^2 + 2x + 3x^2}{h} \\
&= \lim_{h \to 0} \frac{h(-2 - 6x - 3h)}{h} \\
&= \lim_{h \to 0} (-2 - 6x) \\
&= -2 - 6x
\end{align*}
\]
Find the equation of the tangent line to the graph of \( y = -\frac{1}{x^2} \) at \( x = 2 \).

Let \( f(x) = -\frac{1}{x^2} \).

\[
\begin{align*}
\therefore f'(x) &= \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} \\
&= \lim_{h \to 0} \frac{-\frac{1}{(x+h)^2} + \frac{1}{x^2}}{h} \\
&= \lim_{h \to 0} \frac{-x^2(x+h)^2 + x^2(x+h)^2}{h \cdot x^2(x+h)^2} \\
&= \lim_{h \to 0} \frac{-x^2 + x^2}{h \cdot x^2(x+h)^2} \\
&= \lim_{h \to 0} \frac{0}{h \cdot x^2(x+h)^2} \\
&= \frac{0}{x^2} \\
&= 0
\end{align*}
\]

\[
\therefore \text{Slope of tgt. line at } x = 2 \text{ is } f'(2) = -\frac{1}{4} = \frac{1}{m}.
\]

Also \( f(2) = -\frac{1}{2^2} = -\frac{1}{4} \), so pt. on the graph is \((2, -\frac{1}{4})\).

\[
\begin{align*}
\therefore \text{The slope of the tgt line at } x = 2 \text{ is } y - y_1 &= m(x - x_1) \\
y + \frac{1}{4} &= \frac{1}{4}(x - 2) \\
y &= \frac{1}{4}x - \frac{3}{4}
\end{align*}
\]

Find the equation(s) of the tangent lines drawn from the point \((-2, 6)\) to the graph of \( y = 2x^2 \). Make sure to show all of your calculations.

Let \( f(x) = 2x^2 \).

Find \( f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} \)

\[
\begin{align*}
&= \lim_{h \to 0} \frac{2(x+h)^2 - 2x^2}{h} \\
&= \lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 - 2x^2}{h} \\
&= \lim_{h \to 0} \frac{4x + 2h}{h} \\
&= \lim_{h \to 0} \frac{4x}{h} \\
&= 4x
\end{align*}
\]

\[
\therefore f'(x) = 4x
\]

\[
\therefore \text{Slope of tgt } PQ = f'(a) = 4a
\]

Also, slope of \( PQ = \frac{2a^2 - 6}{a + 2} \) —— (2)

Set \( 1 \) & \( 2 \) equal: \( \frac{2a^2 - 6}{a + 2} = 4a 
\]

\[
\begin{align*}
2a^2 - 6 &= 4a^2 + 8a \\
0 &= 2a^2 + 8a + 6 \\
0 &= a^2 + 4a + 3 = (a+3)(a+1) \\
0 &= -3 \text{ or } a = -1
\end{align*}
\]

\( a = -3 \) corresponds to point \( Q \).

\( a = -1 \) corresponds to point \( R \) (see the figure).

Eqs of \( PQ \):

\[
\begin{align*}
y - 18 &= -12(x + 3) \\
y - 2 &= -4(x + 1)
\end{align*}
\]

Eqs of \( PR \):

\[
\begin{align*}
y &= -12x - 18 \\
y &= -4x - 2
\end{align*}
\]

\[
\therefore \text{The required equations of the two tangents are } y = -12x - 18 \text{ and } y = -4x - 2.
\]
Find the equation(s) of the tangent lines drawn from the point 
(-2, 6) to the graph of \( y = 2x^2 \). Make sure to show all of your 
calculations.

Let \( f(x) = 2x^2 \)

Find \( f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \)

\[
= \lim_{h \to 0} \frac{2(x+h)^2 - 2x^2}{h}
\]

\[
= \lim_{h \to 0} \frac{4xh + 4h^2 - 2x}{h}
\]

\[
= \lim_{h \to 0} \frac{4x + 4h}{1}
\]

\[
= 4x
\]

\[
f'(x) = 4x
\]

\[
\text{Slope of } PQ = f'(a) = 4a
\]

Also, \( \text{Slope of } PQ = \frac{2a^2 - 6}{a + 2} \)

Set \( 1 \) & \( 2 \) equal:

\[
\frac{2a^2 - 6}{a + 2} = 4a
\]

\[
2a^2 - 6 = 4a^2 + 8a
\]

\[
0 = 2a^2 + 8a + 6
\]

\[
0 = a^2 + 4a + 3 = (a+3)(a+1)
\]

\[
a = -3 \quad \text{or} \quad a = -1
\]

\[a = -3 \text{ Corresponds to point } Q\]

\[a = -1 \text{ Corresponds to point } R \] (see the figure)

Eqn of \( PQ \):

\[
y = -12(x+3)
\]

Eqn of \( PR \):

\[
y = -4(x+1)
\]

\[
\text{The required equations of the two tangents}
\]

are \( y = -12x - 18 \) and \( y = -4x - 2 \). //

Consider the function \( f(x) = \frac{x-3}{x^2 - 2x - 3} = \frac{(x-3)}{(x-3)(x+1)} \)

(a) Find \( \lim_{x \to -1} f(x) \)

\[
= -\infty
\]

(Does not exist)

(b) Find all the vertical asymptotes

of the graph of \( f \) using limit calculations.

By (a), the graph has a V.A. at \( x = -1 \). However, the graph does not have a V.A.

at \( x = 3 \), because \( f(x) \) does not approach \( +\infty \) or \( -\infty \) as \( x \to 3^- \) or \( x \to 3^+ \).

The only V.A. is \( x = -1 \). //