The Birth of Calculus

**Tangent line to**

\[ y = f(x) \text{ at point } P \]

\[ f(x) \]

\[ f(x+h) \]

\[ x \]

\[ x+h \]

\[ y = f(x) \]

\[ f(x+h) - f(x) \]

\[ h \]

Diff. Quo. \( = \frac{f(x+h)-f(x)}{h} \)

The slope of the secant line \( PQ \)

The average rate of change of \( f(x) \) from \( x \) to \( x+h \)

Now, make \( h \) smaller and smaller. So, as \( h \to 0 \) one gets:

\[ f'(x) = \lim_{{h \to 0}} \frac{f(x+h)-f(x)}{h} \]

The slope of the tangent line at \( P \)

The instantaneous rate of change of \( f(x) \) at \( P \)

**Algebraic Meaning of the Derivative**

**Geometric Meaning**

**Physical Meaning**