IMPORTANT: Do not attempt to write or workout any answer on this piece of paper, as it will not be graded.

1. $ABC$ is a right triangle with the right angle at $B$. Given that $A = 34^\circ$ and $a = 23.5\ cm$, find $c$. (two decimal places).

2. $ABC$ is a right triangle with the right angle at $B$. Given that $b = 12\ cm$ and $a = 5\ cm$, find the approximate value of $C$ (two decimal places), and the exact value of $c$.

3. Find all solutions of the equation $2\csc\theta + 4 = 0$ where $-180^\circ \leq \theta \leq 540^\circ$.

4. Find exact solutions of the equation $3\cos\theta - 6\cos^3\theta = 0$ where $-2\pi \leq \theta \leq \pi$.

5. From the top of a lighthouse $270\ ft$ tall, the angle of depression of a small boat on the ocean surface is $42^\circ$. Find how far is the boat from the top of the lighthouse (two decimal places).

6. Two vertical buildings are located on the ground, facing each other. The shorter building is $30$ feet tall. From the bottom of the shorter building, the angle of elevation to the top of the taller building is $50^\circ$. From the top of the shorter building the angle of elevation to the top of the taller building is $20^\circ$. Find the angle of depression from the top of the shorter building to the bottom of the taller building (two decimal places).

7. A hill makes an angle of $10^\circ$ with the horizontal level. A vertical building is standing on the top of the hill. From a point on the hillside, $20$ feet downhill from the base of the building, the angle of elevation to the top of the building is $43^\circ$. Find the height of the building (two decimal places).

8. Prove the identity: \[
\frac{8\tan^3\beta + \sec\beta\tan^2\beta + \sec\beta}{\sec\beta + 2\tan\beta} = 1 - 2\sec\beta\tan\beta + 5\tan^2\beta
\]

9. Find all solutions of the equation $\sec\alpha + \tan\alpha = 2$ where $-360^\circ \leq \alpha \leq 360^\circ$. 

1. \( \tan 34^\circ = \dfrac{23.5}{c} \)  
   \[ c \left( \tan 34^\circ \right) = 23.5 \]  
   \[ c \approx 34.84 \text{ cm} \]

2. \[ \cos C = \dfrac{5}{12} \]  
   \[ C \approx \cos^{-1} \left( \dfrac{5}{12} \right) \]  
   \[ C \approx 65.38^\circ \]

3. \[ 2 \sec \theta + 4 = 0 \]  
   \[ 2 \sec \theta = -4 \]  
   \[ \sec \theta = -2 \]  
   \[ \sin \theta = -\dfrac{1}{2} \]  
   \[ \theta = -30^\circ, -150^\circ, 210^\circ, 330^\circ \]  
   Only 4 solutions.

4. \[ 3 \cos \theta - 6 \cos^3 \theta = 0 \]  
   \[ 3 \cos \theta \left( 1 - 2 \cos^2 \theta \right) = 0 \]  
   \[ \cos \theta = 0 \text{ or } 1 = 2 \cos^2 \theta \]  
   \[ \cos \theta = 0 \text{ or } \cos \theta = \pm \dfrac{1}{\sqrt{2}}. \]  
   Now break into 3 different tracks!

   - \( \cos \theta = 0 \)  
     \[ \theta = -\dfrac{\pi}{2}, -\dfrac{3\pi}{2}, \dfrac{\pi}{2} \]

   - \( \cos \theta = \pm \dfrac{1}{\sqrt{2}} \)  
     \[ \theta = -\dfrac{\pi}{4}, -\dfrac{5\pi}{4}, \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{3\pi}{4}, -\dfrac{\pi}{4}, -\dfrac{7\pi}{4}, \dfrac{\pi}{4}, \dfrac{7\pi}{4} \]

   Only 9 solutions: \( \theta = -\dfrac{\pi}{2}, -\dfrac{3\pi}{2}, -\dfrac{3\pi}{4}, -\dfrac{5\pi}{4}, \dfrac{\pi}{4}, \dfrac{3\pi}{4}, -\dfrac{\pi}{4}, -\dfrac{7\pi}{4}, \dfrac{\pi}{4}, \dfrac{7\pi}{4} \).
Math 165 (Fall 2016)

Key Test 2

\[ \sin 42^\circ = \frac{270}{d} \]
\[ : \quad d = \frac{270}{\sin 42^\circ} \approx 403.51 \text{ ft} \]

\[ \begin{align*}
\text{From } \triangle BDE: \quad \tan 20^\circ &= \frac{h}{d} \\
\therefore h &= d \tan 20^\circ \\
\text{From } \triangle ACE: \quad \tan 50^\circ &= \frac{h+30}{d} \\
\therefore h+30 &= d \tan 50^\circ \\
\end{align*} \]

Subtract eqn (i) from eqn (ii)
\[ 30 = d (\tan 50^\circ - \tan 20^\circ) \]
\[ : \quad d = \frac{30}{\tan 50^\circ - \tan 20^\circ} \approx 36.24 \text{ ft} \]

Now, from \( \triangle ABC \), \( \tan \theta = \frac{30}{36.24} \ldots \)
\[ \theta = \tan^{-1} \left( \frac{30}{36.24} \right) \approx 39.62^\circ \]
\[ \therefore \text{The angle of depression from the top of the shorter bldg to the bottom of the taller bldg is about } 39.62^\circ \]

\[ \begin{align*}
\text{Use Law of Sines for the oblique } \\
\triangle ABC: \quad \frac{h}{\sin 33^\circ} &= \frac{20}{\sin 47^\circ} \\
\therefore h &= \frac{20 \sin 33^\circ}{\sin 47^\circ} \approx 14.88 \text{ ft} \\
\end{align*} \]
\[ \therefore \text{The bldg is about } 14.88 \text{ ft tall} \]
8. Prove: \[
\frac{8 \tan^3 \beta + \sec \beta \tan^2 \beta + \sec \beta}{\sec \beta + 2 \tan \beta} = 1 - 2 \sec \beta \tan \beta + 5 \tan^2 \beta
\]

\[
L.H.S = \frac{8 \tan^3 \beta + \sec \beta \tan^2 \beta + \sec \beta}{\sec \beta + 2 \tan \beta}
\]

\[
= \frac{8 \tan^3 \beta + \sec^3 \beta}{\sec \beta + 2 \tan \beta}
\]

\[
= \frac{8 \tan^3 \beta + (\sec \beta)^3}{\sec \beta + 2 \tan \beta}
\]

\[
= \frac{8 \tan^3 \beta + (\sec \beta)^3 + (2 \tan \beta)^3}{\sec \beta + 2 \tan \beta}
\]

\[
= (\sec \beta + 2 \tan \beta)(\sec^2 \beta - 2 \sec \beta \tan \beta + 4 \tan^2 \beta)
\]

\[
= 1 + \tan^2 \beta - 2 \sec \beta \tan \beta + 4 \tan^2 \beta
\]

\[
= 1 - 2 \sec \beta \tan \beta + 5 \tan^2 \beta = R.H.S
\]

\[
\therefore L.H.S = R.H.S
\]

9. Solve: \[
\sec \alpha + \tan \alpha = 2 - 1 - 360^\circ \leq \alpha \leq 360^\circ
\]

\[
\sec \alpha = 2 - \tan \alpha
\]

Square both sides: \[
(\sec \alpha)^2 = (2 - \tan \alpha)^2
\]

\[
\therefore \sec^2 \alpha = 4 - 4 \tan \alpha + \tan^2 \alpha
\]

\[
1 + \tan^2 \alpha = 4 - 4 \tan \alpha + \tan^2 \alpha
\]

\[
4 \tan \alpha = 3
\]

\[
\tan \alpha = \frac{3}{4}
\]

Ref \(g = \tan^{-1}(\frac{3}{4}) \approx 36.87^\circ
\]

\[
\therefore \alpha = -143.13^\circ, -323.13^\circ, 36.87^\circ, 216.87^\circ
\]

All these are solutions of the squared eqn (2).

However, not all these necessarily solutions of the original unsquared equation (1).

So, we must check each on on orig. eqn (1).

We can find \(\alpha = -143.13^\circ\) and \(-216.87^\circ\) do NOT satisfy (1) (Use Calculator)

However, other 2 solutions satisfy.

Final soltn: \(\alpha \approx -323.13^\circ, 36.87^\circ\)