1. Find the derivatives of the following functions. Simplify.
   (a) \( f(x) = \frac{1}{3x^3} = \frac{1}{3} x^{-3} \)
   \[ f'(x) = \frac{1}{3} (-3x^{-4}) = -\frac{1}{x^4} \]
   \( g(x) = \ln \left( \frac{2}{x} \right) = \ln 2 - \ln x \)
   \[ g'(x) = \frac{-1}{x} \]

2. Find the equation of the tangent line to the graph of \( f(x) = 3 \sqrt{13-x^2} \) at \( x = 2 \). Leave the exact answer in the slope-intercept form.
   \[ f'(x) = 3 \cdot \frac{1}{2} (13-x^2)^{-\frac{1}{2}} (-2x) \]
   \[ f'(x) = -3x \sqrt{13-x^2} \]
   \[ f'(2) = -3(2) = -6 \]
   \[ y - y_1 = m(x - x_1) \]
   \[ y - 3 = -2(x - 2) \]
   \[ y = -2x + 13 \]

3. Find the x-coordinates of the relative minima and relative maxima of the function \( y = -4x^3 + 3x^2 + 5 \).
   Provide the exact answers.
   Domain = \( \mathbb{R} \)
   \[ f'(x) = -12x^2 + 6x \]
   \[ f'(x) = 6x(-2x + 1) \]
   \[ f'(x) = 6x \left( -2x + 1 \right) \]
   \[ f'(x) = 6x(2x - 1) \]
   \[ x = 0, \frac{1}{2} \]
   Critical points: \( x = 0, \frac{1}{2} \)
   Never
   Sign of \( f' \):
   \[ - \quad + \quad + \quad + \]
   Rel. Min. at \( x = \frac{1}{2} \)
   Rel. Max at \( x = 0 \)

4. Find the x-coordinates of the points on the graph of \( f(x) = x^2e^{-3x} \) at which the tangent line is horizontal.
   Provide the exact answers.
   Domain = \( \mathbb{R} \)
   \[ f'(x) = 2xe^{-3x} + x^2e^{-3x} \]
   \[ f'(x) = x(e^{-3x}(-3) + 2e^{-3x}) \]
   \[ f'(x) = x(e^{-3x}(-3 + 2)) \]
   \[ f'(x) = x(e^{-3x}(-3 + 2)) = 0 \]
   \[ x = 0 \] or \( e^{-3x} = 0 \) or \(-3x + 2 = 0\)
   \[ x = 0 \] No solutions
   \[ x = \frac{2}{3} \] D.N.E

5. Find the intervals where the function \( y = (x-3)^2(2x+1)^3 \) is increasing or decreasing. Provide the exact answer in the interval notation.
   Domain = \( \mathbb{R} \)
   \[ f'(x) = (x-3)^2(3(2x+1)^2 + 2(x-3)(2)(2x+1)) \]
   \[ f'(x) = 2(x-3)(2x+1)^2 \left[ 3(x-3) + (2x+1) \right] \]
   \[ f'(x) = 2(x-3)(2x+1)^2(5x-8) \]
   \[ x = 3; x = -\frac{1}{2}; x = \frac{8}{5} \]
   \[ f \text{ is inc. on } (-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \frac{8}{5}) \cup (\frac{8}{5}, \infty) \]
   \[ f \text{ is dec. on } (-\frac{1}{2}, \frac{8}{5}) \]

\[ x = \frac{8}{5}, 3 \]
6. Find the derivative of the function \( f(t) = \frac{3}{1 + 2^{-t}} \) and simplify so that all exponents are positive. Use the simplified derivative to find the slope of the tangent line to the graph of \( f \) at \( t = 2 \). Provide the exact answer.

\[
\begin{align*}
  f(c(t)) &= 3(1 + 2^{-t})^{-2} \\
  f'(c(t)) &= 3(-1)(1 + 2^{-t})^{-3} \cdot 2^{-t} \ln 2 \\
  \therefore f'(c(t)) &= \frac{3 \ln 2}{2^t (1 + 2^{-t})^2} = \frac{3 \ln 2}{2^t (2^t + 1)^2} \\
  m &= f'(2) = \frac{3(\ln 2) \cdot 2}{(4 + 1)^2} = \frac{12 \ln 2}{25} 
\end{align*}
\]

7. Find the x-coordinates of the points on the graph of \( y = \frac{2x - 1}{x + 3} \) at which the tangent line has a slope of 2. Provide the exact answers.

\[
\begin{align*}
  f'(x) &= \frac{(x + 3)(2) - (2x - 1) \cdot 1}{(x + 3)^2} \\
  f'(x) &= \frac{2x + 6 - 2x + 1}{(x + 3)^2} \\
  \therefore f'(x) &= \frac{7}{(x + 3)^2} \\
  \therefore (x + 3)^2 &= \frac{7}{2} \\
  \therefore x + 3 &= \pm \frac{\sqrt{14}}{2} \\
  \therefore x &= -3 \pm \frac{\sqrt{14}}{2} = \frac{-6 \pm \sqrt{14}}{2} 
\end{align*}
\]

8. Find the critical points of the function \( f(x) = 2x \sqrt[3]{x^2 - 1} \). Provide the exact answers.

\[
\begin{align*}
  f'(x) &= \frac{2x \cdot \frac{1}{3} (x^2 - 1)^{-2/3} (2x) + 2 \cdot (x^2 - 1)^{1/3}}{3 (x^2 - 1)^{2/3}} \\
  &= \frac{4x^2 + 2(x^2 - 1)^{1/3}}{3 (x^2 - 1)^{2/3}} \\
  \therefore f'(x) &= \frac{10x^2 - 6}{3 (x^2 - 1)^{2/3}} = \frac{2(5x^2 - 3)}{3 (x^2 - 1)^{2/3}} \\
  \text{EOR} \quad x &= \pm \frac{\sqrt{3}}{5}, \quad x = \pm 1 
\end{align*}
\]

9. The revenue \( R(x) \) and the cost \( C(x) \) for the production of \( x \) units is given by \( R(x) = 2x - \frac{x^2}{25000} \) and \( C(x) = 2100 + 0.25x \). Find the marginal average profit for the production of 100 units (two decimal places).

\[
\begin{align*}
  P(x) &= R(x) - C(x) \\
  P(x) &= 2x - \frac{x^2}{25000} - \left[ 2100 + 0.25x \right] \\
  P(x) &= 1.75x - \frac{x^2}{25000} - 2100 \\
  \therefore P'(100) &= \frac{-1}{25000} + \frac{2100}{x^2} \\
  \therefore P'(100) &= \frac{-1}{25000} + \frac{2100}{(100)^2} \\
  \therefore P'(100) &= 0.21 
\end{align*}
\]