1. Find the derivatives of the following functions:

(a) \( f(x) = 5x^2 + 2 \)
\[
\begin{align*}
\frac{df}{dx} &= 5(2x) + 0 \\
\therefore \quad \frac{df}{dx} &= 10x
\end{align*}
\]

(b) \( g(x) = \frac{1}{\sqrt{6}} \)
\[
\begin{align*}
\frac{dg}{dx} &= 0
\end{align*}
\]

2. Find the equation of the tangent line to the graph of \( f(x) = x^3 - 3x^2 + 6x - 4 \) at \( x = -2 \).

\[
\begin{align*}
f'(-2) &= (-2)^3 - 3(-2)^2 + 6(-2) - 4 \\
&= -8 - 12 - 12 - 4 \\
&= -36
\end{align*}
\]
\[
\text{pt: on the graph } (x, y) = (-2, -36)
\]
\[
\text{eqn. of tgl: } y - y_1 = m(x - x_1)
\]
\[
y - (-36) = m(x - (-2))
\]
\[
y + 36 = m(x + 2)
\]
\[
y = mx + 30
\]

3. Given \( f(x) = \frac{3}{2\sqrt{x}} \), use the definition of the derivative to find \( f'(4) \). Do not use the short-cut rules for the derivatives to do this problem. Provide the exact answer.

\[
\begin{align*}
f'(x) &= \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{3}{2\sqrt{x + h}} - \frac{3}{2\sqrt{x}} \\
&= \lim_{h \to 0} \frac{3\sqrt{x} - 3\sqrt{x + h}}{2\sqrt{x + h} \cdot 2\sqrt{x}} \\
&= \lim_{h \to 0} \frac{3\sqrt{x} - 3\sqrt{x + h}}{2\sqrt{x}(2\sqrt{x} + 2\sqrt{x + h})} \\
&= \lim_{h \to 0} \frac{3}{2\sqrt{x}(2\sqrt{x} + 2\sqrt{x})} \\
&= \frac{3}{4x}\sqrt{x}
\end{align*}
\]

\[
\therefore \quad f'(4) = \frac{-3}{32}
\]

4. Find \( \lim_{x \to 4} \frac{2x^2 - 32}{12 - 3x} \) algebraically.

\[
\begin{align*}
&= \lim_{x \to 4} \frac{2(x + 4)(x - 4)}{3(4 - x)} \\
&= \lim_{x \to 4} \frac{2(x + 4)(x - 4)}{-3(x - 4)} \\
&= \frac{2(4 + 4)}{-3} \\
&= \frac{-16}{3}
\end{align*}
\]

5. Find \( \lim_{x \to 1} \frac{x - 3}{x^3 - 1} \) algebraically.

\[
\begin{align*}
&= \lim_{x \to 1} \frac{\frac{1}{x - 3} + \frac{1}{2}}{(x - 1)(x^2 + x + 1)(x - 3)} \\
&= \lim_{x \to 1} \frac{2 + (x - 3)}{(x - 1)(x^2 + x + 1)(x - 3)} \\
&= \lim_{x \to 1} \frac{(x - 1)}{(x - 1)(x^2 + x + 1)(x - 3)} \\
&= \lim_{x \to 1} \frac{1}{(x^2 + x + 1)(x - 3)} \\
&= \frac{1}{(3)(2)(-2)} \\
&= -\frac{1}{12}
\end{align*}
\]
6. Find \( \lim_{x \to \infty} \frac{2x^2 - x - 3}{1 + x + 5x^2} \) algebraically.

\[
\lim_{x \to \infty} \frac{2x^2 - x - 3}{x^2 + x + 5x^4/x^2} = \lim_{x \to \infty} \frac{2 - \frac{1}{x} - \frac{3}{x^2}}{\frac{1}{x^2} + \frac{1}{x} + 5} = \frac{2 - 0 - 0}{0 + 0 + 5} = \frac{2}{5}
\]

7. Find the x-coordinates of the points on the graph of \( f(x) = 6x^2 + \frac{3}{2x^2} \) at which the tangent line is horizontal.

Provide the exact answers.

\[
f(x) = 6x^2 + \frac{3}{2x^2}
\]

\[
f'(x) = 12x + \frac{3}{2}(-2x^{-3})\]

\[
f''(x) = 12 - \frac{3}{x^3}
\]

Set \( f'(x) = 0 \) and solve for \( x \)

\[
12x - \frac{3}{x^3} = 0
\]

\[
12x = \frac{3}{x^3}
\]

\[
12x = \frac{3}{x^3}
\]

\[\therefore x = \pm \sqrt[3]{\frac{1}{2}} = \pm \frac{1}{\sqrt[3]{2}} \quad \text{(must simplify)}\]

8. (a) Find \( \log_{3.2} \sqrt[4]{27} \) by hand. No calculators.

Set:

\[
9^x = \sqrt[4]{27}
\]

\[
(3^3)^x = (27)^{1/4}
\]

\[
3^{3x} = 3^{3/4}
\]

\[
2x = 3/4
\]

\[
x = 3/8
\]

\[
\log_{3.2} \sqrt[4]{27} = \frac{3}{8}
\]

(b) Solve: \( \log_3 (x + 2) + \log_3 (2x + 5) = 1 \) (exact solutions)

\[
\log_3 \left(\frac{x+2}{2x+5}\right) = 1
\]

\[
\frac{x+2}{2x+5} = 3
\]

\[
x+2 = 6x+15
\]

\[
x+2 = 6x+15
\]

\[
-13 = 5x
\]

\[
x = -\frac{13}{5}
\]

9. The revenue \( R(x) \) and the cost \( C(x) \) for the production of \( x \) units is given by \( R(x) = 2x - \frac{x^2}{25000} \) and \( C(x) = 2100 + 0.25x \). Find the marginal profit for the production of 15,000 units (two decimal places)

\[
P(x) = R(x) - C(x)
\]

\[
P'(x) = R'(x) - C'(x)
\]

\[
P'(x) = 2 - \frac{2x}{25000} - 0.25
\]

\[
P'(x) = 1.75 - \frac{x}{12500}
\]

\[
P'(15000) = 1.75 - \frac{15000}{12500}
\]

\[
P'(15000) = 1.75 - 1.20
\]

\[
P'(15000) = 0.55
\]
1. Find the derivatives of the following functions:
   (a) \( f(x) = 5x^2 + 2 \)  
   (b) \( g(x) = \frac{1}{\sqrt[6]{x}} \)

2. Find the equation of the tangent line to the graph of \( f(x) = x^3 - 3x^2 + 6x - 4 \) at \( x = -2 \).

3. Given \( f(x) = \frac{3}{2\sqrt{x}} \), use the definition of the derivative to find \( f'(4) \). Do not use the short-cut rules for the derivatives to do this problem. Provide the exact answer.

4. Find \( \lim_{x \to 4} \frac{2x^2 - 32}{12 - 3x} \) algebraically.

5. Find \( \lim_{x \to 1} \frac{1}{x-3} + \frac{1}{2} \) algebraically.
6. Find \( \lim_{x \to \infty} \frac{2x^2 - x - 3}{1 + x + 5x^2} \) algebraically.

7. Find the \( x \)-coordinates of the points on the graph of \( f(x) = 6x^2 + \frac{3}{2x^2} \) at which the tangent line is horizontal. Provide the exact answers.

8. (a) Find \( \log_9 \sqrt[3]{27} \) by hand. No calculators. (b) Solve: \( \log_3(x + 2) = 1 + \log_3(2x + 5) \) (exact solutions)

9. The revenue \( R(x) \) and the cost \( C(x) \) for the production of \( x \) units is given by \( R(x) = 2x - \frac{x^2}{25000} \) and \( C(x) = 2100 + 0.25x \). Find the marginal profit for the production of 15,000 units (two decimal places)