1. (a) Convert 750° to radians (exact answer)  
(b) Convert 6 radians to degrees (two decimal places)

2. In the triangle $ABC$, $B = 52^\circ$, $c=12.5$ cm, and $a= 6.2$ cm. Find $b$ (two decimal places).

3. In the triangle $ABC$, $a= 3.8$ cm, $B = 82^\circ$, and $C = 25^\circ$. Find $b$ (two decimal places).

4. In the triangle $ABC$, $a = 2.4$ in, $b = 6.5$ in, and $c = 7.1$ in. Find $B$ (two decimal places)

5. Find the exact values of the first three trigonometric functions of an angle of $\frac{119\pi}{6}$ radians.

6. The radius of a circular sector is 30 inches and its central angle is $210^\circ$. Find the exact value of its arc length.

7. The area of a circular sector is 10 sq. cm. and its arc length is 8 cm. Find its central angle in degrees. (two decimal places)

8. Find all exact solutions of the equation $4\cos^2\theta = 1$ where $-11\pi / 6 \leq \theta \leq 3\pi / 2$.

9. The longest diagonal of a parallelogram is $\sqrt{67}$ inches. One of the angles of the parallelogram is $120^\circ$ and the perimeter of the parallelogram is 18 inches. Find the exact area of the parallelogram.
1. (a) $750^\circ = \frac{25\pi}{6} \text{ rad} = \frac{25\pi}{180^\circ} \text{ rad}$
    (b) $6 \text{ rad} = \frac{6 \times 180^\circ}{\pi} \approx 343.77^\circ$

2. \[ b^2 = a^2 + c^2 - 2ac \cos B \]
   \[ b^2 = 6.2^2 + 12.5^2 - 2 \times 6.2 \times 12.5 \times \cos (52^\circ) \]
   \[ \therefore b = \sqrt{6.2^2 + 12.5^2 - 2 \times 6.2 \times 12.5 \times \cos (52^\circ)} \approx 9.96 \text{ cm} \]

3. \[ A = 180^\circ - (82^\circ + 25^\circ) \approx 73^\circ \]
   \[ \frac{b}{\sin 82^\circ} = \frac{3.8}{\sin 73^\circ} \]
   \[ \therefore b = \frac{3.8 \sin 82^\circ}{\sin 73^\circ} \approx 3.93 \text{ cm} \]

4. \[ b^2 = a^2 + c^2 - 2ac \cos B \]
   \[ 2ac \cos B = a^2 + c^2 - b^2 \]
   \[ \therefore \cos B = \frac{a^2 + c^2 - b^2}{2ac} \]
   \[ \therefore B = \cos^{-1} \left( \frac{2.4^2 + 7.1^2 - 6.5^2}{2 \times 2.4 \times 7.1} \right) \approx 65.89^\circ \]

5. \[ \frac{119\pi}{6} \text{ rad} = \frac{20\pi}{6} \text{ rad} = \frac{10\pi}{30^\circ} \]
   \[ \sin \left( \frac{119\pi}{6} \right) = \frac{1}{2} \sin 30^\circ = -\frac{1}{2} \]
   \[ \cos \left( \frac{119\pi}{6} \right) = \frac{\sqrt{3}}{2} \cos 30^\circ = \frac{\sqrt{3}}{2} \]
   \[ \tan \left( \frac{119\pi}{6} \right) = \frac{\sqrt{3}}{3} \tan 30^\circ = -\frac{1}{\sqrt{3}} \]

6. \[ \theta = \frac{r}{\theta} \]
   \[ r = 30 \text{ cm} \]
   \[ \theta = 210^\circ = \frac{7\pi}{6} \text{ rad} \]
   \[ \theta = \frac{35\pi}{6} \text{ inches} \]

7. \[ A = \frac{1}{2} r^2 \theta \]
   \[ \theta = 20^\circ = 64 \]
   \[ \theta = \frac{1}{2} \left( \frac{3}{8} \right)^2 \theta \]
   \[ \theta = \frac{64}{20} \frac{\text{ rad}}{\pi} = \frac{16}{5} \frac{\text{ rad}}{\pi} \approx 183.35^\circ \]
8. \(4 \cos^2 \theta = 1\)
   \[
   \cos \theta = \frac{1}{4}
   \]
   \[
   \therefore \cos \theta = \pm \frac{1}{2}.
   \]
   Ref \(\theta = \frac{\pi}{3}\)

\[
\theta = -\frac{2\pi}{3}, -\frac{4\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{\pi}{3}, -\frac{\pi}{3}, -\frac{5\pi}{3}, \frac{\pi}{3}
\]

Only 7 solutions: \(\theta = -\frac{2\pi}{3}, -\frac{4\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{\pi}{3}, -\frac{\pi}{3}, -\frac{5\pi}{3}\)

9. \(x + y = 9\) —— 1

Use Law of Cosines for \(\Delta ABC\):
\[
(567)^2 = x^2 + y^2 - 2xy \cos(120°)
\]
\[
67 = x^2 + y^2 - 2xy \left(-\frac{1}{2}\right)
\]
\[
\therefore x^2 + y^2 + xy = 67
\]

By 1, \(y = 9 - x\).
Plug back in 2
\[
x^2 + (9-x)^2 + x(9-x) = 67
\]
\[
x^2 + 81 - 18x + x^2 + 9x - x^2 = 67
\]
\[
x^2 - 9x + 14 = 0
\]
\[
(x-7)(x-2) = 0
\]
\[
\therefore x = 7 \text{ or } x = 2.
\]
Since \(x\) is the longest side, we can pick \(x\) to be 7

\[
x = 7 \therefore y = 9-x = 9-7 = 2
\]

Now we have \(x = 7 \text{ cm} \) and \(y = 2 \text{ cm}\)

\[
\therefore \text{Area of } ABCD = 2 \times \text{Area of } \Delta ABC = 2 \times \frac{1}{2} \times 7 \times 2 \times \sin(120°) = \frac{7 \sqrt{3}}{2}
\]

\[
\therefore \text{Area of } ABCD = 7\sqrt{3} \text{ cm}^2
\]

~ END OF TEST 3 ~