SHOW ALL WORK

1. Find the distance between the points (2, -3) and (-4, -5). Provide the exact answer in a simplified form.
\[ d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = \sqrt{(-4-2)^2 + (-5+3)^2} = \sqrt{(-6)^2 + (-2)^2} = \sqrt{36 + 4} = \sqrt{40} = 2\sqrt{10} \text{ units} \]

2. State the Pythagorean Theorem in precise terms. DONE IN CLASS (SEE NOTES!)
ABC is a right triangle with the right angle at B. Given that \( b = 4\sqrt{3} \) cm, and \( a = 2 \) cm, find the exact value of \( c \) (simplify). Also find the area of the triangle.

Use Pyth. Thm.: \[ b^2 = a^2 + c^2 \]
\[ (4\sqrt{3})^2 = (2)^2 + c^2 \]
\[ 48 = 4 + c^2 \]
\[ c^2 = 44 \]
\[ c = \sqrt{44} \text{ cm} \]
Area \( A \) = \( \frac{1}{2} \) (base) \times (height)
\[ A = \frac{1}{2} \times 2\sqrt{11} \times 2 \]
\[ A = 2\sqrt{11} \text{ cm}^2 \]

3. (a) Convert 143° 23' 15'' into degrees and decimal degrees (two decimal places)
\[ 143° 23' 15'' = 143 + \frac{23}{60} + \frac{15}{3600} \]
\[ \approx 143.39° \]

(b) Convert 58.383° into degrees, minutes, and seconds.
\[ 58.383° = 58° + (0.383 \times 60)' \]
\[ \approx 58° 22' 5'' \]

4. Given that the point \( P(6, -8) \) is on the terminal side of some angle \( \theta \) in the standard position, find the exact values of all the trigonometric functions of \( \theta \).
\[ r = \sqrt{x^2 + y^2} \]
\[ r = \sqrt{6^2 + (-8)^2} \]
\[ r = \sqrt{36 + 64} \]
\[ r = \sqrt{100} \]
\[ r = 10 \]
\[ \sin \theta = \frac{y}{r} = \frac{-8}{10} = -\frac{4}{5} \]
\[ \csc \theta = -\frac{5}{4} \]
\[ \cos \theta = \frac{x}{r} = \frac{6}{10} = \frac{3}{5} \]
\[ \sec \theta = \frac{5}{3} \]
\[ \tan \theta = \frac{y}{x} = -\frac{8}{6} = -\frac{4}{3} \]
\[ \cot \theta = -\frac{3}{4} \]

5. Given \( \sec \alpha = -\frac{3}{\sqrt{5}} \) with \( \alpha \) in quadrant II, find the exact value of \( \tan \alpha \). Use only the trigonometric identities to solve this problem.

\[ \sec^2 \alpha = 1 + \tan^2 \alpha \]
\[ \left(-\frac{3}{\sqrt{5}}\right)^2 = 1 + \tan^2 \alpha \]
\[ \frac{9}{5} = 1 + \tan^2 \alpha \]
\[ \tan^2 \alpha = \frac{9}{5} - 1 = \frac{4}{5} \]
\[ \therefore \tan \alpha = \pm \sqrt{\frac{4}{5}} \]

But \( \alpha \) is in the quadrant II, so \( \tan \alpha \) is negative.
\[ \therefore \tan \alpha = -\frac{\sqrt{4}}{\sqrt{5}} \]
\[ \therefore \tan \alpha = -\frac{2}{\sqrt{5}} \]
6. Given \( \sec \beta = \frac{-3}{2} \) with \( \beta \) in quadrant IV, find the exact values of all the trigonometric functions of \( \beta \).

Use any method to solve this problem.

\[
\begin{align*}
\sec \beta &= \frac{-3}{2} = \frac{3}{-2} = \frac{r}{y} \\
&= y = -2; \quad r = 3 \\
x^2 + y^2 &= r^2 \\
x^2 + (-2)^2 &= (3)^2 \\
x^2 + 4 &= 9 \\
x^2 &= 5 \\
x &= \pm \sqrt{5}
\end{align*}
\]

\[
\begin{align*}
\sin \beta &= \frac{y}{r} = -\frac{2}{3} \\
\cos \beta &= \frac{x}{r} = \frac{\sqrt{5}}{3} \\
\tan \beta &= \frac{y}{x} = -\frac{2}{\sqrt{5}} \\
\cot \beta &= \frac{x}{y} = -\frac{\sqrt{5}}{2} \\
\sec \beta &= \frac{r}{x} = \frac{3}{\sqrt{5}} \\
\csc \beta &= \frac{r}{y} = -\frac{3}{2}
\end{align*}
\]

Note:

\( r \) cannot be negative!

7. Given \( \sin \theta = \frac{\sqrt{2}}{5} \) with \( \theta \) in quadrant II, find the exact values \( \sec \theta \) and \( \cot \theta \). Use only the trigonometric identities to solve this problem.

\[
\begin{align*}
\sin^2 \theta + \cos^2 \theta &= 1 \\
\left(\frac{\sqrt{2}}{5}\right)^2 + \cos^2 \theta &= 1 \\
\frac{2}{25} + \cos^2 \theta &= 1 \\
\cos^2 \theta &= \frac{23}{25} \\
\cos \theta &= \frac{\sqrt{23}}{5} \\
\cot \theta &= -\frac{\sqrt{23}}{12} \quad \text{or} \quad \cot \theta = -\frac{\sqrt{46}}{2}
\end{align*}
\]

8. Draw an angle of 630° in the standard position. Then use a suitable \( x - y - r \) calculation to find all trig functions of 630°. Must show the \( x, y, r \) values, and the steps of your calculation, not just the final answer.

\[
\begin{align*}
\sin(630°) &= \frac{\sqrt{3}}{2} = -1 \\
\cos(630°) &= \frac{0}{1} = 0 \\
\tan(630°) &= \frac{-1}{0} = \text{undefined} \\
\cot(630°) &= \frac{0}{\sqrt{3}} = 0 \\
\sec(630°) &= \frac{1}{0} = \text{undefined} \\
\csc(630°) &= \frac{\sqrt{3}}{1} = -1.
\end{align*}
\]

9. Evaluate \( 4\cos^2 360° + 2\cot^3 270° - 3\cos^3 540° \). Show each line of your calculation carefully and methodically.

\[
\begin{align*}
&= 4(\cos 360°)^2 + 2(\cot 270°)^3 - 3(\cos 540°)^3 \\
&= 4(1)^2 + 2(0)^3 - 3(-1)^3 \\
&= 4 + 0 + 3 \\
&= 9
\end{align*}
\]

**Note:**

\( \cot(270°) \) is **not** undefined.

It is in fact equal to zero!!