1. $ABC$ is a right triangle with the right angle at $B$. Given that $A = 23^\circ$ and $c = 3.5\text{ cm}$, find $b$. (two decimal places).

2. $ABC$ is a right triangle with the right angle at $A$. Given that $a = 4\text{ cm}$, and $b = 3\text{ cm}$, find $c$ and $B$ (approximate answers – two decimal places).

3. In the triangle $ABC$, $A = 34^\circ$, $B = 40^\circ$, and $c = 12.3\text{ cm}$. Find $b$ (two decimal places).

4. Find the exact values of the first three trigonometric functions of an angle of $840^\circ$.

5. Solve the equation $6\sec^2\theta - 8 = 0$ where $-180^\circ \leq \theta \leq 630^\circ$.

6. Solve the equation $\cot\theta = \sqrt{2}$ where $0^\circ \leq \theta \leq 180^\circ$.

7. Two vertical buildings are situated across each other on the ground. From the bottom of the shorter building, the angle of elevation to the top of the taller building is $30^\circ$, and from the top of the shorter building the angle of depression to the bottom of the taller building is $20^\circ$. If the taller building is 40 feet taller than the shorter building, find the angle of elevation from the top of the shorter building to the top of the taller building (two decimal places).

8. From the top of a lighthouse 130 ft tall, the angle of depression of a small boat on the ocean surface is $43^\circ$. Find how far is the boat from the bottom of the lighthouse (two decimal places).

9. A balloonist $P$ is directly above a straight road 2 miles long that joins two villages $A$ and $B$. When the balloon $P$ is at a height of 0.5 miles, it was known that the angle $APB$ is a right angle. Find that angle of depression from $P$ to village $A$ (village farthest from $P$) at that instant (two decimal places). See the given figure.
5. \[ 6 \sec^2 \theta - 8 = 0 \]
\[ \therefore 6 \sec^2 \theta = 8 \]
\[ \sec^2 \theta = \frac{8}{6} = \frac{4}{3} \]
\[ \therefore \cos^2 \theta = \frac{4}{3} \]
\[ \therefore \cos \theta = \pm \frac{\sqrt{3}}{2} \]

Final Solutions: 9 Solutions.
\[ \theta = -150^\circ, 150^\circ, 210^\circ, 510^\circ, 570^\circ, -30^\circ, 30^\circ, 330^\circ, 390^\circ \]

6. \[ \cot \theta = \sqrt{2} \text{ where } 0^\circ \leq \theta \leq 180^\circ \]
\[ \cot \theta \text{ is negative in quadrant II,} \]
\[ \text{So only solutions between } 0^\circ \text{ and } 180^\circ \]
\[ \text{come from } 
\theta = \tan^{-1} \left( \frac{1}{\sqrt{2}} \right) \approx 35.26^\circ \]

7. \[
\text{Let } BD (= AE) = d.
\]
\[
\text{From } \triangle BCD: \quad \tan 30^\circ = \frac{CD}{d}
\]
\[
\therefore CD = d \tan 30^\circ \quad (1)
\]
\[
\text{From } \triangle ADE: \quad \tan 20^\circ = \frac{DE}{d}
\]
\[
\therefore DE = d \tan 20^\circ \quad (2)
\]

Use (1) and (2) now:
\[
CD - DE = 40
\]
\[
\therefore d \tan 30^\circ - d \tan 20^\circ = 40
\]
\[
d \left( \tan 30^\circ - \tan 20^\circ \right) = 40
\]
\[
\therefore d = \frac{40}{\tan 30^\circ - \tan 20^\circ} \approx 187.45
\]

Now:
\[
\tan \theta = \frac{40}{d} = \frac{40}{187.4589...}
\]
\[
\theta = \tan^{-1} \left( \frac{40}{187.4589...} \right) \approx 12.05^\circ
\]
\[
\tan 43^\circ = \frac{130}{d}
\]
\[
\therefore d \cdot \tan 43^\circ = 130
\]
\[
\therefore d = \frac{130}{\tan 43^\circ} \approx 139.41 \text{ ft}
\]

Let \( AC = x \). So \( BC = 2 - x \)

From \( \triangle APC \):
\[
\tan \theta = \frac{0.5}{x}
\]

From \( \triangle BPC \):
\[
\tan \theta = \frac{2-x}{0.5}
\]

By (1) and (2):
\[
\frac{0.5}{x} = \frac{2-x}{0.5}
\]
\[
(0.5)^2 = xc(2-x)
\]
\[
\frac{1}{4} = 2x - x^2 \quad \text{or} \quad 1 = 8x - 4x^2
\]
\[
4x^2 - 8x + 1 = 0
\]
\[
x = \frac{8 \pm \sqrt{64 - 16}}{8} = \frac{8 \pm 4\sqrt{3}}{8} = \frac{2 \pm \sqrt{3}}{2}
\]
\[
\therefore x \approx 0.1340 \quad \text{or} \quad x \approx 1.8660
\]
\[
\therefore BC = 2 - x \approx 1.8660 \quad \text{or} \quad BC = 2 - x \approx 0.1340
\]

Since \( AC > BC \), the correct choice is \( AC = x \approx 1.8660 \)

Now, by (1):
\[
\tan \theta = \frac{0.5}{x} = \frac{0.5}{1.8660}
\]
\[
\theta \approx \tan^{-1}(\frac{0.5}{1.8660}) \approx 15.00^\circ
\]