1. (a) Convert $340^\circ$ to radians (exact answer).  
   (b) Convert $3.5$ radians to degrees (approximate answer, 
       two decimal places)

2. The arc length of a circular sector is $12$ inches and its central angle is $210^\circ$. Find the exact value of the radius.

3. Find the exact value of $\cot\left(\frac{14\pi}{3}\text{ rad}\right)$. Show all your work.  
   [really easy!]

4. Find the amplitude, period, phase shift, vertical shift, and graph any cycle of $y = 3 \cos(5x)$. Find the exact $x$-coordinates of the subdivision points of any one cycle. Show all intermediate graphs. Use the class format.

5. Find the amplitude, period, phase shift, vertical shift, and graph any cycle of $y = -3 \sin\left(\frac{1}{3}x + \frac{\pi}{2}\right) + 1$. Find the exact $x$-coordinates of the subdivision points of any one cycle. Show all intermediate graphs. Use the class format.

6. Find the exact value of $\cos(-555^\circ)$. Simplify your answer, and rationalize the denominator.

7. Prove the identity: $\sec^2 \alpha + \cos^2 \alpha - 2 = \sin^4 \alpha \sec^2 \alpha$

8. Given $\cos(s) = \frac{2}{3}$ and $\sin(t) = -\frac{1}{6}$, $s$ and $t$ in quadrant IV, find the exact value of $\cos(s - t)$.

9. Prove the identity: $\cos(\alpha + \beta)\cos(\theta - \beta) = \sin(\alpha + \beta)\sin(\theta - \beta) + \cos(\alpha + \theta)$
1. (a) $340^\circ = \frac{340 \times \pi}{180^\circ} \text{ rad} = \frac{17\pi}{9} \text{ rad}$  
(b) $3.5 \text{ rad} = 3.5 \times 180^\circ \approx 200.54^\circ$

2. $\omega = 12 \text{ in} \ ; \ \theta = 210^\circ = \frac{210 \times \pi}{180^\circ} \text{ rad} = \frac{7\pi}{6} \text{ rad} \ ; \ \gamma = ?$

Use: $\omega = r \theta$

$12 = \frac{7\pi}{6} \gamma$

$\therefore \gamma = \frac{72}{7\pi}$ inches

3. \( \cot \left( \frac{14\pi}{3} \text{ rad} \right) \)

\[ \frac{14\pi}{3} \text{ rad} = \frac{14\pi}{3} \times \frac{180^\circ}{\pi} = 840^\circ \]

\[ \cot \left( \frac{14\pi}{3} \text{ rad} \right) = \cot \left( 60^\circ \right) = -\cot \left( 60^\circ \right) = -\frac{1}{\sqrt{3}} \]

4. $y = 3 \cos(5x)$

I. $A = 3$ ; $B = 5$ ; $C = 0$ ; $D = 0$

II. $amp = |A| = 3$ ; period $= \frac{2\pi}{B} = \frac{2\pi}{5}$ ; $P:S = C = 0$ ; $V:S = D = 0$.

III. Parent: $y = \cos x$

\[
\begin{array}{c}
\text{left-edge} = 0 \\
\text{Right-edge} = 0 + \frac{2\pi}{\frac{5}{2}} = \frac{2\pi}{5}
\end{array}
\]

IV. Build it: (a) $y = 3 \cos(5x)$

V. Graph it:

Subdiv. pts are: $0, \frac{\pi}{10}, \frac{\pi}{5}, \frac{3\pi}{10}, \text{ and } \frac{2\pi}{5}$
5. \[ y = -3 \sin \left( \frac{1}{3} x + \frac{\pi}{2} \right) + 1 = -3 \sin \left[ \frac{1}{3} \left( x + \frac{3\pi}{2} \right) \right] + 1 \]

I: \( A = -3; \ B = \frac{1}{3}; \ C = -3\pi/2; \ D = 1 \)

\[ \text{amp} = |A| = 3 \]
\[ \text{period} = \frac{2\pi}{B} = \frac{2\pi}{\frac{1}{3}} = 6\pi \]
\[ p.s = C = -\frac{3\pi}{2} \]
\[ v.s = D = 1 \]

III. parent: \( y = \sin x \)

IV. Build it: (a) \( y = 3 \sin \left( \frac{1}{3} x + \frac{\pi}{2} \right) \)
(b) \( y = -3 \sin \left( \frac{1}{3} x + \frac{\pi}{2} \right) \)
(c) \( y = -3 \sin \left( \frac{1}{3} x + \frac{\pi}{2} \right) + 1 \)

V. Graph it:

subdivision pts are: \(-\frac{3\pi}{2}, 0, \frac{3\pi}{2}, \frac{3\pi}{2}, \text{and} \frac{9\pi}{2}\)

6. \( \cos (-555^\circ) \)
= \( \cos 15^\circ \)
= \( -\cos (45^\circ - 30^\circ) \)
= \( -\left( \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ \right) \)
= \( -\left( \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} \right) \)
= \( -\left( \frac{\sqrt{6} + \sqrt{2}}{4} \right) \)
\[ \therefore \cos (-555^\circ) = -\frac{\sqrt{6} + \sqrt{2}}{4}. \]
Prove: \( \sec^2 \alpha + \csc^2 \alpha - 2 = \sin^4 \alpha \cdot \sec^2 \alpha \).

\[ \text{R.H.S.} = \sin^4 \alpha \cdot \sec^2 \alpha = (1 - \cos^2 \alpha)^2 \cdot \frac{1}{\cos^2 \alpha} = \frac{1 - 2 \cos^2 \alpha + \cos^4 \alpha}{\cos^2 \alpha} \]

\[ = \frac{1}{\cos^2 \alpha} - \frac{2 \cos^2 \alpha}{\cos^2 \alpha} + \frac{\cos^4 \alpha}{\cos^2 \alpha} \]

\[ = \sec^2 \alpha - 2 + \cos^2 \alpha = \text{L.H.S.} \]

\[ \therefore \text{L.H.S.} = \text{R.H.S.} \]

8) Given: \( \cos \alpha = \frac{2}{3} \); \( \sin \theta = -\frac{1}{3} \); \( \theta \) & \( \theta \) in IV

Find: \( \cos (\alpha - \theta) \)

\[ \cos (\alpha - \theta) = \cos \alpha \cdot \cos \theta + \sin \alpha \cdot \sin \theta \]

\[ = \left( \frac{2}{3} \right) \left( \frac{\sqrt{35}}{6} \right) + \left( -\frac{1}{3} \right) \left( -\frac{\sqrt{5}}{3} \right) \]

\[ = \frac{2 \sqrt{35} + \sqrt{5}}{18} \]

\[ \therefore \cos (\alpha - \theta) = \frac{2 \sqrt{35} + \sqrt{5}}{18} \]

Find \( \sin \alpha \):

\[ \sin^2 \alpha + \cos^2 \alpha = 1 \]

\[ \sin^2 \alpha + \left( \frac{2}{3} \right)^2 = 1 \]

\[ \sin^2 \alpha + \frac{4}{9} = 1 \]

\[ \sin^2 \alpha = \frac{5}{9} \]

\[ \sin \alpha = -\frac{\sqrt{5}}{3} \]

Find \( \csc^2 \theta \):

\[ \csc^2 \theta = \frac{35}{36} \]

\[ \csc \theta = \frac{\sqrt{35}}{6} \]

9) Prove: \( \cos (\alpha + \beta) \cdot \cos (\alpha - \beta) = \sin (\alpha + \beta) \cdot \sin (\alpha - \beta) + \cos (\alpha + \theta) \)

\[ \text{R.H.S.} = \sin (\alpha + \beta) \cdot \sin (\alpha - \beta) + \cos [(\alpha + \beta) + (\alpha - \beta)] \]

\[ = \sin (\alpha + \beta) \cdot \sin (\alpha - \beta) + \cos (\alpha + \beta) \cdot \cos (\alpha - \beta) - \sin (\alpha + \beta) \cdot \sin (\alpha - \beta) \]

\[ = \cos (\alpha + \beta) \cdot \cos (\alpha - \beta) \]

\[ = \text{L.H.S.} \]

\[ \therefore \text{L.H.S.} = \text{R.H.S.} \]

\[ \text{The solution is pretty short!} \]