1. \( \triangle ABC \) is a right triangle with the right angle at \( B \). Given that \( b = 4\sqrt{5} \text{ cm}, \ c = 2 \text{ cm}, \) find the exact value of \( a \) (simplify).

2. (a) Convert \( 326^\circ 34' 43'' \) into degrees and decimal degrees (two decimal places) \hspace{5cm} (b) Convert \( 127.241^\circ \) into degrees, minutes, and seconds.

3. Given that the point \( P(5, -12) \) is on the terminal side of some angle \( \theta \) in the standard position, find the exact values of the last three trigonometric functions of \( \theta \).

4. Given \( \cot \beta = \frac{5}{3} \) with \( \sec \beta < 0 \), find the exact values of the first three trigonometric functions of \( \beta \). Use any method to solve this problem. Show complete work.

5. Given \( \csc \theta = \frac{-\sqrt{7}}{2} \) with \( \theta \) in quadrant III find the exact value of \( \sec \theta \). Use only the trigonometric identities to solve this problem.
6. Find any exact solution for $\theta : \sin(2\theta + 5^\circ) \sec(23^\circ - 6\theta) - 1 = 0$. Show each line of your work correctly.

7. Draw an angle of $630^\circ$ in the standard position. Then use a suitable $x - y - r$ calculation to find all trig functions of $630^\circ$. Must show the $x$, $y$, $r$ values, and the steps of your calculation, not just the final answer.

8. Evaluate $2\sin^3(-90^\circ) - 4\cos^2180^\circ - 5\cot^2990^\circ$. Show each line of your calculation carefully and methodically.

9. The equation of the terminal side of some angle $\theta$ in the standard position is given by $2x + 3y = 0$, $x \leq 0$. Find the exact values of the first three trigonometric functions of $\theta$. Simplify your answers.
1. $ABC$ is a right triangle with the right angle at $B$. Given that $b = 4\sqrt{5}$ cm, $c = 2$ cm, find the exact value of $a$ (simplify).

$$b^2 = a^2 + c^2$$

$$b^2 = (4\sqrt{5})^2$$

$$b^2 = a^2 + 2^2$$

$$b^2 = a^2 + 4$$

$$\sqrt{b^2} = \sqrt{a^2 + 4}$$

$$a = \pm \sqrt{76}$$

$$a = \pm \sqrt{4 \times 19} = 2\sqrt{19}$$

2. (a) Convert $326^\circ 34' 43''$ into degrees and decimal degrees (two decimal places)

$$326 + \frac{34}{60} + \frac{43}{3600}$$

$$\approx 326.58^\circ$$

(b) Convert $127.241^\circ$ into degrees, minutes, and seconds.

$$127^\circ + (0.241 \times 60)'$$

$$= 127^\circ + 14' + (0.46 \times 60)''$$

$$\approx 127^\circ 14' 28''$$

3. Given that the point $P(5, -12)$ is on the terminal side of some angle $\theta$ in the standard position, find the exact values of the last three trigonometric functions of $\theta$.

$$x = 5; \quad y = -12; \quad r = ?$$

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{(5)^2 + (-12)^2}$$

$$r = \sqrt{25 + 144} = 13$$

4. Given $\cot \beta = \frac{5}{3}$ with $\sec \beta < 0$, find the exact values of the first three trigonometric functions of $\beta$. Use any method to solve this problem. Show complete work.

$$\cot \beta = \frac{5}{3}$$

$$\Rightarrow \sec \beta = \frac{1}{\cos \beta} = -\frac{3}{5}$$

$$\Rightarrow \sin \beta = \frac{y}{r} = -\frac{3}{\sqrt{34}}$$

5. Given $\csc \theta = \frac{-\sqrt{7}}{2}$ with $\theta$ in quadrant III find the exact value of $\sec \theta$. Use only the trigonometric identities to solve this problem.

$$\sin \theta = \frac{1}{\csc \theta} = -\frac{2}{\sqrt{7}}$$

$$\cos \theta = \pm \frac{\sqrt{7}}{7}$$

$$\cos \theta = -\frac{\sqrt{7}}{7}$$

$$\csc \theta = \frac{1}{\sin \theta} = \frac{-\sqrt{7}}{2}$$

$$\sec \theta = \frac{1}{\cos \theta} = -\sqrt{7}$$

$$\implies \csc \theta = \frac{3}{7}$$

$$\cos \theta = \pm \frac{3}{7}$$

$$\sec \theta = \frac{-\sqrt{7}}{2}$$
6. Find any exact solution for \( \theta : \sin(20^\circ + 5^\circ) \csc(23^\circ - 6\theta) - 1 = 0 \). Show each line of your work correctly.

\[
\begin{align*}
\sin(20^\circ + 5^\circ) \csc(23^\circ - 6\theta) & = 1 \\
\sin(20^\circ + 5^\circ) & = \frac{1}{\csc(23^\circ - 6\theta)} \\
\sin(20^\circ + 5^\circ) & = \sin(23^\circ - 6\theta) \\
\text{Set: } 2\theta + 5^\circ & = 23^\circ - 6\theta \\
8\theta & = 18^\circ \\
\therefore \theta & = \frac{18^\circ}{8} = \frac{9^\circ}{4}
\end{align*}
\]

7. Draw an angle of 630° in the standard position. Then use a suitable \( x - y - r \) calculation to find all trig functions of 630°. Must show the \( x, y, r \) values, and the steps of your calculation, not just the final answer.

\[
\begin{align*}
\cos 630^\circ & = \frac{x}{r} = \frac{0}{1} = 0 \\
\csc 630^\circ & = \frac{r}{y} = \frac{1}{-1} = -1 \\
\sec 630^\circ & = \frac{r}{x} = \frac{1}{0} \text{ undefined} \\
\tan 630^\circ & = \frac{y}{x} = \frac{-1}{0} \text{ undefined} \\
\cot 630^\circ & = \frac{x}{y} = \frac{0}{-1} = 0
\end{align*}
\]

8. Evaluate \( 2\sin^3(-90^\circ) - 4\cos^2180^\circ - 5\cot^2990^\circ \). Show each line of your calculation carefully and methodically.

\[
\begin{align*}
& = 2[\sin(-90^\circ)]^3 - 4[\cos 180^\circ]^2 - 5[\cot 990^\circ]^2 \\
& = 2(-1)^3 - 4(-1)^2 - 5(0)^2 \\
& = 2(-1) - 4(1) - 5(0) \\
& = -2 - 4 - 0 \\
& = -6
\end{align*}
\]

9. The equation of the terminal side of some angle \( \theta \) in the standard position is given by \( 2x + 3y = 0 \), \( x \leq 0 \). Find the exact values of the first three trigonometric functions of \( \theta \). Simplify your answers.

\[
\begin{align*}
2x + 3y & = 0 \\
3y & = -2x \\
y & = -\frac{2}{3}x \\
\text{We know } x & \leq 0 \\
\therefore \text{ Let } x & = -3 \\
y & = 2 \\
r & = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + (2)^2} = \sqrt{13} \\
\therefore r & = \sqrt{13}
\end{align*}
\]

\[
\begin{align*}
\sin \theta & = \frac{y}{r} = \frac{2}{\sqrt{13}} \\
\cos \theta & = \frac{x}{r} = \frac{-3}{\sqrt{13}} \\
\tan \theta & = \frac{y}{x} = \frac{2}{-3} = -\frac{2}{3}
\end{align*}
\]