1. $\triangle ABC$ is a right triangle with the right angle at $B$. Given that $A = 54^\circ$ and $c = 15.3 \text{ cm}$, find $b$. (two decimal places).

2. $\triangle ABC$ is a right triangle with the right angle at $A$. Given that $a = 8 \text{ cm}$ and $c = 5 \text{ cm}$, find the approximate value of $C$ (two decimal places), and the exact value of $\cot B$.

3. Find the exact values of the first three trigonometric functions of an angle of $-570^\circ$.

4. Find a solution for $\theta$ in the interval $[0^\circ, 90^\circ]$ such that $3\cot \theta - 1135 = 0$. Provide the approximate answer (four decimal places).

5. Find all solutions of the equation $6\tan^2 \theta - 2 = 0$ where $-270^\circ \leq \theta \leq 540^\circ$.

6. From the top of a lighthouse $260 \text{ ft}$ tall, the angle of depression of a small boat on the ocean surface is $31^\circ$. Find how far is the boat from the bottom of the lighthouse (two decimal places).

7. A hillside makes an angle of $16^\circ$ with the horizontal, and there is a $30 \text{ ft}$ tall vertical building standing on the top of the hill. From a point $45 \text{ ft}$ downhill from the base of the building, along the hill, find the angle of elevation to the top of the building (two decimal places).

8. Two vertical buildings are located on the ground, facing each other. The shorter building is $30$ feet tall. From the bottom of the shorter building, the angle of elevation to the top of the taller building is $50^\circ$. From the top of the shorter building the angle of elevation to the top of the taller building is $20^\circ$. Find height of the taller building (two decimal places).
1. \[ \cos 54^\circ = \frac{15.3}{b} \]
   \[\therefore b = \frac{15.3}{\cos 54^\circ} \approx 26.03 \text{ cm} \]

2. \[ \sin C = \frac{5}{8} \]
   \[\therefore C = \sin^{-1} \left( \frac{5}{8} \right) \]
   \[64 = b^2 + 25 \]
   \[b^2 = 39 \]
   \[\therefore b = \sqrt{39} \]

3. \[ \sin (-570^\circ) = \sin 30^\circ = \frac{1}{2} \]
   \[\cos (-570^\circ) = \cos 30^\circ = -\frac{\sqrt{3}}{2} \]
   \[\tan (-570^\circ) = \tan 30^\circ = -\frac{1}{\sqrt{3}} \]

4. \[3 \cot \theta = 113.5 \]
   \[\cot \theta = \frac{113.5}{3} \]
   \[\therefore \tan \theta = \frac{1}{\cot \theta} = \frac{3}{113.5} \]
   \[\therefore \theta \approx 0.1514^\circ \]

5. \[6 \tan^2 \theta - 2 = 0 \]
   \[6 \tan^2 \theta = 2 \]
   \[\tan^2 \theta = \frac{1}{3} \]
   \[\therefore \tan \theta = \pm \frac{1}{\sqrt{3}} \]

   \[\theta = -210^\circ, -30^\circ, 150^\circ, 330^\circ, 510^\circ \]

Final answer: \[\theta = -210^\circ, -30^\circ, 150^\circ, 330^\circ, 510^\circ, -150^\circ, 30^\circ, 210^\circ, 390^\circ \]

Final solutions!
\[\tan 31^\circ = \frac{260}{d}\]
\[d = \frac{260}{\tan 31^\circ}\]
\[d \approx 432.71 \text{ ft}\]

**Step 1: Find BC**
\[\sin 16^\circ = \frac{BC}{45}\]
\[BC = 45 \sin 16^\circ \approx 12.4036 B101\ldots\]

**Step 2: Find AB**
\[\cos 16^\circ = \frac{AB}{45}\]
\[AB = 45 \cos 16^\circ \approx 43.25677\ldots\]

**Step 3: Find \(\theta\) (= B \hat{A} D)**
\[\tan \theta = \frac{30 + 12.4036\ldots}{43.25677\ldots}\]
\[\theta = \tan^{-1} \left( \frac{42.4036\ldots}{43.25677\ldots} \right) \approx 44.43^\circ\]

**Step 1: Use \(\triangle ACD\)**
\[\tan 50^\circ = \frac{d + 30}{x}\]
\[x = \frac{d + 30}{\tan 50^\circ}\]

**Step 2: Use \(\triangle BDE\)**
\[\tan 20^\circ = \frac{d}{x}\]
\[x = \frac{d}{\tan 20^\circ}\]

Set (1) and (2) equal:
\[\frac{d + 30}{\tan 50^\circ} = \frac{d}{\tan 20^\circ}\]
\[(d + 30) \tan 20^\circ = d \tan 50^\circ\]
\[d \tan 20^\circ + 30 \tan 20^\circ = d \tan 50^\circ\]
\[30 \tan 20^\circ = d (\tan 50^\circ - \tan 20^\circ)\]
\[d = \frac{30 \tan 20^\circ}{\tan 50^\circ - \tan 20^\circ}\]
\[\approx 13.19077\ldots\]
\[\approx 43.19 \text{ ft}\]