# ABSTRACT

ALLAIN, RHETT. Investigating the Relationship Between Student Difficulties with the Concept of Electric Potential and the Concept of Rate of Change. (Under the direction of Robert J. Beichner)

An important aspect of curricula development is not only understanding what difficulties students have but also, why they have these difficulties. The purpose of this study was to investigate the possibility that students have difficulty with the concept of electric potential because they have difficulty with the concept of rate of change. The study sample consisted of over 300 students from various colleges and universities.

To investigate this relationship, I created a diagnostic instrument that contains 14 items on rate of change and 11 items on electric potential this is the *Rate And Potential Test* (RAPT). These items were taken from other diagnostic instruments or modified from questions used in other research studies. I also conducted think-aloud interviews to insure that students were interpreting the items correctly.

My findings can be broken into two areas. The first area deals with findings about the relationship between student difficulties with rate of change and electric potential. The second area of findings deals with the appropriateness of the RAPT for investigating this relationship. I found that there is a correlation between the way students answer rate of change items and the way they answer electric potential items. I also found that some students in upper-level undergraduate courses made mistakes similar to introductory students. The RAPT was found to be a reliable (KR20 = 0.83) instrument and it did not matter if it was given on paper or via *WebAssign*, a web-based homework delivery system.

**INVESTIGATING THE RELATIONSHIP BETWEEN STUDENT DIFFICULTIES WITH THE CONCEPT OF ELECTRIC POTENTIAL AND THE CONCEPT OF RATE OF CHANGE**

by

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A dissertation submitted to the Graduate Faculty of North Carolina State University in partial fulfillment of the requirements for the Degree of Doctor of Philosophy

**PHYSICS**

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# BIOGRAPHY

I was born in Mobile, AL in 1970 and spent the early years of my life in the southeastern United States before my family settled in Naperville, Illinois. I graduated with honors from Waubonsie Valley High School and received a Bachelors of Science in Physics from Benedictine University in May 1992. I began graduate studies at the University of Alabama in the fall of 1992.

While attending the University of Alabama, I assisted in the development and construction of an electron drift velocity monitor to be used in the Superconducting Super Collider. In 1994, I spent the summer at CERN located in Geneva, Switzerland where I analyzed data from the Microvertex Detector. In addition to research, I taught introductory physics and astronomy labs. It was the teaching of these labs and the interaction with the students that gave me my first taste for teaching. In December 1995, I received a Masters of Science in Physics.

Upon completion of the Master's program at the University of Alabama, I began my teaching career at Judson College located in Marion, Alabama. I was responsible for the teaching and planning all physics courses and labs offered on campus. I also served as adjunct professor of physics at Marion Military Institute.

I was married to Ashley Allain on August 5, 1995 at St. Francis of Assisi University Parish. In June 1997, we moved to Raleigh, North Carolina where I entered the doctoral program in Physics at North Carolina State University. At NCSU, I have assisted in the teaching of introductory physics classes and participated in physics education research surrounding the SCALE-UP project.

My wife and I are blessed with a beautiful daughter, Abigail Ruth, who was born April 18, 2000. We attend the Catholic Community of St. Francis of Assisi where we are involved with youth ministry and social outreach programs.

# ACKNOWLEDGEMENTS

The completion of this project could not have been possible without the support, patience and generosity of many people. I would like to begin by thanking God for giving me the wisdom and endurance to see me through this project. I hope that I can share my gifts with others.

I owe thanks to Eric Brewe, Eric Ayars, Richard Mowat, Fred Lado, Beth Reig, Kwong Chung, and Brian Pyper for their assistance in the data collection process. Most Physics Education Research projects do not depend on expensive equipment, but rather large numbers of students. I appreciate the generosity of these individual for sharing their students and even class time with me.

I would like to thank my advisor, Robert Beichner, for all of his hard work and support. I have benefited from his guidance and encouragement in my research. The other members of the Physics Education Research and Development group at North Carolina State University: David Abbott, Jeanne Morse, and Scott Bonham have also been a great help. They have encouraged me and helped me focus my research to what it is today. I would also like to thank Melissa Dancy and Duane Deardorff. I have used their disseratations as models for my own, especially in the area of formatting.

During my last year at North Carolina State, I was supported by a NFS Fellowship. With the Fellowship, I was able to concentrate on analysis of data and on writing my dissertation. The Fellowship also included funds that allowed me to travel to national meetings at which I made contacts with others to assist in data collection.

I would like to thank my wife, Ashley Allain. Her love, support, and patience have been some the greatest and most wonderful gifts. She has been very helpful in assisting me with the statistical analysis and advice on writing. I would like to thank my daughter, Abigail. Her laughter and new tricks have brought me happiness and motivation. I would also like to thank my family whose prayers and support have helped me attain this goal.

# Table of Contents

# List of Tables

# List of Figures

# Chapter 1

# Overview of Research

The goal of physics education research is to determine how to assist students in their understanding of physics. This includes determining what difficulties students have with the concepts, why they have these difficulties and how to address these difficulties. This study will focus on what difficulties students have with the concepts of electric potential and investigate possible reasons for these difficulties. The primary means of this investigation is through the creation and use of a diagnostic instrument developed to probe the relationship between the concepts of rate of change and electric potential. This instrument is the Rate and Potential Test (RAPT). In this chapter, I will first give an overview of the physics education research philosophy. I will also present some "products" of physics education research in the form of research-based curricula as well as an overview of some of the instruments designed to assess student understanding.

## 1.1 Overview of Physics Education Research

In the beginning of physics education, the general consensus was if the material was clearer and the lectures had better demonstrations, then the students would understand[1]. Therefore, the focus of early physics education efforts was toward developing new textbooks, films, and demonstrations. Many physicists have come to realize that no matter how clear a lecture is, most students cannot appropriately learn physics in a passive mode. This can be seen in a quote from a long time physics education researcher, Arnold Arons:

> *"I am firmly convinced that a major portion of our residual difficulties resides not so much in the quality or clarity of our delivery systems as in the widely prevalent illusion that students will master arts of thinking and reasoning as well as huge volumes of concepts, principles, models, and theories through passive inculcation (i.e., by reading or listening to sufficiently lucid exposition) and through the doing of conventional end-of-chapter homework problems."*

According to Arons, students need to be actively engaged to learn physics. This can be seen in many curricular materials developed for students at the elementary school level.[2] Arons asserts the successfulness of these curricula centers around the researchers developing the materials while they were closely engaged with the students that would be using this material. Arons compares this to the development of the secondary level curricula in which the developers had little or no contact with their intended audience[3]. As a result, students could not adequately ascertain the effectiveness of the programs. This was not really physics education research. In order to be considered research, the developers would have to examine the students and determine what affect the new materials had on learning.

To actively engage students in physics, there must be knowledge of the difficulties students encounter with the material. Once student difficulties are identified, appropriate activities can be developed that will better facilitate the learning of these concepts. The new activities can then be examined for their effect on student learning and re-evaluated. From this comes the famous PER circle of research[4] as shown in Figure 1.1.

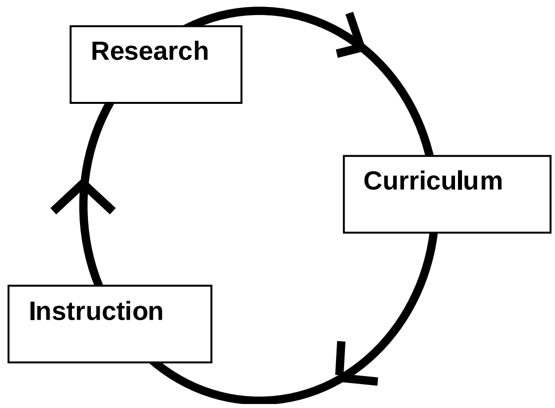



Figure 1.1 – The Physics Education Research circle

Since the PER research cycle is a circle, the research aspect does not have to be the beginning of improving instruction. The idea to develop and investigate a particular topic often comes from classroom observations or discussions of homework problems with students. These settings often provide subtle clues that reveal to the instructor incorrect thinking strategies that the student may possess. Nor does the cycle have to begin with informal observations; it could just as easily start with the systematic investigation of a particular topic. The research aspect of the cycle consists of obtaining evidence that supports preliminary ideas. The methods of this research include formal and informal student interviews, diagnostic instruments, and analysis of student responses on test questions or homework.

### 1.1.1 Research–Based Curricula

Once the student difficulties are identified, the next step is to develop or modify existing curricula to address these difficulties. There are several examples of research-based curricula that are currently available.

*Tutorials in Introductory Physics*[5] (TIP) is an activity-based workbook designed for use during the recitation portion of a lecture. TIP is not meant to replace lecture, but rather supplement it with conceptual based activities and problems. The materials do not focus on traditional physics problems and how to solve them, but rather on the conceptual understanding needed to solve such problems. It begins by introducing students to simple cases of particular phenomena and then leads to cases of those phenomena that students usually predict incorrectly. The methods of *Tutorials in Introductory Physics* lead students to cases that conflict with common beliefs. In order for TIP to be successful, the developers must know what difficulties students have with the material so that conflicting cases can be constructed. Even after the curriculum has been implemented, research is conducted to determine how well it addresses the known student difficulties as well as uncover new student difficulties. This information is used to revise TIP.

*Workshop Physics*[6] (WP) is another curriculum where students discover phenomena through investigative methods. WP is a very hands-on approach to learning physics. Most units start with a real world application such as breaking a board with karate. Students formulate a hypothesis and test this hypothesis with semi-guided activities. These activities are heavily dependent on the use of computers for data collection, data analysis, simulations, and mathematical modeling. One of the major advantages of WP is that students learn using the scientific method with tools similar to those used in real world research. The disadvantage of WP is that it requires a smaller class size than is common in many schools as well as a large equipment requirement. The development of WP also requires knowledge of student difficulties. The activities are then based on these student difficulties.

*Interactive Lecture Demonstrations*[7] (ILD) are activities that are developed to accompany lecture demonstrations. ILD's are used as activities to be inserted in traditional lecture courses. This does not exclude them from being used in active-engagement courses. Each ILD follows the "predict, observe, explain" protocol. The ILD method can best be seen with an example on Newton's third law. The instructor first shows a collision between two frictionless carts of unequal mass on a track. The students then make predictions as to the comparisons of the forces between the carts and discuss their predictions with their peers. This is followed by class discussion summarizing the general opinions of the class as well as the explanations for these predictions. To check student predictions, the instructor performs the same demonstration of the collision between two carts, except this time there is a force probe connected to each cart. The data collected from each force probe is displayed to the class so that they can compare their predictions with the results. Demonstrations are usually chosen such that most students will initially predict incorrectly. The final part of the ILD has the students attempt to explain why their predictions were incorrect. In order to design appropriate ILDs, the developer must also have an understanding of not only were students have difficulties, but also how they will likely predict the phenomena.

**1.1.2 Diagnostic Tests**

The research community has also developed several diagnostic instruments that are used to identify student difficulties and misconceptions. These instruments prove to be a valuable asset to the PER community as they can be used extensively in the research aspect of curriculum development. Diagnostics also serve as a means to bring an awareness of student difficulties to the general physics community, as was done with the Force Concept Inventory[8] (FCI). The FCI consists of seemingly straightforward questions about force and motion. Most instructors would say the test is too simple to give to their class and that most students would answer all of the questions correctly. However, after administration of the FCI, the instructor discovers that most students do not score well. This is usually a shock to the instructor, but provides a more accurate description of the conceptual level of the students. Many instructors assume that if students can solve typical homework type problems, then they must also have a developed conceptual understanding of the material also. This is not the case, as many students merely memorize various solutions to problems and attempt to re-apply those solutions to new problems. Eric Mazur[9] illustrates an example of students being capable of solving typical homework problems but lacking conceptual understanding through a simple investigation. Mazur gave a pair of questions on a midterm exam; one question was a typical numerical problem that is commonly found in exams, the other was a conceptual problem on the same topic. Both problems were worth 10 points. The average score on the numerical problem was 6.9 while the average score on the conceptual problem was only 4.9. There were also a number of students that scored a 10 on the numerical problem while scoring a 0 on the conceptual problem.

Diagnostic instruments can complement the data obtained from student interviews. Through an interview, the researcher or instructor is better able to obtain a reasonable “picture” of an individual student’s difficulties with a given topic. But even though the researcher is able to probe more deeply into the individual student’s cognitive strategies, the interview does not reveal how common these particular difficulties are. One solution to this problem is to use a diagnostic instrument in conjunction with the student interviews. The diagnostic instrument has the advantage of being able to examine many more students during a given time interval at the cost of loss of individual details. It also can serve to identify common student difficulties or quantify the prevalence of known difficulties. This knowledge is useful in the development of appropriate activities created to address these difficulties.

Most of the diagnostic instruments available today focus on mechanics topics covered in the first semester of a typical introductory course. The most widely used mechanics diagnostics are the FCI, the *Force and Motion Concept Inventory*[10] (FMCE), and the *Test of Understanding Graphs-*

*Kinematics*[11] (TUG-K). The FCI probes students' conceptions of force and motion, as does the FMCE. The TUG-K examines student difficulties with graphs as they are used in kinematics.

One of the reasons the FCI is so popular is that it can be used for several different purposes. As stated earlier, the FCI can be used to enlighten instructors to the nature of their students' conceptual understanding as was done in Mazur's class and many others like his. The FCI can essentially serve as a "wake up call" to many instructors who feel that if students can do traditional problems, they have conceptual understanding also.

Secondly, the FCI may serve as an evaluation tool to measure the effectiveness of a particular curriculum. This is possible because the first semester of the introductory physics course focuses mainly on forces and their effect on motion. While the topics covered on the FCI and the topics covered in a particular semester of a physics course are never exactly the same, they are close enough to obtain a measure of the effectiveness of the material. An important measure when studying the effectiveness of a curriculum is that of the gain, or Hake factor <*h*>, named after Richard Hake of Indiana University. The Hake factor is defined as the fraction of the possible gain. Mathematically, that is;

<*h*> = (post average % - pre average %)/(100 – pre average%)

Hake[12] conducted a study with the FCI that investigated the gains on the FCI for traditional and active learning based courses. His data came from more than 6,000 pre and post instruction results for the FCI from 62 introductory physics courses. By determining the FCI gains for the various courses, he found that courses that actively engage the students in their learning produce significantly higher gains. The actively engaged courses were those that used PER-based curricula such as those described previously.

A third use of the FCI is to use it to develop new curricula. This, for the most part, has been accomplished already in the curricula previously discussed. Although curricula currently exists derived in part from student responses on the FCI, it is not complete. The FCI can be used to evaluate the effectiveness of the new curricula and determine if students are still having difficulties with a given topic. For instance, if there are a high percentage of students that are incorrectly answering questions on the FCI related to force and motion, then the curricula could be modified to address this issue more closely. Relating to the research cycle mentioned above, this would be part of the "research" phase that takes place after instruction to determine the success of the curriculum. As noted earlier, for a thorough job, more than the FCI would be needed. Interviews would be a necessary accompaniment to the FCI data. There also exists another diagnostic that complements the FCI, it is the Force and Motion Concept Evaluation

(FMCE). The FMCE is not as popular as the FCI, but gives a more detailed view of student difficulties with the concept of force.

### 1.1.3 Other Diagnostic Instruments

Other diagnostic instruments exist besides the FCI. There are also instruments for use in the second semester of the introductory physics course such as Determining and Interpreting Resistive Electric Circuits Concepts Test (DIRECT), the Conceptual Survey of Electricity and Magnetism (CSEM), as well as various smaller diagnostics on waves and optics. Even though there are second semester diagnostics, there is not an electromagnetic equivalent of the FCI. It may be impossible to develop such an instrument for several reasons. One reason is that when students take the FCI, most students can answer all the questions and they can all likely state a reason or explanation for the choice they picked. This is because all people naturally obtain a "sense" of how the world around us works. Brown and Clement[13] found that even though students answered incorrectly on a Newton's third law diagnostic, they indicated that they were confident of their answers. Of course, this does not mean that everybody's "sense" is correct, but it does exist nevertheless. If you ask any person off of the street which will fall with a greater acceleration, a bowling ball or a marble, most people will be able to give an answer and a reason for their answer.

This is not always true with the concepts in electricity and magnetism. If you ask a typical person where the excess charge is located on a spherical conductor, most people will not even have an answer. If this same question is asked of college students taking physics, you may receive the correct answer, but the reason is likely to be that is what the text says or that is how it was described in the lecture. People do not seem to have a natural "sense" for electrical and magnetic concepts. This is most likely because we do not deal directly with electrical effects in the same way that we deal with forces. The lack of direct connection with observable phenomena has both positive and negative aspects. The positive aspect is that students may not be influenced by incorrect observations as they often are in mechanics. As an example, students often believe that a net force is required to keep an object moving at a constant speed, which is "motion implies force". This belief is supported by students' everyday experiences of moving objects. If you want to move a crate at a constant speed, you must push it, if you want to drive your car at a constant speed you must keep your foot on the gas pedal. The down side to the lack of direct connections to observable phenomena is that they cannot be used as building blocks to student understanding. An example of this can be seen in Clement's bridging techniques[14]. Clement used phenomena students did understand as analogies to phenomena they did not understand in the area of Newtonian mechanics. One key element to Clement's approach is to use students' common-sense ideas. In electricity and magnetism, students rarely enter the course with common-sense ideas on the subject, thus the bridging technique would have no basis.

The study of electric fields and electric potentials is the first step beyond comparing everyday observations with theory. That is, the study of fields and potentials requires a higher-level of abstract thinking. Students can no longer simply interact with objects that display the phenomena such dropping a ball in the study of kinematics. For students to understand electric fields and potentials, they must first understand the concept of the vector field. To summarize, students have more direct access to the concepts contained in mechanics, whereas they must access the concepts in electricity and magnetism through a layer of math. This is not to say that there are no directly observable phenomena in EM, but rather to fully understand these phenomena the student must use higher-level mathematics.

A large portion of mechanics is the study of force and how it relates to motion. It could also be said that a large portion of electricity and magnetism is the study of electric and magnetic fields and how they affect matter. The significant difference is that the concept of field is quite a difficult one that students in the introductory class do not usually study completely. Thus, there is an absence of a unifying concept in electricity and magnetism that could be used in a diagnostic instrument to evaluate the entire semester. Instead the diagnostic instruments focus on pieces of the semester, such as DC circuits, optics, or wave motion.

#### 1.1.4 Areas of Focus for Physics Education Research

It is beneficial to examine the research the physics education community has done thus far. The *American Journal of Physics* produces resource letters for various topics. By examining the resource letter on physics education research[15], a sense of the accomplishments can be obtained. The resource letter contains an index of published empirical studies on student difficulties with various phenomena. These indexed articles had to meet several criteria. The focus of the study had to be on the phenomena being studied and the students' understanding of that phenomena. The research was to be conducted in a scientific manner, that is, it must be systematic and reproducible.

A count of the number of studies cited in the Resource Letter for each area of study in physics can reveal where the "gaps" are in the understanding of student difficulties. Figure 2 is a representation of such a count. It can immediately be seen that the most studied area is that of student understanding in mechanics. This is an obvious place to start for researchers because mechanics is usually the first topic covered in the introductory physics course at the college level. Also from figure 2, it can be seen that there are comparatively few studies on student difficulties in electricity and magnetism. The majority of the traditional second semester college physics course is the study of electricity and magnetism, so this is a surprisingly small number of studies. It is also in this area that students find much difficulty[16]. The concepts involved in electricity and magnetism are highly abstract. A student cannot physically hold or even directly see a field. In order for a student to have any interaction with an electric or magnetic field, it must be observed through the fields interactions with other objects.

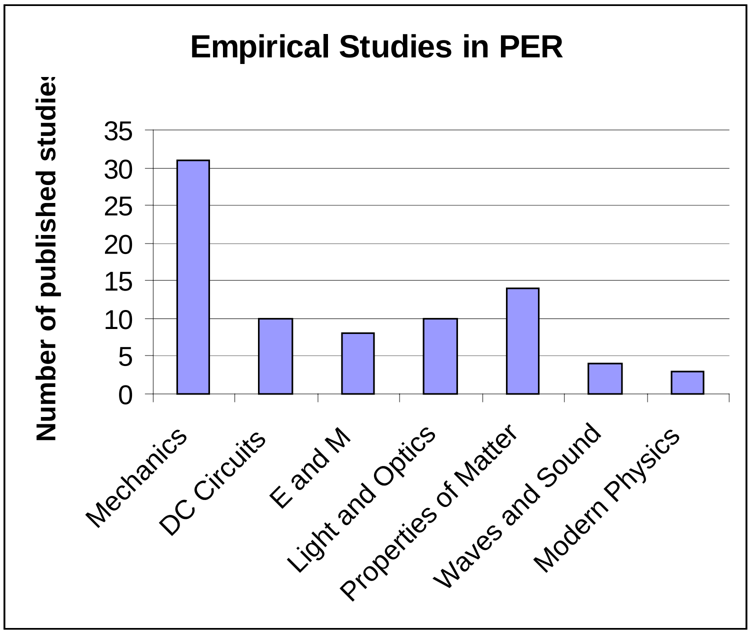



Figure 1.2 – A Count of Empirical Studies in PER

## 1.2 Purpose of the Study

This study can be thought of as the first step towards addressing student difficulties with electric potential. It is important not only to know what these difficulties are, but it would be useful to identify a possible source for these difficulties. The step that should be taken after this study is to develop curricula that addresses these difficulties. In this study, I will investigate the possibility that students have difficulty with electric potential because they lack understanding of the concept of rate of change. If this is indeed the case, it will offer one possible path for curriculum development.

I chose to investigate student difficulties with electric potential for several reasons. Imagine attempting to understand the photo-electric effect without first having an understanding of electric potential. Or attempting to explain the movement of electrons in a circuit without knowing how electric potential relates to electric field and electric force. The concept of the potentials is a fundamental one in physics. Although most texts treat the field as more fundamental than potential, this is not the case in quantum mechanics. The concept of electric potential is an essential building block for many later concepts such as electromagnetic induction and DC circuits. Although electric potential is such an important concept, student difficulties with electric potential have been largely neglected. There has been some study on the difficulties students have with the concepts of energy as it relates to mechanics[17], but this must be extended into the area of electricity.

## 1.3 Research Questions

In this project, I have chosen to focus on the following research questions.

1. Is the Rate and Potential Test (RAPT) an appropriate instrument to investigate the relationship between student models in rate of change and electric potential?
2. Is there a relationship between the way students answer rate of change questions and the way they answer electric potential questions?
3. Do students in active-engagement courses perform differently on the RAPT than students in traditional courses?
4. Do upper-level undergraduate students use similar models as the introductory physics students?

## Summary

In this chapter, I have reviewed the goals and philosophy of physics education research. I have explained the need to investigate student difficulties with electric potential. The specific research questions are also stated.

[1] A. Arons (1993). Uses of the Past: Reflections on United States Physics Curriculum Development. Interchange, 24, 105-128.

[2] Some examples of curricular materials for elementary class are Elementary Science Study (ESS), Science Curriculum Improvement Study (SCIS), Science a Process Approach (SAPA). A. Arons (1993). Uses of the Past: Reflections on United States Physics Curriculum Development. Interchange, 24, 105-128.

[3] see reference 1

[4] The Physics Education Group at University of Washington, http://www.phys.washington.edu/groups/peg

[5] L. C. McDermott, P.S. Shaffer, and the Physics Education Group at the University of Washington, Seattle (1998) Tutorials in Introductory Physics. Upper Saddle River, NJ: Prentice Hall.

[6] P.W. Laws (1997) Workshop Physics. New York, NY: John Wiley and Sons.

[7]Sokoloff, D.R. & Thornton, D.K. (1997) Using Interactive Lecture Demonstrations to create an active-learning environment. The Physics Teacher, 35 340-347.

[8] Hestenes, D., Wells, M., & Swackhamer, G. (1992) Force Concept Inventory. The Physics Teacher, 30, 141-158.

[9] Full details of the investigation can be found in the book: Mazur, E. (1997). Peer instruction: A user's manual. Upper Saddle River, NJ: Prentice Hall.

[10] Thorton, R.K., & Sokoloff, D.R. (1998). Assessing student learning of Newton's laws: The Force and Motion Conceptual Evaluation and the evaluation of active learning laboratory and lecture curricula. American Journal of Physics, 66, 338-352.

[11] Beichner, R.J. (1994) Testing student interpretation of kinematics graphs. American Journal of Physics, 62, 750-762.

[12] Hake, R.R. (1998). Interactive-engagement versus traditional methods: A six-thousand-student survey of mechanics test data for introductory physics courses. American Journal of Physics, 66, 64-74.

[13] Brown, D., & Clement, J. (1987). Misconceptions concerning Newton's law of action and reaction. Proceedings of the second international seminar on misconceptions and educational strategies in science and mathematics. Ithaca, NY: Cornell University.

[14] Clement, J. (1993). Using bridging analogies and anchoring intuitions to deal with students' preconceptions in physics. Journal of Research in Science Teaching, 30, 1241-1257.

[15] McDermott, L.C., & Redish, E.F. (1999). RL-PER1: Resource Letter on Physics Education Research. American Journal of Physics, 67, 755-767.

[16] Arons, A.B. (1997). Teaching Introductory Physics. New York, NY: John Wiley and Sons

[17] Lawson, R.A., & McDermott, L.C. (1987) Student understanding of the work-energy and impulse-momentum theorems. American Journal of Physics, 55, 811-817.; O'Brien Pride, T., Vokos, S., & McDermott, L.C. (1998). The challenge of matching learning assessments to teaching goals: An example from the work-energy and impulse-momentum theorems. American Journal of Physics, 66, 147-157.

# Chapter 2

# Background

## 2.1 Previous Studies on Student Understanding of Electric Potential

While there has not been a comprehensive study of student concepts in the area of electric potential, there have been several studies that address the issues related to electric potential. There has not been a formal research study on student difficulties with electric potential.

### 2.1.1 Electric Field Lines

Törnkvist, Pettersson, and Tranströmer[1] from the Royal Institute of Technology in Stockholm, Sweden looked at how students understand electric field lines. The study began by administering a test[2] that consisted of a drawing (figure 2.1) of electric field lines between three objects. The field lines the students saw had some errors that the students were asked to identify. The researchers focused on three errors: a set of lines that formed a complete circle, a field line that had a kink in it, and two field lines crossing each other. This question was given to 566 second year, non-major students as part of their final exam in an electricity and magnetism course. The researchers categorized the student responses by the type of explanation they offered (correct explanation, naïve explanation, or stating basic principles).

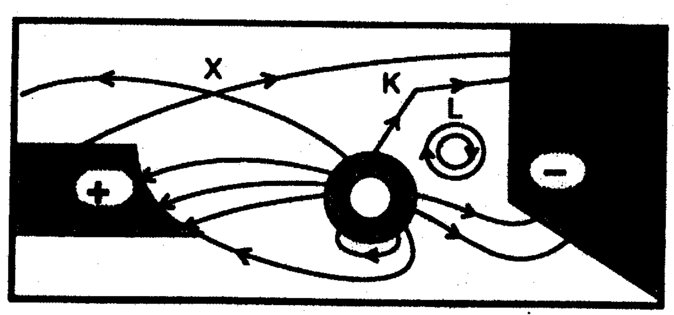



Figure 2.1 – Field Lines containing errors. The version the students saw did not contain the letters.

After a simple analysis of student response, 87 students were interviewed. The interviews took place two years after the written exam. The students were randomly chosen from the current EM course, thus these students had never taken the original exam. The interview consisted of nine questions that the students were to discuss. The questions all focused on some aspect of electric field lines, such as determining the direction of force on a particle, drawing lines that would account for a given force, drawing trajectories for a given particle in a given field, and modifying a field line pattern to indicate an increase in quantity or amount of source charges.

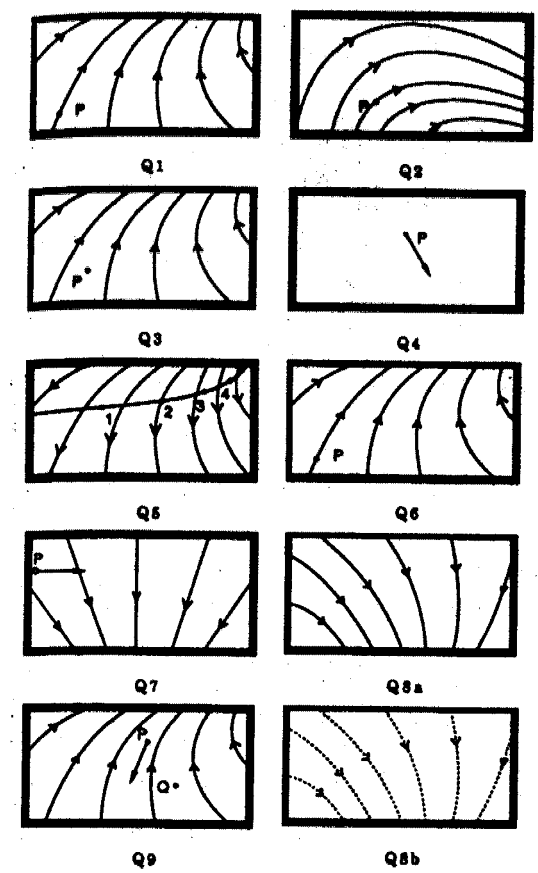



Figure 2.2 – Illustrations for Questions 1 through 9

- *Q1 – Q3: Draw a force vector on the given charge in the point in the given field.*
- *Q4: Draw field lines that can account for the given force or in the given point.*
- *Q5: Draw force vectors on the given charge for the given positions on the given positions on the given trajectory 1-2-3-4 in the given field.*
- *Q6: Draw a likely trajectory for a particle with zero initial velocity in the given point in the given field.*
- *Q7: Draw likely trajectories for two particles having the same mass but opposite charge and with the initial velocity indicated in the given point in the given field.*
- *Q8: Modify the given field line pattern when the source charges are increased by a factor 1.5 (Use the drawing Q8b, where the original field-lines are given as dotted curves.)*
- *Q9: Draw a force vector on the charge in the point Q when the force vector on the same charge in another point P is given.*

From their investigation, the researchers reported two major findings. First, students have problems applying the mathematical concepts of uniqueness, continuity, proportionality and isomorphic mapping.

- **Uniqueness:** The field can have only one value at each location in space.
- **Continuity:** The field must be continuous, that is spatial derivatives of the components of the field must exist at all points.
- **Proportionality:** The magnitudes of the field lines (spacing between lines) must be proportional to the field itself.
- **Isomorphic mapping:** Every point in the field line plot corresponds to a value of the field.

Second, students do not treat field lines as purely mathematical constructs, but give them more physical significance. This may not be too surprising as Faraday himself first thought of magnetic field lines as having a material existence[3]. Faraday called these lines of magnetic force, and attributed them with an elastic property. When two opposite magnetic poles were placed near each other, Faraday said that the elastic lines of force were stretched to produce an attraction. When two similar poles were placed near each other, the lines of force were compressed to produce a repulsive force. Students rarely attribute this elastic property to field lines, rather they often treat them as trajectories of particles.

The study at the Royal Institute of Technology in Stockholm does not directly investigate student concepts of electric potential, but it does show that students have difficulty with electric concepts because they are intangible. This difficulty could be true for electric potential as well. This confusion between field lines and lines of trajectories may exist with experts as well. In an informal study at North Carolina State University, a dipole was shown with a positive charge on the line bisecting the dipole. Experts were asked what the motion of the positive charge would be if released from rest. A common answer from these experts was that it would follow a path similar to an electric field line. The confusion may stem from the relation between the acceleration and the velocity because the force the particle feels is directly related to its acceleration. So the field lines are proportional to the “acceleration” lines. This may be an advanced form of the confusion[4] between velocity and acceleration.

An important limitation of the study from the Royal Institute of Technology is that the researchers focused solely on electric field lines as a representation of the electric field. The problems the researchers found may be related only to students’ reading of field lines and may not be also associated with the students’ understanding of electric field. These problems could be a form of confusion by representation rather than difficulties with the concept itself. It should be noted that although the representation of field lines is included in many introductory physics texts, it is not a necessary representation. An alternative method of representation is that of the plot of the electric field as vectors with the length of the vector representing the magnitude of the field, or the color of the vector representing the magnitude. Another limitation of this study is that students from the United States were not studied, so it is not guaranteed that the findings are applicable to US students.

### 2.1.2 Student Concepts of the Electric Field

A research study by Rainson, Tranströmer, and Viennot[5] also looked at student concepts of the electric field. The study focused on students in France and Sweden taking courses in electricity and magnetism at various levels. The lowest level of students studied understood the electric field as being defined as $F=qE$ and $E=V/d$ while the most advanced students had been introduced to Maxwell's equations and dielectrics. The researchers classified the students into one of four categories depending on their level of understanding of the electric field. Initially, 10 interviews were conducted on four questions regarding various aspects of the electric field. After the interviews were conducted, these four questions were given in written form to 1145 students in France and Sweden.

*In the situation represented in the drawing below, the two equipotential surfaces are infinite planes, perpendicular to the plane of the figure. Describe the electric field between the two surfaces.*

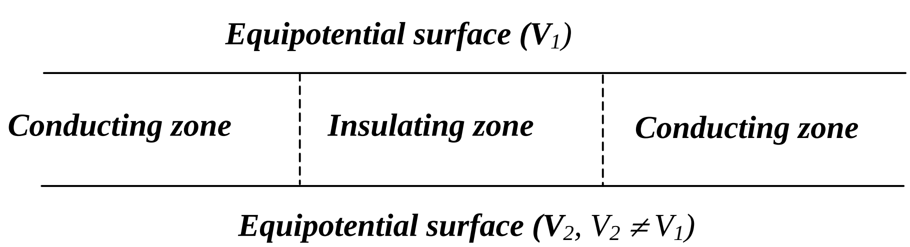



Figure 2.3 – "Neapolitan ice cream" question

Of the four questions, one was related to electric potential. This question, referred to as the "Neapolitan ice cream question", is a comparison of electric fields in various regions between two plates held at a fixed potential difference (see fig 2.3).

The goal of the "Neapolitan ice cream" question was to probe students' ideas about the electric field in conducting and insulating materials. The researchers had noted that often students would say there can be an electric field in a conductor, but not in an insulator. They describe this reasoning as *field if mobility* which means that if charges are able to move, then there can be an electric field. The idea of *field if mobility* can be labeled as a facet[6] as described by Jim Minstrell. Other incorrect responses for the above question include saying that $E$ must be zero inside a conductor even though there is a change in potential from one side to the other. All but the lowest level of students ($S_1$) had covered the relationship between E and current density ( $\boldsymbol{j} = \gamma\boldsymbol{E}$).

Another question asked students to compare both the magnitude and direction of the electric field inside the wires of a simple circuit. The goal of this question was to examine students' ideas about the surface charges needed to create the electric field inside the wires. The question also asked what causes the electric field inside the wires. Some common responses were; "the battery", "voltage of the battery", "E = -grad V", or "the current caused the electric field". This suggested to the researchers that students interpret formulas as a sign of causation, that is the quantities on the right cause the quantities on the left. The researchers also concluded that for most students, electrostatics and circuits are unconnected. It should also be noted that surface charges as the cause of the electric field is rarely covered in introductory courses[7].

### 2.1.3 Electric Potential and Ohm's Law

There is also a study that investigates how students treat electric potential when it is encountered in circuits. Cohen, Eylon and Ganiel[8] created a diagnostic questionnaire that covered basic concepts in circuits. This diagnostic was developed from interviews the researchers conducted with students on simple circuits. The questionnaire was administered to 145 high school students and 21 teachers from Israel. The level of the class was roughly that of a first-year college course in the US. To further investigate student responses, 14 students were interviewed after taking the questionnaire.

One concept investigated was students' understanding of potential difference. A multiple-choice question that addressed this was one in which the student is asked what causes the potential difference across a resistor with current flowing through it. Most students (70%) answered this correctly as the change in energy of electrons moving at the two ends of the resistor. Students also demonstrated that they could manipulate $V=IR$ in a question that asks what $V$ will be when $I = 0$. However, when given the case of $I = 0$ and asking what $V$ must be, most students (80%) said that $V$ must also be 0. In this case, it may seem that students are taking $V=IR$ as the definition of electric potential difference.

The researchers also found that close to a third of the students answered questions in a manner that suggest that a battery is a source of constant current rather than constant potential (for an ideal battery). Through interviews, the researchers also found that students commonly treat current as the fundamental concept that causes potential difference. This is similar to the findings by Rainson, Tranströmer, and Viennot in which they found that students treat formulas as a sign of causation where *V* is the cause of *IR*. One reason for this mode of thinking is that students are introduced to current at a much earlier age than when they are introduced to electric potential. Students may tend to treat current as a more concrete concept than electric potential because current is the flow of electrons, something the students can easily visualize.

## 2.2 Student Difficulties with the Concept of Rate of Change

The concept of rate of change is important in the understanding of the relationship between the electric field and the electric potential. The electric field is the gradient of the electric potential ( $\vec{E} = -\nabla V$ ), so that the *x*-component of the electric field would be $E_x = -\frac{dV}{dx}$

The electric field in the *x* direction is dependent on the spatial rate of change of the electric potential in the *x* direction. Due to the relationship between electric field and electric potential, it is possible that student difficulties relating the two could be based on student difficulties with the concept of rate of change. The following in an overview of studies of student difficulties with the concept of rate of change.

### 2.2.1 Conceptions of Rate of Change in High School Students

Hauger[9] investigated students' knowledge and conceptions about rate of change in high school and college students. He interviewed 12 high school students, 15 college students in the second semester of calculus, and 10 college math majors. The study focused on three different categories of rate of change. Hauger investigate three types of rates of change; global, interval, and instantaneous.

- **Global rate of change** deals with the overall shape of a graph of a quantity. Global descriptions are qualitative and deal with the general trend of the quantity such as increasing or decreasing over a given interval.
- **Interval rate of change** is a quantitative value for the change of a quantity over a given interval. Interval rate of change corresponds to the average change over this interval.
- **Instantaneous rate of change** is a quantitative value for the rate of change of a quantity at a particular point. Instantaneous rate of change is the derivative of that quantity with respect to time or distance.

The focus of Hauger's interviews were on the resources that students have available to them when dealing with instantaneous rate of change. Specifically, he wanted to investigate what features of interval rate of change students used to determine the instantaneous rate of change of a function. In the interviews, the researcher asked the students to make a graph and a table showing the distance between two people as they walked towards each other. For the first case, the people were walking towards each other while increasing their speed. The second case had the people approaching at a constant speed. The third had the people approaching while slowing down. During the interview, students were also presented with a graph and a table for the height of a ball that was thrown up into the air. The students were asked to describe the height of the ball over time as well as identify intervals where the ball is moving faster and slower. In another task, Hauger presented the students with a graph and table of values and asked them questions to investigate their knowledge of rate of change. A final task presented the students with a graph and table of values for the relationship between the radius and area of circles

Hauger also conducted a detailed investigation of four precalculus students and how they performed on a tasks involving rate of change. The students were given the following tasks.

**Task 1**

*Two people start at opposite corners of a room and walk toward each other. As they walk, they both slow down as they get closer to each other, pass, and then they both speed up as they get further apart. This takes a total of eight seconds. The opposite corners of the room are 20 feet apart.*

- *a. On the graph paper supplied, draw a graph showing the distance between the two people at each moment in time. Describe your graph*
- *b. How does your graph show the two people slowing down? Speeding up?*
- *c. Now complete the table of values for your graph.*
- *d. How does your table of values show the two people slowing down? Speeding up?*

**Task 2**

*These same two people decide again to start at opposite corners of the room and walk toward each other but this time they both decide to maintain the same steady pace the whole way. Again it takes a total of eight seconds for each to walk the 20 feet.*

- *a. On the same graph paper you used for Task 1, draw a graph showing the distance between the two people at each moment in time. Describe your graph.*
- *b. How does your graph show the steady pace of the two people?*
- *c. Now complete the table of values for your graph.*
- *d. How does your table of values show the steady pace of the two people?*

**Task 3**

*These same two people decide once more to start at opposite corners and walk toward each other. But this time as they walk, they both speed up as they get closer to each other, pass, and then they both slow down as they get farther apart. Once again it takes eight seconds for each to walk the 20 feet.*

a. *On the same graph paper you used for Tasks 1 and 2, draw a graph showing the distance between the two people at each moment in time. Describe your graph.*
b. *How does your graph show the two people speeding up? Slowing down?*
c. *Now complete the table of values for your graph.*
d. *How does your table of values show the two people speeding up? Slowing down?*

The focus of the research study was on the resources students use to determine the instantaneous rate of change. Hauger determined that students tend to use their knowledge of interval rate of change to think about instantaneous rate of change. That is they answered instantaneous rate of change questions by taking a small interval and using interval rate of change reasoning. Hauger also found that students continue to construct knowledge of rate of change after taking a course in introductory calculus.

### 2.2.2 Graphical representations of rate of change

One area that is often studied that involves rate of change is that of students' ability to make and analyze graphs. Nemirovsky and Rubin[10] looked at the difficulties high school students have in explaining the relationship between a function and its derivative. The researchers interviewed high school students who had not yet taken calculus and presented an analysis based on the interviews of six of these students. Two seventy five minute interviews were conducted for each student. The interviews were based on 15 problems the students were to construct functions and their derivatives in one of three different contexts. The researchers used the three different contexts of motion, fluids, and number-change in an attempt to study rate of change in graphs and see if context had a large influence. For each context, the students created graphs using computers. For the context of motion, a small car and a motion detector were used to produce graphs that would be examined. Fluid was studied with a bag that air could be put into or removed at different rates with an air flow monitor connected to a computer. For each context, parallel problems were presented. Comparing a velocity graph to a position graph in the motion context was parallel to comparing a flow rate vs. volume graph as well as B vs. x corresponding to A vs. x for number-change.

The primary finding that the researchers came to was that students have a tendency to assume a resemblance between the appearance or behavior of a function and its derivative in all three contexts. They suggest that there are two possible sources for this type of mistake. It could be a representational mistake, that is a problem with reading graphs. The other possibility is that it could be a conceptual problem in that students do not distinguish between velocity and position or fluid flow and volume.

Since students have a tendency to make graphs of a quantity and its rate of change similar, the researchers discuss the types of resemblances that graphs can exhibit. These resemblances can be present in any of the three contexts, but for the sake of simplicity the types of resemblances will be explained in the context of motion.

- **Simple replication:** An example would be a student that produces a velocity graph that looks exactly like a position graph.

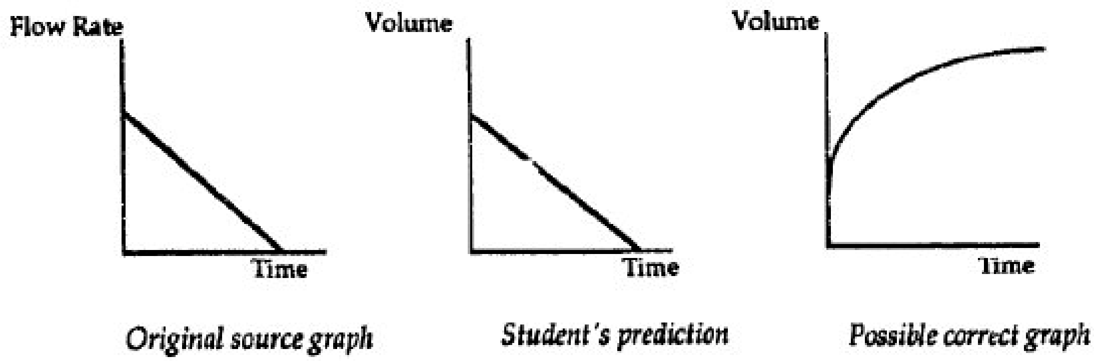



Figure 2.4 – Example of simple replication[11]

- **Same direction of change:** An increasing position graph corresponds to an increasing velocity graph. This does not mean that they have to be identical, just exhibit the same general trend.

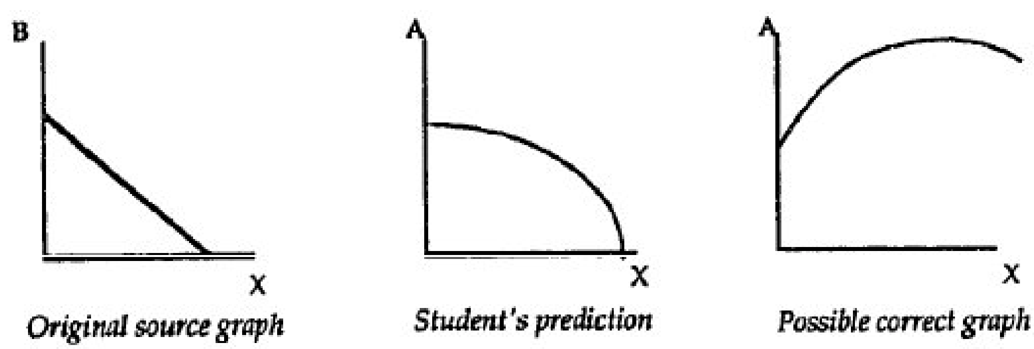



Figure 2.5 – Example of same direction of change[12]

- **Same shape:** A graph of the rate of change of a quantity will have the same shape as a graph of that quantity. If the position graph for an object is a straight line, then the velocity graph will also be a straight line.

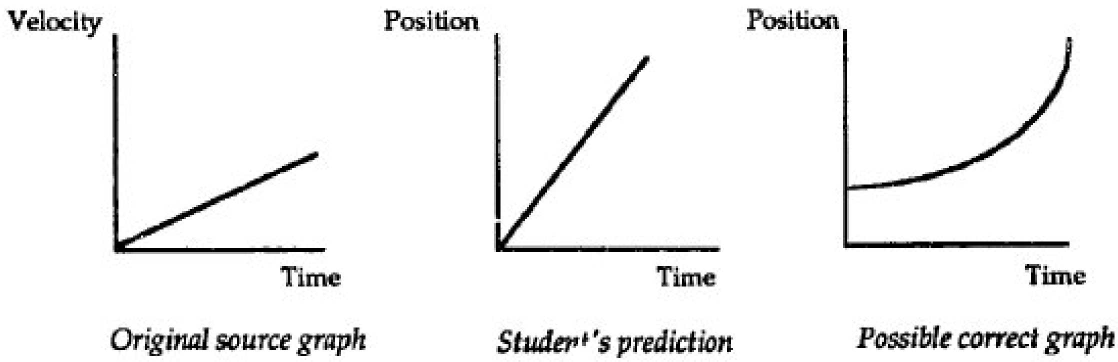



Figure 2.6 – Example of same shape[13]

- **Same sign:** The graph of the rate of a quantity will have the same sign as the graph of the quantity. If the position graph has negative values, the velocity graph will also have negative values.

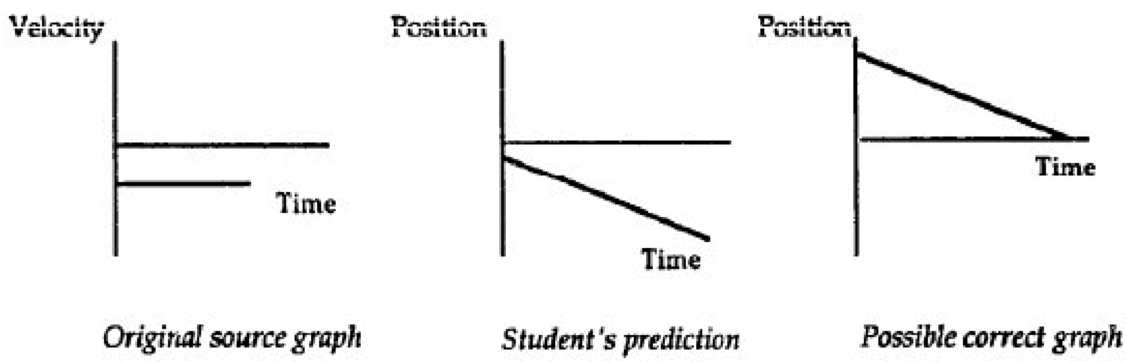



Figure 2.7 – Example of same sign[14]

- **Same geometrical transformation:** The graph of the rate of a quantity will be some geometrical transformation of the graph of that quantity. There are three different geometrical transformations; translation, rotation, or reflection.

**Translation**

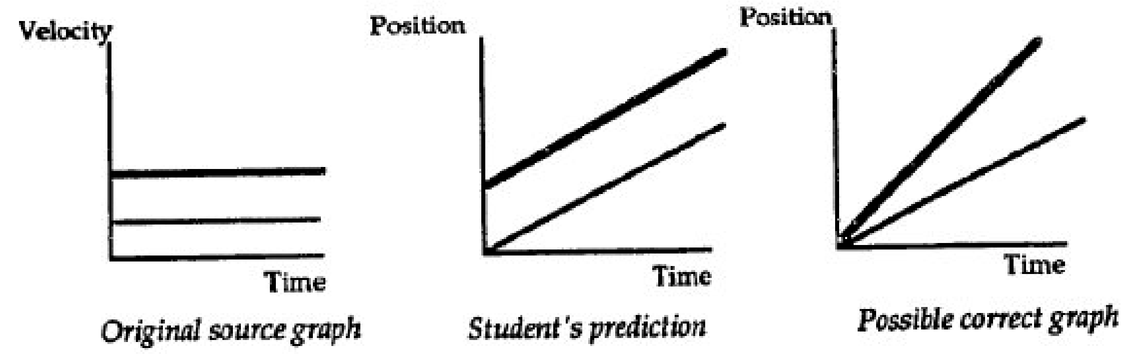



Figure 2.8 – Example of same geometrical transformation: translation[15]

**Rotation**

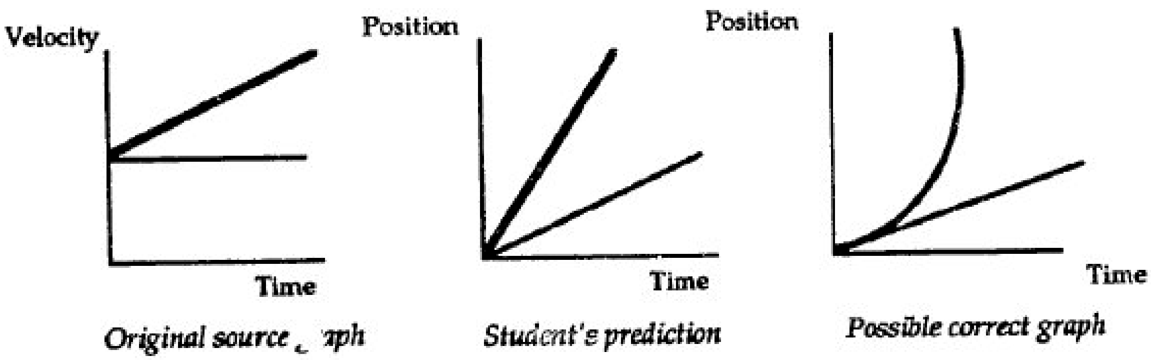



Figure 2.9 – Example of same geometrical transformation: rotation[16]

**Reflection**

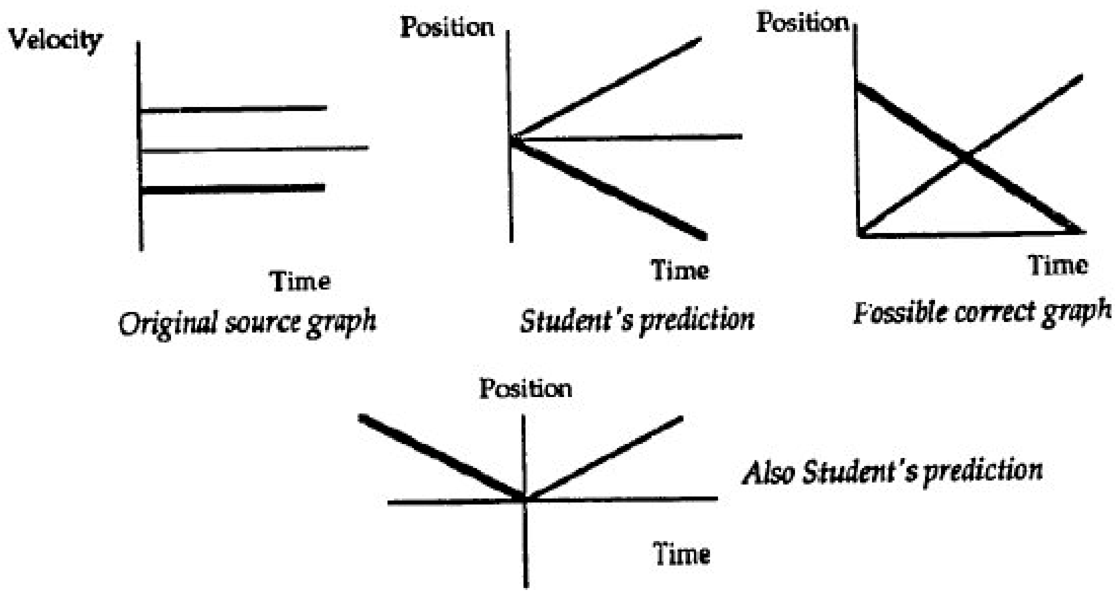



Figure 2.10 – Example of same geometrical transformation: reflection[17]

The researchers also suggested some possible reasons why students are prone to using resemblances when making graphs of a quantity or its rate of change. Students see resemblances as a tool for making sense of complicated situation and they see this tool as something simple and reliable. Which type of resemblance the students use depends on the cues they receive from the problem.

- **Syntactic cues:** These cues are based on graphical features, they are independent of the student's knowledge of the quantity and it relation to its rate of change. In this case, a student would use a resemblance to create a rate of change graph regardless of the nature of the quantity.
- **Semantic cues:** Are not based on graphical features, but rather the student's ideas about the quantity. An example of this would be a student incorrectly relating velocity and position regardless of what features are shown on the graph.
- **Linguistic cues:** Many students have trouble correctly interpreting the words in a problem. A common problem occurs with the words more and less. Students often think of "more distance" as implying "more velocity". And similarly, students interpret "less flow rate" as implying "less volume". The main problem students have in interpreting "more" and "less" is that more/less can mean comparing two entities or two different times for the same entity. An example can be seen with the phrase "more velocity". If there are two cars in a problem, a student could interpret "more velocity" as meaning one car has more velocity than the other. The student might also interpret "more velocity" as meaning the car has more velocity than it did a while ago.

Another difficulty the researchers discovered was students' confusion between slope and steepness of a graph. It is possible to have two graphs representing the same function, but displayed differently by having the axis scaled differently. It appears that many students are cueing on the steepness of a graph, rather than its slope. The steepness depends only on how the graph is displayed, where the slope is the numerical value for the rate of change of that quantity.

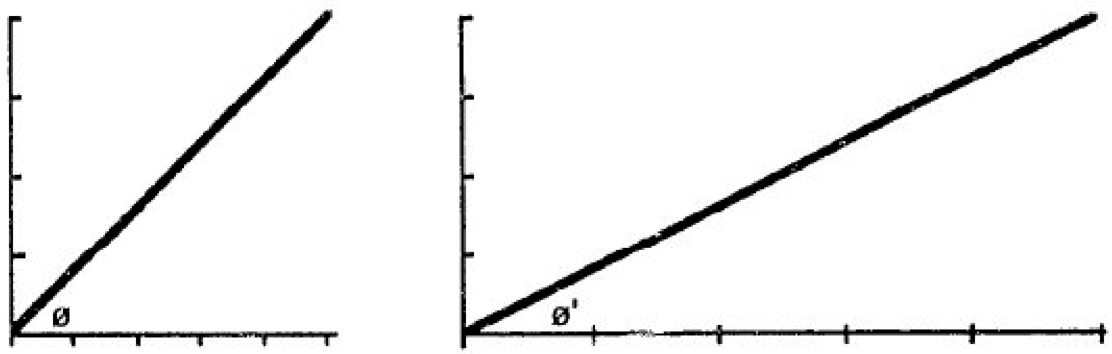



Figure 2.11 – Two graphs displaying the same function[18]. Both lines have the same slope, but different steepness.

**2.2.3 Difficulties Interpreting Graphs**

Students have exhibited some difficulties in interpreting graphs in the context of kinematics. McDermott[19] conducted a study that categorized the common graph related errors made by students. The researchers examined written responses of several hundred introductory physics student at the University of Washington. The most noticeable error that they found was students' confusion between the slope and the height of the graph. This confusion can be seen in the students' responses to the following problem:

The figure below shows a position versus time graph for the motions of two objects A and B that are moving along the same meter stick.

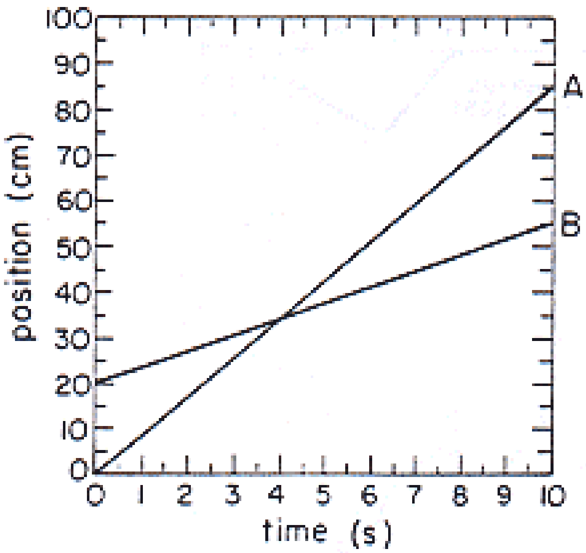



Figure 2.12 – Position versus time graph for problem given to students.

a) At the instant $t$ = 2 s, is the speed of object A greater than, less than, or equal to the speed of object B? Explain your reasoning
b) Do the objects A and B ever have the same speed? If so, at what times? Explain your reasoning.

The researchers found that many students use the height of the lines rather than the slope of the lines to answer questions about the speed of the objects. For part a, it was common for students to say that object B had a greater speed because it had a greater position at time $t$ = 2 seconds. Some of the students also answered that the two objects have the same speed at time $t$ = 4 seconds even though this is the time that the objects have the same position but different speeds.

## 2.3 Models of Student Concepts

In order to study what difficulties students have in a particular area, it is useful to have a general model of how students understand and use concepts. The following is an overview of different proposed models of student concepts.

### 2.3.1 Phenomenological Primitives

Andrea diSessa[20] has studied the basic mental pieces that make up a concept. In short, diSessa has tried to develop the concept of a concept. The main resource for diSessa's development of concepts came from interviews with 20 students in introductory physics at MIT. The students were interviewed at different intervals over a three year period and asked carefully constructed questions that probed their understanding of physics. The outcome diSessa was looking for was a chart of the knowledge system, he wanted to know the elements of this knowledge system and how they were used to explain physics concepts. diSessa wished to explore 4 aspects of this knowledge system,

- To understand and describe the elements or pieces of knowledge
- How the system works, that is how students use the system to obtain an answer or explanation
- How the system is developed and how it evolves over time
- To describe the different levels and relations between the knowledge elements

The term diSessa coined for the basic knowledge element is *phenomenological primitive* known as p-prim for short. From the name itself, a short description of a p-prim can be obtained. The phenomenological portion refers to the idea that the p-prim is an interpretation of reality and based on some observed phenomenon. By observed, diSessa refers to phenomena that a person either sees occur in everyday life such as seeing an apple fall, or experiences such as falling out of a tree. The p-prim is primitive because they are so basic as to be self-explanatory in their meaning as well as comprising the basic elements of cognitive thinking. By self-explanatory, diSessa explains that the *p-prim* can be used to explain other phenomena, but they are not explained themselves in the knowledge system. It is not argued that the p-prim is necessarily the most basic bit of reasoning, but rather the smallest observable bit of reasoning. The p-prim is a basic bit in reasoning as the atom is a basic bit in matter, whereas the atom is not really the smallest and most basic piece, most likely the p-prim is not either. An important point is that the *p-prim* is the smallest *measurable* piece of student reasoning. This is similar to the hidden variables controversy that arose in quantum mechanics[21]. But as in quantum physics, there is little point investigating variables that can not be measured and thus confirmed by experimentation. It is also important to note that the p-prim is not content specific, but rather a general rule for dealing with various content. An example of a *p-prim* that is not content specific is *Ohm's p-prim* which can be used in explaining (correctly or incorrectly) phenomena in both circuits and mechanics.

*Ohm's p-prim* is the most commonly discussed *p-prim.* It can be interpreted as "more requires more". It gets its name from Ohm's law, an obvious example; "in order to obtain a larger change in potential across an element, more current is required". *Ohm's p-prim* can also be interpreted as "more change in potential" and "more resistance" result in no change in current, this is the basis of compensatory reasoning. It is important to note that *Ohm's p-prim* is not a misconception and it is not even incorrect. *Ohm's p-prim* is a bit of reasoning that can be used correctly or incorrectly. An example of a correct use of *Ohm's p-prim* is in explaining that more force is required to accelerate a larger mass. An incorrect use of *Ohm's p-prim* would be stating that if the velocity of an object increases, its acceleration must increase also.

Another important feature of p-prims is that they are intuitive, it would be difficult for a person to explain why a certain p-prim is the way it is. Since p-prims are intuitive, they can be very robust and thus difficult to change. A later study by diSessa[22] investigated what needs to change when students learn. In this study it was suggested that when students go through conceptual change, it is not the p-prims that change (for they are not incorrect), but rather the links between p-prims. According to the conceptual model proposed by diSessa, the difference between an expert and novice would not be in the p-prims, but in the connections. A novice would have *p-prims* linked only at the local level, with fewer global connections. The expert would have a global structure linking many different p-prims on many different levels.

**2.3.2 Facets**

The model of concept described by diSessa is very provoking, but not so easy to observe. It is quite difficult to observe a particular p-prim in action because that p-prim can be used in a wide variety of contexts. Jim Minstrell[23] chooses to study student knowledge at a different "grain-size". Minstrell uses a larger unit of knowledge than does diSessa, he looks at concepts as they apply to various content domains. It is important to note that Minstrell's and diSessa's ideas are not conflicting, just as the study of atoms and the study of molecules are not conflicting. Minstrell's main motivation for studying knowledge structures is that he believes student reasoning is correct at some level. Clement[24] suggests that situations that students understand can be used as bridges to bring students to understand more complex situations. In order to move away from terms such as "misconceptions", Minstrell uses the term *facets* to describe pieces of student knowledge. A facet can be one of several forms of reasoning, it can be a strategic model, a content specific bit of knowledge, or generic bits of knowledge. A generic bit of reasoning would be quite similar to diSessa's p-prims. Minstrell uses the example of "more means…more" as an example of a generic bit of reasoning and he notes how similar this is to *Ohm's p-prim*. Since the two terms represent the same thing, I will refer to a generic bit of reasoning as a *p-prim*. "Heavier objects fall faster than lighter objects" would be an example of a content-specific bit of reasoning. A facet can also be a strategy that a student employs. An example of a strategic facet would be "average velocity can be determined by adding the initial and final velocity and dividing by two." For purposes of clarity, the term "facet" will be used here to represent the content-specific bit of reasoning.

An important aspect of facets is that they are based on observations and thus are never entirely wrong. The example of the heavier object falling faster than a lighter object is based on observation. If a ping-pong ball and a bowling ball are dropped, the heavier object will have a greater acceleration and speed when it hits the ground. Thus for some instances, this facet is true. A facet can therefore be used to bridge students to more correct facets. Another consequence of facets being based on observations is that students tend to hold strongly to the facets they possess. When told that heavy objects and light objects fall at the same rate, many students will not believe this from their own personal experience. Of course, few students have actually dropped a crumpled piece of paper and a flat piece of paper and compare the observation to the prediction that they should fall at the same rate.

Minstrell has organized the facets dealing with phenomena into *clusters*. Each cluster contains a facet that represents the correct model of thinking, this is called the goal facet. The cluster also contains all commonly used student modes of thinking as well as a facet that represents the general incorrect approach students use – this is called the mental model facet. The following table is an example of a cluster of facets on forces involved in interactions. Minstrell used the numbering system shown in the table in an effort to catalog the facets. The facet labeled with the number *xx0* is always the goal facet. The facet labeled *xx9* is the mental model facet.

Table 2.1 – Facet Cluster for Newton's Third Law

| | |
|---|---|
| 470 | **Goal Facet:** All interactions involve equal magnitude and oppositely directed action and reaction forces that are acting on the separate, interacting bodies. |
| 472 | Action and reaction forces are equal and opposite forces on the same object. |
| 475 | The stronger/firmer/harder object will exert the larger force. |
| 476 | The object moving the fastest will exert the greater force. |
| 477 | The more active/energetic object will exert the greater force. |
| 478 | The bigger/heavier object will exert the larger force. |
| 479 | **Mental Model Facet:** In an interaction between objects, the one with more of a particular perceptually salient characteristic will exert the larger force. |

### 2.3.3 Conceptual Model Measurement

It has been observed that students are not always consistent in the facet they use. A relevant study can be seen in Michael Wittmann's[25] dissertation, which looked at how students come to understand wave phenomena. Wittmann noted that students often employ a mixture of conceptual models to explain various wave phenomena. This could be seen in diagnostic instruments that contained several questions regarding similar phenomena. Wittmann placed student responses on the diagnostic instrument into one of three models, the first was the conceptual consensus model (CCM) and the second was the particle pulses model (PPM). A third model was that of random guessing. The CCM is consistent with the goal facet for waves and the particle pulses model would be the mental model facet. The particle pulses model is the facet that treats waves as particles. Wittmann found that students were for the most part using both the PPM and CCM in their responses.

Traditional analysis of pre and post results of a diagnostic instrument do not take into account the individual inconsistencies that students have. Lei Bao proposed a new method for analyzing diagnostic instruments in his dissertation[26]. The traditional method of analyzing instruments uses the Spearman assumption[27] which says that students have a true value of understanding, but the instrument provides a measured value that is some combination of the true value plus some random value. Bao's approach is to say that there is no true value for the student, but rather the student's value is a mixed state of conceptual models plus a random variable. The new method places student responses in an n-dimensional model space with a dimension for each possible conceptual model the student could use. Edward Redish[28] presented a sample analysis of the Force Concept Inventory[29]. The FCI is a commonly used diagnostic instrument that probes student understanding of force and motion. There are five questions on the FCI that deal with force and how it relates to motion. To create the model space, three models are proposed:

- **Model 1:** a force is needed to maintain motion
- **Model 2:** unbalanced force produces a change in velocity
- **Model 3:** other ideas or random guessing

In terms of facets, model 1 would be the mental model facet, and model 2 would be the goal facet. The reason a student answers a question a particular way should fall into one of these three models. The main point is that this is not necessarily how a student will answer all the questions dealing with force and motion. Each multiple-choice response for the five force and motion questions is assigned to one of the three models. From the student responses, a student response vector is created. If the student answered all five questions using model 1, then that student's response vector would be directly along the model 1 axis. It is important to note that the student response vector is a measure of the interaction between the diagnostic instrument and the student. It is not a measure of the internal conceptual properties of the student. This stresses the importance of creating valid and reliable diagnostic instruments. From the student response vectors a density matrix for all the students in a particular class can be created. The density matrix represents the relationships between the different models that students use. A traditional comparison of pre and post scores does not include information about how consistent a student was in their answer, or how consistent the class as a whole was. The density matrices for pre and post do contain information about how consistent the class was in their answers.

## 2.4 Research on Student Understanding

Although there have been no research studies focused primarily on student understanding of electric potential, there have been many studies of student understanding of various other topics. Most studies follow a similar research method, so an overview of two studies will be described.

### 2.4.1 Student Understanding of Single-slit and Double-slit Interference

The University of Washington has investigated student understanding in many topical areas. One such area of investigation is student understanding of interference[30]. The basic goal of the research was to determine what problems students have with interference phenomena. Students studied were mainly introductory calculus-based physics students, and modern physics students at the University of Washington. Of the students studied, most were either science majors or engineers and only 5% were physics majors. At the beginning of the course, most students were familiar with the concept that light travels in straight lines and in all directions from a source. In previous courses, students had also studied reflection, refraction, and ray diagrams. The introductory course covers the representations necessary for students to explain and predict polarization, diffraction, and interference. For students in modern physics, it is assumed that the wave nature of light is understood.

The primary research tool used at the University of Washington is the *Individual Demonstration Interview* (IDI). The IDI as described by Trowbridge and McDermott[31] is similar to the "clinical interview" as conducted by Jean Piaget[32]. In the *Individual Demonstration Interview,* a student is confronted with a simple apparatus that demonstrates some physical phenomena (such as motion in one dimension). The student is then asked questions relating to the apparatus or the phenomena and possibly asked to predict the outcome of the demonstration. The demonstration is then performed and the results discussed. One of the key elements of the *Individual Demonstration Interview* is that it consists of only simple equipment so that students can focus on the phenomena being displayed rather than the equipment used. The interviews last from 20 to 30 minutes and are either video taped or audio taped, then transcribed and analyzed. An example of an *Interactive Demonstration Interview* from Trowbridge and McDermott can be seen in their investigation of student understanding of velocity[33]. The demonstration consisted of two aluminum channels with balls that could roll in them. One channel lay flat on a table while the other was inclined but still straight. The two balls were released and students were asked to compare the motions. Specifically students were asked to indicate when, if ever, the balls had the same speed. In this particular study, it was determined that many students confused position and velocity as the most common answer was the point where the balls passed.

For the study of student understanding of interference, the *individual demonstration interviews* were used as the initial source of student data. Student volunteers with grades at or above the average were chosen from the introductory class and the modern physics class. The interviews were conducted after traditional instruction had been completed. The *individual demonstration interview* consisted of an unlit bulb, a mask with a rectangular hole, and a screen. The students were asked three questions:

- What would you see on the screen if the mask were placed between the bulb and the screen and the bulb was lit in a dark room?
- How would your prediction change if the bulb were moved farther and farther from the mask?
- Assuming the bulb to be very far from the mask and very bright, how would your prediction change if the slit were made narrower and narrower?

Although these were the three primary questions, the interview protocol was dynamic in that the interviewer would pose new questions related to student responses in order to more deeply probe their understanding. The interviewer also made certain that the student used some simplifying assumptions. If the student answered that the bulb would not be bright enough to see a pattern, the interviewer would add that the bulb was bright enough so that you could see a pattern. If the student attempted to treat the bulb as an extended source, the interviewer told the student to treat it as a point source. By forcing the students to use these assumptions, the student's understanding of the basic principles can be investigated.

The results from the *individual demonstration interview* were used to create several written questions that were administered to a large number of students. One such question showed the students a single slit and the diffraction that it would produce. The slit was then halfway covered – that is made narrower. The question posed to the students was what will happen to the diffraction pattern when half the slit is covered? Common student answers were that by narrowing the slit, the diffraction pattern would narrow – a case of students applying geometrical physics to the situation. A similar written question that students were asked involved a double slit and its interference pattern. Students were asked what would happen to the pattern if one slit were covered. Common incorrect responses indicate that students were treating the interference pattern as being half produced by one slit and half by the other slit so that by covering one slit, half of the pattern would change. Analysis of the written responses to this question revealed that students confuse geometrical optics and physical optics. Students would use both models (physical and geometrical) indiscriminately, or both at the same time.

From the interviews, the researchers also found that many students attempted to attribute diffraction to a comparison of wavelength to slit width. They seemed to think that the diffraction occurred when the wavelength was bigger than the slit and thus the light could not "fit" through. (see Figure 2.13) Some students also use the comparison of the slit and wavelength as a sharp transition from geometrical to physical optics. Many say that for wavelengths greater than the slit width, geometrical optics are used and physical optics are used if the wavelength is smaller than the slit. Other students treated the vertical slit as though it were a polarizer.

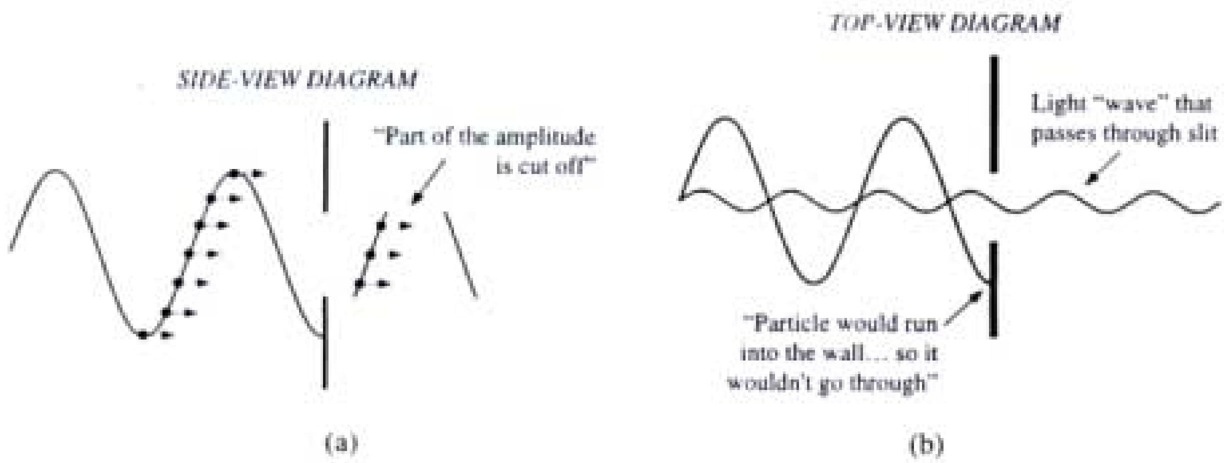



Figure 2.13 – Example of students concepts of a waves passing through an opening

Another finding from the study was that few students understood the importance of the path difference in explaining interference patterns. A written question was created in which students were asked to determine the interference at three points due to two objects vibrating in water. The two vibrating objects are placed 2.5 wavelengths apart (see figure 2.14). Students were asked to determine whether the interference was maximum constructive, maximum destructive, or neither for each of the three points. Some students were able to correctly predict the interference at points A and B, but not for point C (the point at a diagonal to the two sources). Even graduate teaching assistants had trouble determining the interference at point C, only about half of them were able to correctly answer that there would be destructive interference. Some students that did attempt to find the interference at C said that it was constructive. The reason behind their explanation showed that they were thinking about path length instead of path length difference.

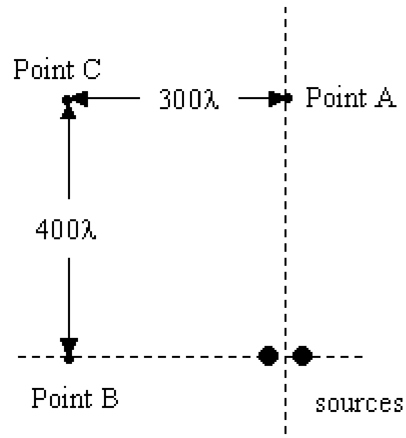



Figure 2.14 – Illustration for path difference question

The researchers also found that students tend to memorize algebraic formulas without understanding them and apply them incorrectly. An example of this can be seen in a question that was given to students at the University of Maryland. The question had light of a given wavelength incident on a double slit. The distance from the center of the pattern produced on a screen to the first dark line is also given. The students were asked to determine the distance between the slits and the screen. Almost half of the students responded in a manner that indicated they were trying to apply the formula $d \sin \theta = m \lambda$ that they had memorized for the location of the maximum.

The research methodology of the University of Washington begins with the *Individual Demonstration Interview.* By studying how students think on a simple apparatus, the research group allows the data from the interviews to determine what aspects of the concept to study. Allowing the data to form the theory rather than using data to confirm a theory is known as grounded theory.[34] The grounded theory approach to data analysis is commonly used with qualitative data as it does not pre-suppose any theory on how the students are thinking. Once the difficulties are determined, written questions are formulated to probe a wider range of students and determine how common these difficulties are.

### 2.4.2 Student Understanding of Mechanical Waves

Another example of investigations on student understanding can be seen in the research done at the University of Maryland. M. Wittmann has conducted an investigation of student understanding of mechanical waves[35]. The research methods at the University of Maryland are similar to those at University of Washington, but there are some differences. Differences in approach depend not only on the location that the research is being conducted, but also the subject matter being studied.

Before investigating student understanding, Wittmann first developed a framework in which to observe student behaviors. The framework he chose was that of the mental models as described above. Two mental models were proposed, the particle pulse mental model and the common consensus mental model. In the particle pulse mental model (PPM), the student treats a wave pulse and gives it attributes as though it were a particle.

The population studied by Wittmann was that of students in the second semester of a three semester introductory physics course at University of Maryland. The course consisted of three hours of lecture a week, a traditional lab, and a small group discussion lead by a graduate teaching assistant. The group discussions were either traditional recitations, or physics tutorials[36] from the University of Washington. The course assumed two things about waves: the small-amplitude model would be used and the media was non-dispersive and deformable. The course objectives for waves were that students would understand that wave propagation is a medium's response to a disturbance and that wave superposition occurs by adding individual wave displacements.

The primary method of developing diagnostic questions is to create a question that probes a certain aspect of the student mental model. The question is then given to students in a free response form. From the student answers to the free response and from interviews, common student responses are put into multiple choices for that same question. The question is then given to students in the form of a multiple-choice multiple-response (MCMR) format. Figure 2.15 shows an example of a diagnostic question in both its free response form and its MCMR form:

***Version 1: Free-Response (FR) format*:** A taut string is attached to a distant wall. A pulse moving on the string towards the wall reaches the wall in time $t_0$ (see diagram). How would you decrease the time it takes for the pulse to reach the wall? Explain your reasoning.

***Version 2: Multiple-Choice, Multiple-Response (MCMR) format*:**
A taut string is attached to a distant wall. A demonstrator moves her hand to create a pulse traveling toward the wall (see diagram). The demonstrator wants to produce a pulse that takes a longer time to reach the wall. Which of the actions *a–k* **taken by itself** will produce this result? More than one answer may be correct. If so, give them all. Explain your reasoning.

a. Move her hand more quickly (but still only up and down once by the same amount).
b. Move her hand more slowly (but still only up and down once by the same amount).
c. Move her hand a larger distance but up and down in the same amount of time.
d. Move her hand a smaller distance but up and down in the same amount of time.
e. Use a heavier string of the same length, under the same tension
f. Use a lighter string of the same length, under the same tension
g. Use a string of the same density, but decrease the tension.
h. Use a string of the same density, but increase the tension.
i. Put more force into the wave.
j. Put less force into the wave.
k. None of the above answers will cause the desired effect.

Figure 2.15 – Question given to students to assess their concepts of wave motion

In the fall semester of 1997, students were given both the free response version of this question and the MCMR version. Students were first given the free response and after they turned that in, they were given the MCMR. When answering the free response version, students rarely gave more than one answer, but the same students usually choose multiple answers on the MCMR. The MCMR choices seem to act as a triggering mechanism that elicits additional responses. On both versions students were told to explain their answers. The researchers compared the responses on the free response version to those on the MCMR. Their primary finding was that students do not use the mental models consistently. As an example of this, nearly three fourths of the students chose the correct answer on the MCMR after instruction, but many also chose the response that the motion of the hand could have an effect on the wave speed as well. This observation correlates with the idea that students hold multiple mental models at the same time.

In this diagnostic question and in others developed for the study of student understanding of waves, the question was derived from known student difficulties. Once the question was created, it was then given as a free response and used in interviews. From the free response and interviews, multiple-choices were chosen and used to create the MCMR. This is different than the approach of the University of Washington in which they use interviews to develop the questions. The interviews at the University of Washington are very open ended. By looking where students have problems in the interviews, the researchers know where and how to focus the questions.

## 2.5 Verbal Protocols

The first level of data collection in the study of student models comes from interviews with students. The goal of the interview is best stated by Ericsson and Simon[37];

> *"The general goal of think-aloud procedures is to capture what actually goes on in a subject's mind."*

This is the same goal of most physics education research, to find out what the student is thinking. Of course, there is no exact means to determine what a student actually thinks, but we can try with both interviews and diagnostic testing. Both interviews and diagnostics have their advantages and disadvantages. Interviews are able to probe more deeply into the student thinking as the subject is not limited to paper and written responses. The interview is also able to be adaptive, if the researcher sees something interesting during the interview, then the interview can be re-directed to investigate it. There are also disadvantages of interviewing as a method for examining how students think. The number of students that a researcher is able to examine is significantly smaller than the number of students that could be examined with a diagnostic test. Thus, without sufficient numbers of interviews it is difficult to determine if the difficulties a subject encounters is common in the population or not. Both interviews and diagnostic tests are needed, to cover the depth and breadth of the research area.

If verbal protocols are to be used as a research instrument, their validity must be investigated. Ericsson and Simon[38] identify two types of verbalization, direct and encoded. Direct verbalization is information exactly as it is stored in the long term memory (LTM) of the subject. Encoded verbalization is information that has been stored in short term memory (STM) and processed one or more times, this is assigned a level. Level 1 encoded verbalization is information with no processing. Level 2 encoded verbalization has one set of processing where level 3 would have two sets of processing. An example of level 2 verbalization would be a description of something visualized. The subject must first processes what he or she sees before being able to describe it verbally. A level 3 verbalization would be an instance of a subject that must analyze information on some level, such as scanning or filtering for something particular.

Ericsson and Simon report that only level 1 and 2 verbalizations can be accurately reported. Thus asking questions that require the subject to analyze, such as "why did you…" would not be able to be reported accurately. There can still be some difficulties using only level 1 and 2 verbalizations.

- Not all information the investigator wants to study will necessarily pass through the STM.
- The subject's ability to report the contents of the STM is limited.
- The subject may be taking mental short cuts instead of reporting the contents of the STM.
- The procedure of processing the contents of the STM may interfere with the performance of the desired task (such as working a physics problem).

Ericsson and Simon also state that reporting the contents of the LTM are not reliable, thus tasks that depend on automatic processes (that come from the LTM) are not an appropriate choice for verbal protocols.

The means in which the interview is conducted can also affect the validity of the verbal protocols[39]. The instructions given to the student can have a large influence over the student's responses. The researcher must be careful not give instructions that lead the subject to further encode the contents of the STM. The mere presence of the researcher and the recording equipment may also become a barrier to the subject accurately reporting the contents of the STM. There can also be problems with the reporting by the subject, the subject could be telling the researcher what he or she thinks the researcher wants to hear. There is also the possibility of researcher bias in the analysis of the interviews.

[1] Törnkvist, S., Pettersson, S.K., & Tranströmer, G. (1993) Confusion by representation: On students' comprehension of the electric field concept. American Journal of Physics, 61, 335-338.
[2] The question was taken from Wilson, A.W., & Hacket, R.Q. (1974) Inquiring into Physics. Paragons, Oxford.
[3] Asimov, I. (1920) The History of Physics. New York, NY: Walker and Company.
[4] Trowbridge, D.E., and McDermott, L.C. (1980) Investigation of student understanding of the concept of velocity in one dimension. American Journal of Physics, 49, 242-253.
[5] Rainson, S., Tranströmer, G. &Viennot, L. (1994) Students' understanding of superposition of electric fields. American Journal of Physics, 62, 1026-1032.
[6] Minstrell, J. (1991). Facets of students' knowledge and relevant instruction. Paper presented at the Conference on Research in Physics Learning: Theoretical Issues and Empirical Studies, Bremen, Germany.
[7] Shabay, R. & Sherwood, B. (1995). Electric and Magnetic Interactions. New York, NY: John Wiley and Sons.
[8] Cohen, R., Eylon, B.& Ganiel, U. (1983) Potential difference and current in simple electric circuits: A study of students' concepts. American Journal of Physics, 51, 407-412.
[9] Hauger, G.S. (1995) Rate of change knowledge in high school and college students. A paper presented at the annual meeting of the American Education Research Association, San Francisco, CA and Hauger, G.S. (1997) Growth of knowledge of rate in four precalculus students. A paper presented at the Annual Meeting of the American Educational Research Association, Chicago, IL.
[10] Nemirovsky, R., & Rubin, A. (1992). Students' tendency to assume resemblances between a function and its derivative. Cambridge, MA TERC (ERIC Document Reproduction Service No. ED ED351193)
[11] see reference 10
[12] see reference 10
[13] see reference 10
[14] see reference 10
[15] see reference 10
[16] see reference 10
[17] see reference 10
[18] see reference 10
[19] McDermott, L.C., Rosenquist, M.L., & van Zee, E.H. (1987) Student difficulties in connecting graphs and physics: Examples from kinematics. American Journal of Physics, 55, 503-513
[20] diSessa, A. (1992) Toward an Epistemology of Physics. Cognition and Instruction, 10, 105-225.
[21] Pinch, T.J. (1979) The hidden-variables controversy in quantum physics. Physics Education, 14, 48-52.
[22] DiSessa, A. (1998) What changes in conceptual change? International Journal of Science Education, 20, 1155-1191.
[23] Minstrell, J. (1991) Facets of students' knowledge and relevant instruction. A paper presented at the Conference on Research in Physics Learning: Theoretical Issues and Empirical Studies, Bremen, Germany.
[24] Clement, J. (1989) Not All Preconceptions are Misconceptions: Finding 'Anchoring Conceptions' for Grounding Instruction on Students' Intuitions. International Journal of Science Education, 11, 554-565.
[25] Wittmann, M. (1999) Making sense of how students come to an understanding of physics: An example from mechanical waves. Ph.D. dissertation, University of Maryland.
[26] Bao, L. (1999) Dynamics of student modeling: A theory, algorithms, and application to quantum mechanics. Ph.D. dissertation, University of Maryland.
[27] Pedhazur, E.J., & Schmelkin, L.P. (1991). Measurement, Design, and Analysis: An Integrated Approach. Hillsdale, NJ: Lawrence Erlbaum Associates.
[28] Redish, E.F. (1999) Diagnosing student problems using the results and methods of Physics Education Research. Plenary talk presented at the International Conference on Physics Teaching, Guilin, China.

[29] Hestenes, D., Wells, M., & Swackhamer, G. (1992). Force Concept Inventory. The Physics Teacher, 30, 141-158.

[30] Ambrose, B.S., Shaffer, P.S., Steinberg, R.N., McDermott, L.C. (1999) An investigation of student understanding of single-slit and double-slit interference. American Journal of Physics, 67, 146-155.

[31] Trowbridge, D.E., & McDermott, L.C. (1980) Investigation of student understanding of the concept of velocity in one dimension. American Journal of Physics, 48, 1020-1028.

[32] Piaget, J. (1970) The Child's Conception of Movement and Speed. New York, NY: Ballantine.

[33] see reference 16

[34] Strauss, A., & Corbin, J. (1990) Basics of Qualitative Research: Grounded Theory Procedures and Techniques. Newbury Park, CA: SAGE Publications.

[35] Wittmann, M., Steinberg, R.N., & Redish, E.F. (1999) Making Sense of How Students Make Sense of Mechanical Waves. The Physics Teacher, 37, 15-21.

[36] McDermott, L.C., Shaffer, P.S. & the Physics Education Group at the University of Washington, Seattle (1998) Tutorials in Introductory Physics. Upper Saddle River, NJ: Prentice Hall.

[37] K. Ericsson and H. Simon (1978). Retrospective verbal reports as data. Cambridge, Mass: MIT Press.

[38] Ericsson, K. & Simon, H. (1980) Verbal Protocols as data. Psychological Review, 87, 215-251.

[39] Shapiro, M.A. (1994). Think-Aloud and Thought-List Procedures in Investigating Mental Processes. In Annie Lang (Ed.), Measuring Psychological Responses to Media (pp. 1-14). Hillsdale, NJ: Lawrence Erlbaum Associates.

# Chapter 3
# Methodology

The purpose of the study was to investigate the connection between introductory physics students' responses to rate of change questions and their responses to questions on electric potential. A multiple-choice instrument, The Rate and Potential Test, was developed to meet this goal. In this chapter, I present the study's research methodology by discussing (a) the research design, (b) the sample, (c) the instrumentation, (d) data collection procedures, and (e) data analysis procedures.

## **3.1 Research Design**

The instrument will be used to study the relationship between students' multiple-choice answers on rate of change questions and their multiple-choice answers on electric potential questions. This relationship will also be investigated through think-aloud interviews with students taking the same instrument. The effect of order of questions will be investigated with the use of two versions of the instrument. The two versions contain the same questions but in different orders.

## **3.2 The Sample**

I collected student responses to both versions of the multiple-choice instrument in the spring of 2001 from different sources. These samples are described below. The schools are described according to their classification by the Carnegie Foundation[1].

*Sample 1*

The first sample consisted of 230 students enrolled at "doctoral/research university – extensive" institution located in the Southeast. The Carnegie foundation describes the "doctoral/research university – extensive" class as: "*These institutions typically offer a wide range of baccalaureate programs, and they are committed to graduate education through the doctorate. During the period studied, they awarded 50 or more doctoral degrees per year across at least 15 disciplines."* These students were in one of three sections of introductory, calculus-based physics courses. All three of these sections center around "traditional lecturing." In other words, class time is spent with the instructor lecturing on topics covered in the text as well as working examples of physics problems.

All the students in this sample received and answered the questions using *WebAssign*[2], an online homework delivery system widely used at the university. The students received extra credit towards their homework grade for answering the questions. The extra credit was given regardless of their score on the instrument. The assignment allowed three submissions, but few students submitted more than once. Two sections received version A of the instrument, and one section received version B.

*Sample 2*

There were two sections of introductory physics at an "associate's college" in the Northwest that also took the paper version of the instrument. The Carnegie Foundation describes "associate's college" as: *"These institutions offer associate's degree and certificate programs but, with few exceptions, award no baccalaureate degrees. This group includes institutions where, during the period studied, bachelor's degrees represent less than 10 percent of all undergraduate awards."* These two sections contained a total of 33 students. The teaching style of the course can also be described as "traditional". One section had version A of the instrument while the other section had version B. The instructor administered both versions of the instrument during class.

*Sample 3*

This sample consisted of 41 students in two sections from a "master's college I" located in the Northwest. The Carnegie Foundation describes "master's colleges and universities I" as: *"These insititutions typically offer a wide range of baccalaureate programs, and they are committed to graduate education through the master's degree. During the period studied, they awarded 40 or more master's degrees per year across three or more disciplines."* One section received the paper version A and one section the paper version B. The instructor administered both versions of the instrument during class. The course can also be described as 'traditional".

*Sample 4*

There was one section of 44 students in an introductory physics course at a "doctoral/research university - extensive" located in the Southwest. These students only took paper version A of the instrument during class. This class is not a traditional class, rather it would be classified as "interactive engagement"[3]. Richard Hake defines interactive engagement as "designed at least in part to promote conceptual understanding through interactive engagement of students in heads-on (always) and hands-on (usually) activities with peers and/or instructors". The assistant instructor described the class as concentrating on energy methods throughout the whole year as well as the use of a variety of representational tools and modeling techniques[4].

*Sample 5*

I investigated a second group of students at a "doctoral/research university – extensive" located in the Southeast. This group consisted of 70 introductory physics students. This section did not take the full instrument, but rather a sub-set of the questions that I will refer to as the sub-test. These questions were found to be the ones in which many students selected the answer associated with the model that treats a quantity and its rate of change as behaving similarly. The sub-test followed an open-ended format allowing the students the opportunity to explain why they selected a particular answer choice. The students were given extra credit for answering the questions on the instrument, which was given during class. The sub-test was given to the students on paper rather than WebAssign.

*Sample 6*

The same sub-set of questions was also given to an upper-level undergraduate physics course on electricity and magnetism a "doctoral/research university – extensive" located in the Southeast. There were 18 students in the course comprised of both undergraduate and graduate students and they took the sub-test version on paper.

## 3.3 Instrumentation

In this section, I will describe the creation of the instrument that was used in this study. The general details about the creation are discussed first followed by details on the creation of each item.

The instrument used in this study was the Rate And Potential Test (RAPT) (See Appendix A). The instrument contains multiple-choice items on both the concepts of rate of change and electric potential. Some of the items are derived from items in other diagnostic instruments or from other research studies. I created the rest of the items from experience with student difficulties in previous interviews on electric potential and rate of change. The purpose of these interviews was to do a preliminary evaluation of the items that would be used in the RAPT. Students were recruited from an introductory physics class and paid $10 for their time. The format of the interviews was that students would go over a selection of rate of change and electric potential questions in a think-aloud protocol.

Both the rate of change and the electric potential items fall into one of four cases of rate of change reasoning. These cases are based on the different situations involving a quantity and its rate of change. There, of course, are other cases that could be included. An example of a case that is not included is the a=a case. In this case, the student compares two different objects and says that when the objects have the same of some quantity, like position, the objects will have the same rate of change, like velocity. This case was not included because there is no reasonable electric potential that uses this case. An electric potential question with the a=a case would need to have two different electric potential functions in the same space and this is not physical reasonable.

### 3.3.1 Rate of Change Cases

These cases are the different situations that students can incorrectly relate a quantity to its rate of change when dealing with items from the RAPT.

*Case: All=All*

In this case, the student uses the strategy that the rate of change of a quantity varies the same way as the quantity itself. A student using this strategy would say that a coin thrown straight up will have a decreasing acceleration on the way up and an increasing acceleration on the way down because the velocity changes that way.

*Case: Zero=Zero*

This is actually a sub-case of the *All=All* case. In the *Zero=Zero* case, the student uses the strategy that says that if a quantity is zero, the rate of change of that quantity must also be zero. Using the same coin toss example, the student using this case would say that at the highest point the acceleration of the coin is zero because the velocity was zero.

*Case: No Zero=No Zero*

This is the complimentary case of *Zero=Zero.* For this case, the student says that if the quantity is not zero, the rate of change of that quantity cannot be zero. A student using this strategy would interpret a horizontal segment of a position-time graph as having a constant, but non-zero velocity because the position is not zero.

*Case: Great=Great*

In this case the student uses the strategy that if the quantity is at its greatest or maximum value, the rate of change of that quantity must also be at its greatest or maximum value. A student using this strategy would use height as a measure to determine the acceleration of a ball on a hill rather than the slope of the hill to determine the acceleration. This is similar to the confusion of slope of a graph and height of a graph, but it is not the same mistake as discussed in Chapter 2.

### 3.3.2 Creation of the Instrument

In creating this instrument, I had to work within time constraints. The instrument needed to be given during a regular class period. If I were to fully explore student difficulties with rate of change and electric potential, the instrument would be too long to be given during a regular class period. Therefore, I decided to limit the questions to those in which the student is asked about the rate of change given information on the quantity. In other words, I would include questions asking the student about the velocity given a position versus time graph but I would not ask about the position given a velocity versus time graph. The reason is that the first type of question involves derivatives, the velocity is the derivative of the position. The derivative is not as complicated as the anti-derivative, which would be used to find the position from a velocity versus time graph. This means there are also no questions in which the student is to find the electric potential from the electric field. The questions that I created are such that the students will not need to fully understand calculus to answer them. They will not need to take derivative nor will they need to integrate. The questions only require an understanding of rate of change at a more basic level.

The questions chosen needed to be tested to insure that students were reading and understanding them correctly. I did not want confusing or ambiguous questions, but rather questions that students would answer based on their beliefs of the relationship between a quantity and its rate of change. The source for most questions was other diagnostic instruments. As explained in the individual item discussions, items were taken from the Test of Understanding Graphs and Kinematics (TUG-K), the Force and Motion Concept Evaluation (FMCE), and the Conceptual Survey of Electricity and Magnetism (CSEM). Some of the items are the identical to items from other diagnostics and some items are modifications of the original items. Further details on each item and how it was modified are explained below. The answers for the multiple-choice test were the same as those in the diagnostic that it came from. For the modified items, the multiple-choice items were modified as well. As an example, some TUG-K items were modified into electric potential items. If the original TUG-K item presented a graph of position versus time, the item and its multiple-choice answers were modified to be electric potential versus distance graphs. I created some of the items without any source except for my experience with student difficulties in the classroom. For these items, the multiple-choice answers contain the correct response as well as other common student answers. Usually it is not a issue of choosing the multiple-choice answers as there are only a finite number of possible answers. An example of these answers can be seen in a question that asks to describe the one-dimensional motion of a proton. There are really only 4 possible answers, it does not move, it moves to the left, it moves to the right, or the motion can not be explained.

To test the format of the questions, I conducted think-aloud interviews with 6 students. The format and details of the think-aloud interviews are discussed in section 3.4.3. Each student answered all the questions in a think-aloud format and afterwards explained the reason behind his or her answers. Another reason for conducting these interviews was to see if any unexpected non-content-related student difficulties arose. None of the 6 students had significant trouble understanding what the questions were asking and none of them exhibited any unexpected difficulties.

In order to verify the multiple-choice answers to the instrument, I administered an open-ended version of the instrument. The open-ended version was given to an introductory physics class. I compared the students' written answers with the multiple-choice distractors in the RAPT. In the comparison I was checking to see if this had been a multiple-choice instrument instead of open-ended, would the students answers have been represented. This was true for the most part.

There are always some students that answer the item in a nonsensical manner, but the number of students that do this is small (< 1%).

Once the instrument was in its final form, experts reviewed it for any unforeseen difficulties and to make sure each question was correct.

### 3.3.3 Item details

The 25 items in the RAPT are classified by their context. That is they are either rate of change items designated by an R, or electric potential items designated by a V. I also grouped the items by situation. There are three items that deal with a coin tossed in the air. Since this deals with rate of change, these three items are labeled R1a, R1b, and R1c.

*Items R1a, R1b, and R1c*

*A coin is tossed straight up into the air. After it is released it moves upward, reaches its highest point and falls back down again. Choose the correct description of the acceleration of the coin for the three phases. Take up to be the positive direction.*

1. *While the coin is moving upward after it is released.*
    a. *The acceleration is in the positive direction and decreasing*
    b. *The acceleration is in the positive direction and constant*
    c. *The acceleration is in the positive direction and increasing*
    d. *The acceleration is zero*
    e. *The acceleration is in the negative direction and decreasing*
    f. *The acceleration is in the negative direction and increasing*
    g. *The acceleration is in the negative direction and constant*
2. *When the coin is at its highest point.*
    a. *The acceleration is in the positive direction and decreasing*
    b. *The acceleration is in the positive direction and constant*
    c. *The acceleration is in the positive direction and increasing*
    d. *The acceleration is zero*
    e. *The acceleration is in the negative direction and decreasing*
    f. *The acceleration is in the negative direction and increasing*
    g. *The acceleration is in the negative direction and constant*
3. *While the coin is moving downward.*
    a. *The acceleration is in the positive direction and decreasing*
    b. *The acceleration is in the positive direction and constant*
    c. *The acceleration is in the positive direction and increasing*
    d. *The acceleration is zero*
    e. *The acceleration is in the negative direction and decreasing*
    f. *The acceleration is in the negative direction and increasing*
    g. *The acceleration is in the negative direction and constant*

These items present the student with a coin tossed into the air and ask for a description of the acceleration at three different locations. The source for these items is the *Force and Motion Concept Inventory*[5], a multiple-choice diagnostic instrument designed to investigate students understanding of the concepts in kinematics and Newton's laws of motion.

*Item R2*

*Two identical balls are placed at the two locations on a hill as shown below.*

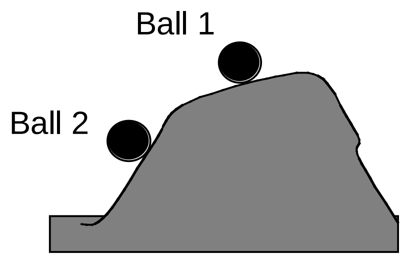



Figure 3.1 – Illustration for item R2

*Both balls are released from rest. At the moment they are released, which ball will have the greater acceleration?*

a. *Ball 1 will have the greater acceleration*
b. *Ball 2 will have the greater acceleration*
c. *Both balls will have the same acceleration*
d. *Not enough information is given to compare the accelerations of the balls*

This item deals with two balls on a hill. The ball that is at a lower level is on a steeper slope and thus will have a greater acceleration. The purpose of this item is to see if students use the strategy that the higher ball will have the greater acceleration. This is also an important item in that it involves a distance rate of change (gradient) rather than time rate of change. The reason that this is important is because the electric field is the gradient of the electric potential rather than a time rate of change as the kinematics items are.

*R3*

*Several tanks are being filled by hoses with adjustable nozzles (so that the amount of water coming out of the hose can be changed). The tanks also have plugs that allow them to be drained. Below are graphs of the amount of water (in gallons) in the tanks as time goes on. Which tank has its water level changing the fastest at some time?*

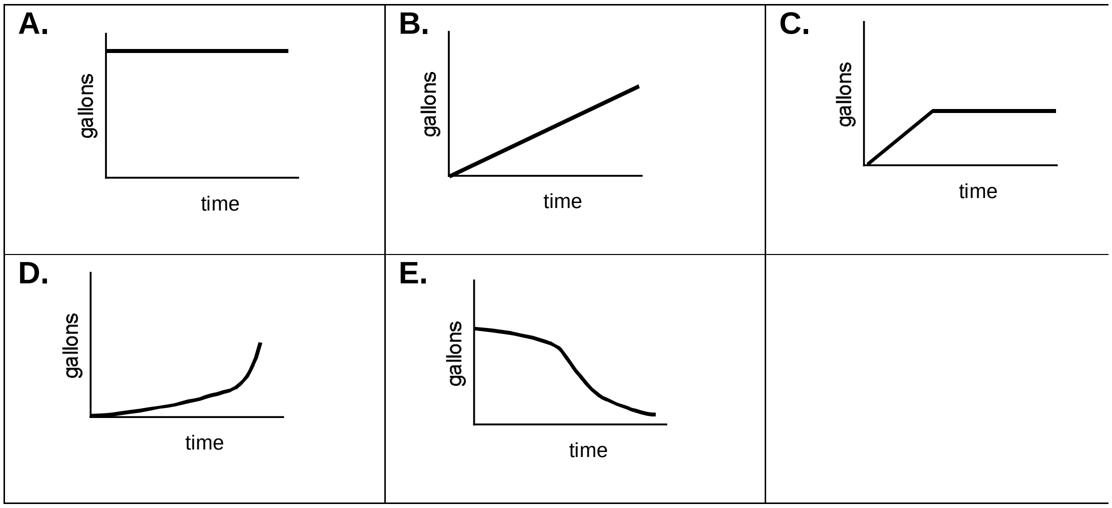



Figure 3.2 – Illustration for item R3

I modified this item from the *Test of Understanding of Graphs in Kinematics*[6] (TUG-K), a multiple-choice instrument designed to investigate how students interpret graphs in the context of kinematics. The original question presents the student with 5 graphs of position versus time for a moving object. Since I want to avoid using only kinematics question to investigate rate of change strategies, I changed this to the volume of water in a tank as it was being filled.

*R4a, R4b, and R4c*

*Questions 6 – 8 refer to two tanks that are being filled by separate water hoses. Below is a graph representing the water in each tank as time goes on.*

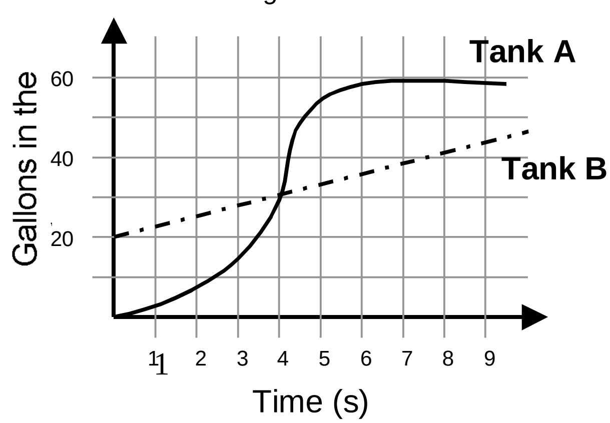



Figure 3.3 – Illustration for item R4a, R4b, and R4c

4. *Which tank is filling up faster at t = 3 seconds?*
   a. *Tank A is filling up faster*
   b. *Tank B is filling up faster*
   c. *Neither tank is filling up faster*
   d. *There is not enough information to determine the answer*

5. *Which tank is filling up faster at t = 4 seconds?*
    a. *Tank A is filling up faster*
    b. *Tank B is filling up faster*
    c. *Neither tank is filling up faster*
    d. *There is not enough information to determine the answer*
6. *Which tank is filling up faster at t = 8 seconds?*
    a. *Tank A is filling up faster*
    b. *Tank B is filling up faster*
    c. *Neither tank is filling up faster*
    d. *There is not enough information to determine the answer*

These items are derived from a question used in a research study[7] at the University of Washington located in Seattle, Washington. The study investigated how students relate velocity and position. The original question showed a position versus time graph for two balls rolling down separate ramps. The key element of the graph was the time at which the two balls were at the same position, although they had different velocities. I wanted to address this same issue, but outside the context of kinematics. This item is a graph of the volume of water in two tanks as a function of time instead. This graph contains the same feature of the two tanks having the same volume at a certain time, but they are being filled at different rates. Another important aspect of the question is that one tank is being filled faster, while the other tank has more water in it at a certain instance. These volume questions are mathematically analogous to the kinematics questions that this is derived from. The velocity is the time rate of change of position as well as the slope of a position versus time graph. In the same respect, how fast a tank is being filled up is the time rate of change of the volume of water in the tank and the slope of a volume versus time graph is also how fast that tank is filling up.

*R5*

*The following is a position-time graph for an object during a 5 second time interval.*

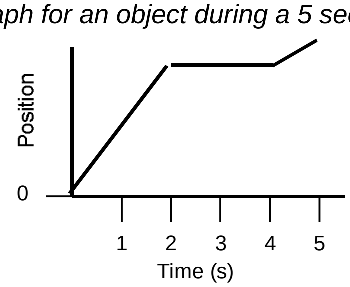



Figure 3.4 – Illustration for item R5

*Which of the following graphs of velocity versus time would best represent the object's motion during the same time interval?*

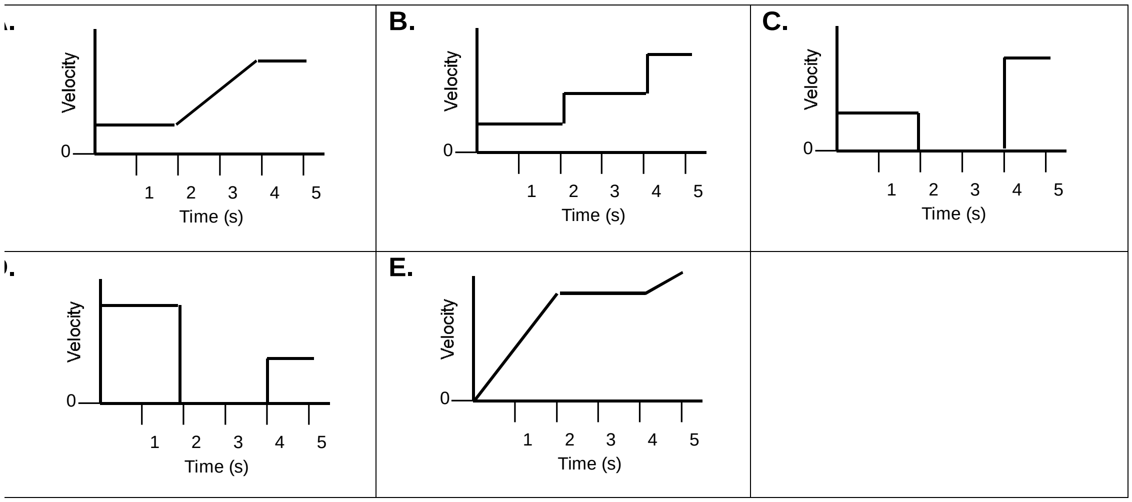



Figure 3.5 – Answers for item R5

This is item is taken directly from the TUG-K. It presents the students with a graph of position versus time and asks them to choose the corresponding graph of velocity versus time.

*R6a, R6b, R6c, R6d*

*Questions 10 – 13 refer to the weather map of part of the U. S. The map displays isobars. Isobars are lines along which the barometric pressure is the same. A map of isobars is useful for locating areas of strong winds. The strongest winds are found in regions where the pressure is changing the most.*

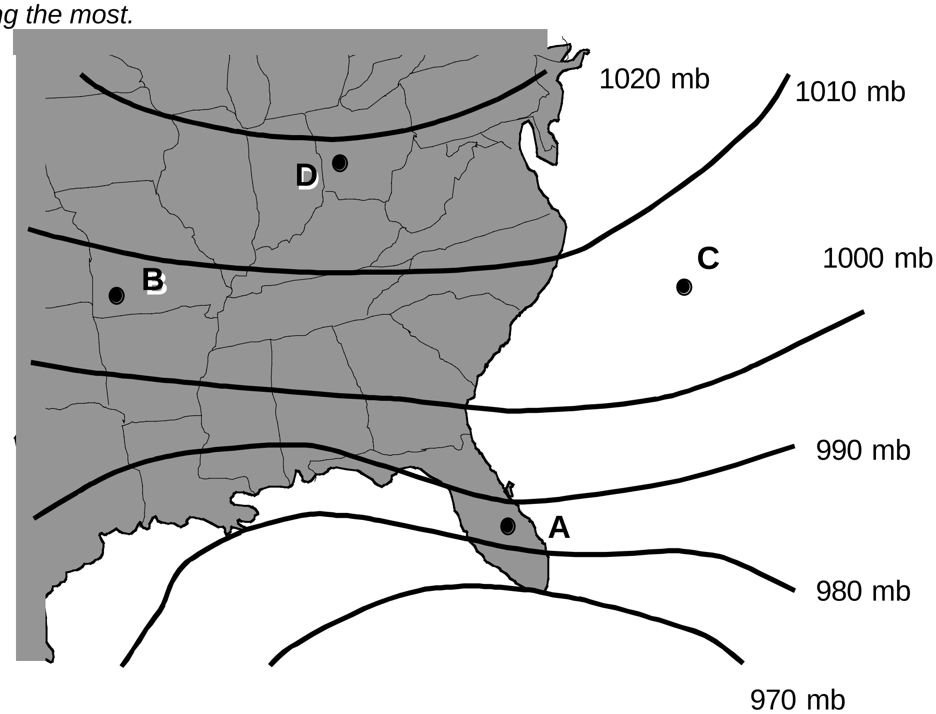



Figure 3.6 – Illustration for items R6a, R6b, R6c, R6d

7. *At which point (A, B, C, D, or E for none) is the wind the strongest?*

*8. At which point (A, B, C, D, or E for none) is the wind the weakest?*
*9. At which point (A, B, C, D, or E for none) is the pressure the greatest?*
*10. At which point (A, B, C, D, or E for none) is the pressure the lowest?*

These four items present the students with map of equipotential lines for a region of space. This item investigates if students have difficulty with equipotential lines because they are not able to read maps of gradients. The R6 items deal with a map of isobars, that is, lines of constant pressure.

*R7*

*A ball is rolling on a flat surface with a constant, non-zero velocity. Which statement describes the magnitude of the acceleration of the ball?*

*a. The acceleration is zero*
*b. The acceleration is constant, but non-zero*
*c. The acceleration is increasing*
*d. The acceleration is decreasing*
*e. There is not enough information to describe the acceleration*

I added this question to balance an electric potential question (V6) that many students were having problems with in a pilot study. The electric potential item that it balances asks about the electric field in a region where the electric potential is constant. The electric field is the rate of change (gradient) of electric potential and the velocity is the rate of change of positioin. This item asks about the acceleration of an object moving at constant velocity.

*V1a, V1b, V1c, and V1d*

*Questions 11 – 14 refer to the region of space with an electric field represented by the equipotential lines as shown below. (An equipotential line is a line along which the value of the electric potential remains the same).*

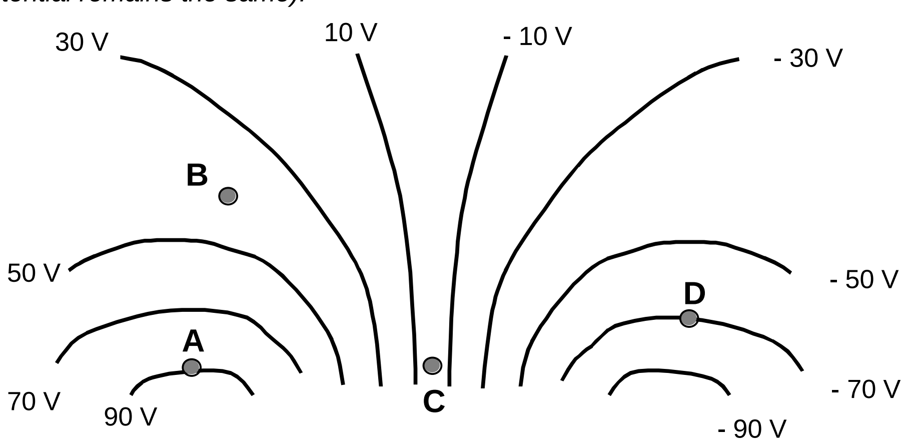



Figure 3.7 – Illustration for items V1a, V1b, V1c, V1d

*11. At which of the points (A, B, C, D, or E for none) is the value of the electric potential the greatest?*

12. *At which of the points (A, B, C, D, or E for none) is the value of the electric potential equal to zero?*
13. *At which of the points (A, B, C, D, or E for none) is the magnitude of the electric field the greatest?*
14. *At which of the points (A, B, C, D, or E for none) is the magnitude of the electric field equal to zero?*

For these items, the student is given an area with an electric potential described by equipotential lines. The first two items (V1a and V1b) ask the student to determine where the potential is the greatest and where it is zero. The purpose of the first two items is to determine if the students can read the equipotential lines to obtain information about the potential. The last 2 items (R1c and R1d) ask the student where the electric field is zero and where it is the greatest.

*V2a and V2b*

*Question 19 and 20 refer to a region of space that has an electric potential described by the equipotential lines as shown below. An equipotential line is a line containing all the points with the same value of the electric potential.*

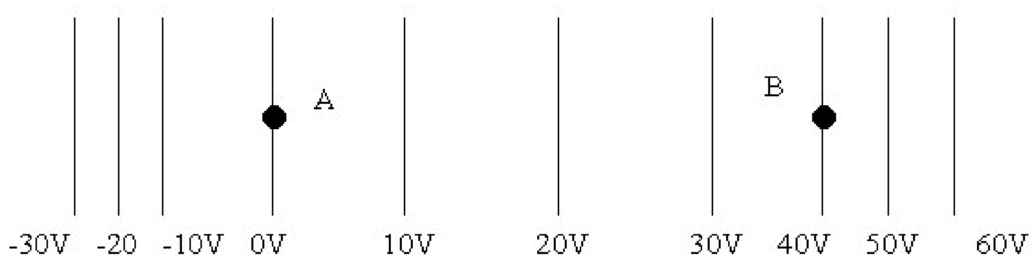



Figure 3.8 – Illustration for items V2a and V2b

15. *A proton is placed at point A. If released from rest, what will its motion be?*
    a. *The proton will not move*
    b. *The proton will move to the right*
    c. *The proton will move to the left*
    d. *The motion can not be determined*

16. *The proton is now moved to point B and released from rest, what will its motion be?*
    a. *The proton will not move*
    b. *The proton will move to the right*
    c. *The proton will move to the left*
    d. *The motion of the proton can not be determined*

I derived these two items from a question on the *Conceptual Survey of Electricity and Magnetism*[8] *(CSEM)*. The CSEM item has students compare the forces at the two positions. Analysis from interviews on the original CSEM item show a common response is that students think that if the electric potential is zero, then there can be no force on a charged particle. As a result, I changed the item to motion of the particles instead of force to match the two balls on a hill question (R2).

*V3*

Five regions with different electric fields are represented below by the graphs of electric potential (V) vs. displacement (x). Which graph represents the region with the largest electric field at some point in the region?

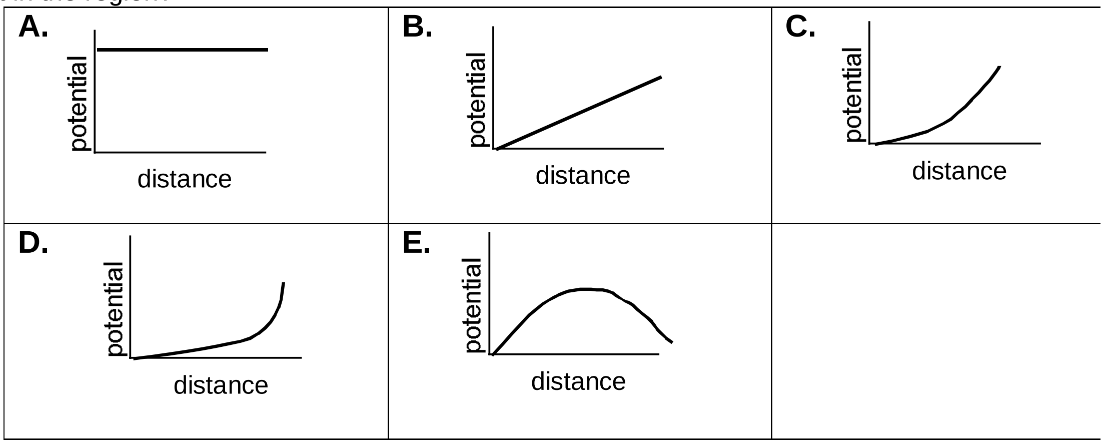



Figure 3.9 – Answers for item V3

This is the same as item R3 except that it deals with graphs of potential versus distance instead of position versus time. Item R3 asks which graph has the greatest velocity at some time and item V3 asks which graph has the greatest electric field at some point. The velocity is the slope of a position versus time graph and the electric field is the slope of a graph of electric potential versus distance.

*V4a and V4b*

*Question 22 and 23 refer to the region of space with an electric potential described by the following graph of electric potential (V) vs. distance (x).*

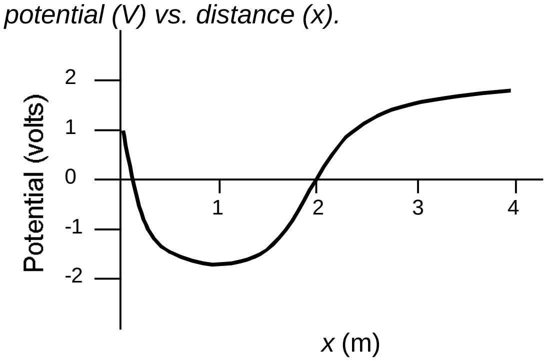



Figure 3.10 – Illustration for items V4a and V4b

17. *A proton is released from rest at x = 2 meters in a region as represented by the above graph. What will be the motion of the proton?*
    a. *The proton will not move*
    b. *The proton will move to the left*
    c. *The proton will move to the right*
    d. *The motion of the proton can not be determined*

18. *The proton is now released at x = 1 meters in the region represented by the above graph. What will be the motion of the proton?*
    a. *The proton will not move*
    b. *The proton will move to the left*
    c. *The proton will move to the right*
    d. *The motion of the proton can not be determined*

This item deals with the same issues as V2a and V2b, but with a different representation. V4a and V4b present the electric potential as a graph of potential versus distance. The graph still contains a location where the potential is zero, but the electric field is not. It also contains a point where the electric field is zero, but the potential is not.

*V5*

*The following is a graph of electric potential (V) vs. distance (x) for a certain region of space.*

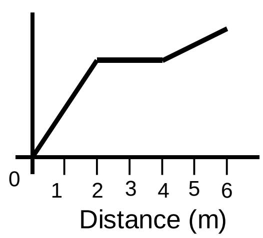



Figure 3.11 – Illustration for item V5

*Which of the following graphs of the* ***magnitude*** *of the electric field (E) vs. distance (x) corresponds to the same region?*

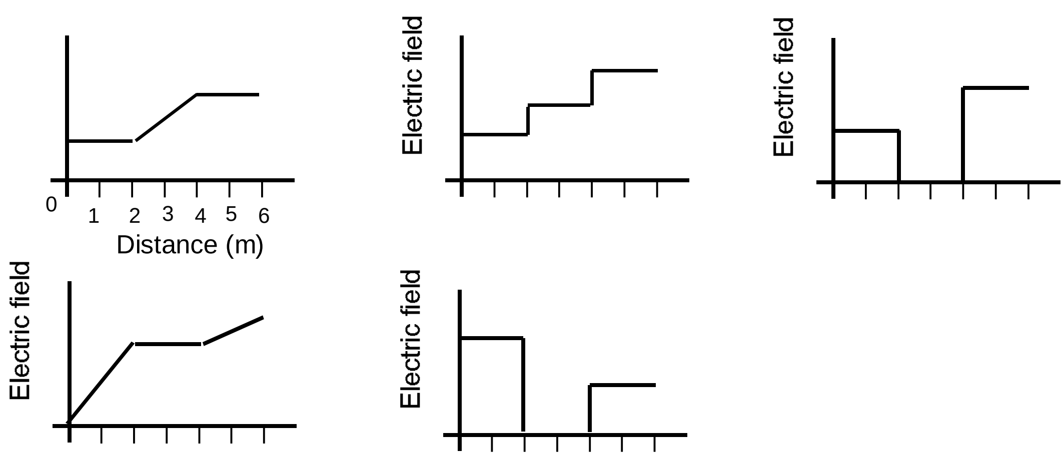



Figure 3.12 – Answers for item V5

This is the same as item R5 except it deals with electric potential and distance rather than distance and time. An important aspect of this item is that it asks for the **magnitude** of the electric field rather than the value. I added this limitation because I did not want students to be confused about the proper sign of the electric field. The electric field is actually the negative of the gradient of electric potential, so these graphs do not show the value of component of the electric field in the *x* –direction. I am interested in the gradient aspect of the relation between electric potential and electric field, not the sign of the relation.

*V6*

19. *The electric potential in a certain region is at a constant, non-zero value. Which statement describes the magnitude of the electric field in this region?*
    a. *The electric field is zero in this region*
    b. *The electric field is constant, but non-zero in this region*
    c. *The electric field is increasing in this region*
    d. *The electric field is decreasing in this region*
    e. *There is not enough information to describe the electric field in this region*

This is the same item as R7 except it deals with electric potential and distance rather than distance and time.

## 3.4 Data Collection

In this section, I will describe the methods used to collect data for the analysis of the study.

### 3.4.1 WebAssign version of the RAPT

I collected data through several different methods. The first was through WebAssign in the spring 2001 semester. Since WebAssign allows students to take the test outside of class, two instructors from sample 1 agreed to give it to their sections of introductory physics. I recreated the instrument as a WebAssign homework assignment with the exact same questions as the paper version. After the students submitted the assignments, their responses were downloaded to an Excel spreadsheet. A total of three introductory physics classes had received the instrument via WebAssign. Ideally I would have each student randomly receive either version A or version B, but this was not technically feasible. WebAssign can only randomize the questions in an assignment when all the questions have the same number of possible answers. Some items in the RAPT have 4 responses and others have 7 responses. This could be a problem if the section that receives version B is composed of students that are either better than or worse than the other two sections. This actually is not so much a problem since other schools have classes that take version B and some classes took a mix of version A and version B. For the WebAssign courses, sections received version A and one section received version B.

### 3.4.2 Paper version of the RAPT

The next phase of data collection involved students taking the paper version of the instrument. These students took either version A or version B. The instructor administering the instrument determined which version a student received.  At most schools, the two versions were given to two separate sections of the course taught by the same instructor.  The students marked their answers on a scantron sheet which was then scanned and added to the Excel spreadsheet with the WebAssign data.  The instructor administered the paper version of the instrument during class time.

**3.4.3 Think-Aloud Interviews**

Another phase of the data collection was the think-aloud interviews. I chose the think-aloud format for the interviews because it required no active participation on the part of the interviewer. This means that I could keep my bias from interfering with the student's thought process. The purpose of the interviews was to obtain supporting evidence for the relation between rate of change understanding and electric potential understanding.  To recruit interview students, I passed around a volunteer sign-up sheet to one section of introductory physics from sample 5. This  section had not taken the instrument.  I told the students these interviews were part of my dissertation research and I was investigating how students think about different topics in physics. I explained the interviews would last about 45 minutes and they would be paid $10 for participating.  About 25 students indicated they were interested in the interviews.  I emailed the volunteers and chose the ones that responded.  A total of 8 interviews were conducted.  In general, it is the above average student that volunteers for an interview.  I asked these students to report their SAT scores to determine how they compare to the average student from this sample.  Of the 8 students, 6 reported scores.  The other two could not remember how they scored on the SAT.  The SAT scores ranged from 1150 to 1350.  The reported average for students entering this university in 1999 is reported to be 1185.  It is better to have above average students than it is to have below average students.  Since I am looking for students that have difficulties with rate of change and electric potential, it seems safe to assume that if above average students have these problems, the average and below average students will have these problems also.

I scheduled the interviews for the convenience of the student.  The interviews were conducted in a room used for interviews by our research group.  Only the student and myself were present during the interviews, which were video taped for analysis.  To introduce the student to the interview, I used the following script:

*"Thank you for participating in this interview. Let me explain what we are going to do. I have given a test to a large number of students. This test looks at how students think about different topics in physics. One thing I need to know is if the student answers a certain way, what is that student thinking. The way to determine this is through interviews, so that is the purpose of this interview. The format we are going to use is called a think-aloud interview. What I am going to ask you to do is to take the test while thinking aloud. Normally when people speak, they have thoughts and they process them. They throw out nonsense thoughts and form complete sentences and then speak them. I don't want you to do that. I want you to just speak the thoughts as they come into your head. Now, I don't expect you to do this perfectly because that is not the way people talk and you are not use to it. What we will do is a think-aloud protocol with a practice question. Here are some tips for doing the think-aloud protocol. First, you want to always be talking because you are always thinking. Also, it is ok to say things that do not make sense or are off topic. If you are reading something, read it aloud. Use whatever test-taking strategies you would normally use. If you normally read the multiple-choice answers before reading the question, that is fine. If you don't know the answer, that is ok also."*

The student then goes through the practice question using the think-aloud protocol. The following question from the Force Concept Inventory (FCI) was used as the practice question.

7. A steel ball is attached to a string and is swung in a circular path in a horizontal plane as illustrated in the accompanying figure.

   At the point P indicated in the figure, the string suddenly breaks near the ball.

   If these events are observed from directly above as in the figure, which path would the ball most closely follow after the string breaks?

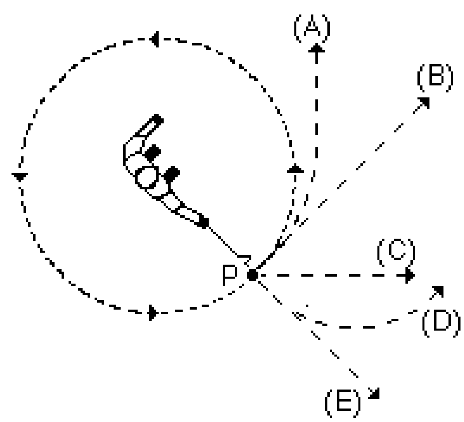



Figure 3.13 – Practice item for think-aloud interviews

The reason I chose this particular question is that it is similar to many of the items used in the RAPT, but it does not cover any of the same concepts as rate of change and electric potential. If the student had questions about the protocol or was not talking enough, we discussed how to do the think-aloud format again. The student then did think-aloud for the entire instrument with no interruption from me. Afterwards I instructed the student to explain the reason for their answers as if they were explaining them to another student.

### 3.4.4 Sub-set question of RAPT

After an initial analysis of the data collected from the interviews, WebAssign, and paper versions of the test, I found which questions students had the most difficulty with and consistently chose the same incorrect response. There were 10 such questions that students answered consistently wrong. These questions were put together in a slightly different format. The multiple-choice answers were followed by a space for the student to explain their answer choice.

In the spring 2001 semester, I gave the sub-set of RAPT items to a section of introductory physics from sample 5. The data was compared to the sub-set of questions from the full set of data for analysis. I also recorded their written explanations for later analysis.

### 3.4.5 Upper-level students

The sub-set of RAPT was also given to a class of 18 upper-level undergraduate students as describe by sample 6. The students were in an advanced, undergraduate class of electricity and magnetism. I recorded the responses as was done with the introductory physics section.

## 3.5 Analysis of the Data

I analyzed two different aspects of the data collected from the RAPT in two different aspects. In this section I will describe the methods used to analyze these two aspects of the instrument. The first set of analysis deals with the instrument itself. I will discuss the analysis of the reliability, item difficulty, and item discrimination for the instrument. The second aspect of analysis deals with how the students' answers on rate of change items relates to their answers on electric potential items.

### 3.5.1 Reliability

In any experiment, some measurements need to be taken to obtain data. Any measuring device has some error associated with it. This would be true if my experiment was to measure the height of all the people in a class using a meter stick. There is also measurement error with the use of a diagnostic instrument and the RAPT is no different. The meter stick measures length, and a reliable meter stick will consistently measure the same student as having the same height. The RAPT does not measure height, but rather it measures the use of rate of change and electric potential models in students. For the RAPT to be reliable, it should report the same score if given to the same student multiple times. The reliability coefficient is a measure of the error associated in the students' scores and its values can range from 0 to 1. A reliability coefficient of zero would indicate the instrument does not consistently measure anything while a coefficient of 1 would indicate the instrument has no measurement error. Doran[9] offers guidelines that interpret the numbers.

Table 3.1 – Guidelines for Interpreting the Reliability Coefficient

| **KR - 20** | **Reliability Description** |
|---|---|
| 0.95 – 0.99 | Very High, Rarely Found |
| 0.90 – 0.95 | High, Sufficient for Measurement of Individuals |
| 0.80 – 0.90 | Fairly High, Possible for Measurement of Individuals |
| 0.70 – 0.80 | Okay, Sufficient for Group Measurement, Not Individuals |
| Below 0.70 | Low, Useful Only for Group Averages and Surveys |

The common method of calculating the reliability coefficient for multiple-choice instruments is the Kuder-Richardson 20 (KR-20) formula[10]. The KR-20 determines the reliability coefficient by measuring the internal consistency of the instruments. If the instrument were split into two separate tests, say even numbered items and odd numbered items, then a reliable instrument would have a high correlation between the scores for the two sub-tests. The KR-20 computes the correlation between sub-test scores for all possible sub-tests. The formula for the KR-20 is

$$KR(20) = \frac{K}{K-1}\left(1 - \frac{\sum p(1-p)}{\sigma^2}\right)$$

Where

$K$ = the number of test items

$p$ = the proportion of students passing a particular item

$\sigma$ = the standard deviation for all scores

The sum is over all the test items.

### 3.5.2 Item Difficulty

To calculate the item difficulty, the students are divided into thirds based on their raw score. The item difficulty is then the average of the proportions of students from the high and low group that answer this item correctly. The high group being students from the top third of the class and the low group from the bottom third.

$$b = \frac{b_H + b_L}{2}$$

Where

$b$ = the item difficulty

$b_H$ = the proportion of students answering the item correct from the high group

$b_L$ = the proportion of students answering the item correct from the low group

An item with a high item difficulty is more difficult for students, that is that most students answer it incorrect. For an ideal instrument, each item would have a difficulty halfway between the random guessing score and 100%[11]. The reason this is the best value for the difficulty is that it leaves more room for student discriminations. If the difficulty is low, then most students will get the item correct and this item is then difficult to use to discriminate between good and poor students. The item can be described[12] by its index of difficulty[13].

**Table 3-0-1**

Table 3.2 – Guidelines for Interpreting the Index of Difficulty

| Index of Difficulty | Item Difficulty Evaluation |
|---|---|
| 0.85 to 1.00 | Very Easy |
| 0.60 to 0.85 | Moderately Easy |
| 0.35 to 0.60 | Moderately Difficult |
| 0.00 to 0.35 | Very Difficult |

### 3.5.3 Item Discrimination

The discrimination index of an item is a measure of how well that item distinguishes between the high scoring students and the low scoring students[14]. One method used to calculate the index of discrimination is the "High and Low Third System"[15] in which the students are broken into three groups according to their overall score on the instrument. The following equation is then used to calculate the index of discrimination

$$D = \frac{(H - L)}{N/2}$$

Where

$D$ = the index of discrimination for a particular item

$H$ = the number of students from the top third of the class that answered the item correctly

$L$ = the number of students from the bottom third of the class that answered the item correctly

$N$ = the total number of students

Thus if every student answered the item correctly, the index of discrimination would be zero meaning the item does not distinguish between high scoring and low scoring students. Table 3.3 presents guidelines for interpreting[16] discrimination indices

Table 3.3 – Guidelines for Interpreting the Index of Discrimination

| Index of Discrimination | Item Discrimination Evaluation |
|---|---|
| 0.40 and up | Excellent Discrimination |
| 0.30 to 0.39 | Good Discrimination |
| 0.10 to 0.29 | Fair Discrimination |
| 0.01 to 0.10 | Poor Discrimination |
| Negative | Item may be miss-keyed or intrinsically ambiguous |

**3.5.4 Correlation Between Sub-Scores**

For each student, I calculated the following; total score, rate score, potential score, rate model score, and potential model score. The rate score is simply the number of rate of change questions the student answered correctly. The potential score is the number of electric potential questions the student answered correctly. The rate model score is the number of times the student's answer to a rate of change question corresponded to the incorrect strategy that treats the rate of change as the quantity. The potential model score is the number of times the student's answer to electric potential questions corresponded to the incorrect strategy that treats the electric field as changing the same as the electric potential.

If students' responses to rate of change questions are related to their responses on electric potential questions, then there will be a correlation between the rate score and the potential score. The same would be true for the rate model score and the potential model score. The correlation coefficient is a measure of how related two variables are. It can have a value from –1 to 1. A correlation coefficient of –1 means that the two variables are perfectly inversely related, 0 means no relation between the variables, and 1 means perfectly related. I used the Pearson product-moment correlation coefficient (*r*)

$$r = \frac{\sum[(x-\bar{x})(y-\bar{y})]}{\sqrt{\sum(x-\bar{x})^2\sum(y-\bar{y})^2}}$$

Where

*r* = the Pearson product-moment correlation coefficient

*x, y* = the individual student scores for that variable

$\bar{x}, \bar{y}$ = the mean scores for that variable

Fisher's Z-transformation[17] is then used to determine if this coefficient is significantly different than zero. Fisher's Z transformation calculates the upper and lower bounds to the coefficient if the sample were increased to infinity. If zero does not lie between these bounds, then it can be said that the correlation coefficient is significantly different from zero at the 95% confidence level. The upper and lower bounds can be found by solving for ρ in the following equation

$$\tfrac{1}{2}\ln\left(\frac{1+r}{1-r}\right) \pm 1.96\sqrt{\frac{1}{n-3}} = \tfrac{1}{2}\ln\left(\frac{1+\rho}{1-\rho}\right)$$

Where

$r$ = the Pearson product-moment correlation coefficient for the given sample size

$n$ = the number of students in the sample

ρ = Pearson product-moment correlation coefficient for sample size of infinity

### 3.5.5 Comparing Two Proportions

One way to compare two different samples is to compare the proportions of students giving a particular response for a given item. A *z*-test can then be used to determine if the two proportions are significantly different. The *z*-score can be calculated with the following formula

$$z = \frac{p_1 - p_2}{\sqrt{P(1-P)(1/n_1 + 1/n_2)}}$$

Where

$$p_x = \frac{\text{responses in sample } x}{n_x} \qquad P = \frac{p_1 n_1 + p_2 n_2}{n_1 + n_2}$$

At the $p$ =0.01 level of significance, the $z$-score must be greater than 2.58 to be significant.

### 3.5.6 Comparing Two Means

There are other comparisons to make besides two proportions. I will also be comparing the means from two samples. This is the case when comparing the scores for two different samples such as the score for version A and the score for version B. For this study, to compare means, I will use the *t*-test assuming variances are unknown but equal. The following equation will be used to calculate the *t*-score.

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_p{}^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where

$\bar{x}_1, \bar{x}_2$ = the two means to be compared

$n_1$, $n_2$ = the number of students in the samples

the pooled variance, $s_p^2$ is calculated with the following formula

$$s_p{}^2 = \frac{(n_1 - 1)s_1{}^2 + (n_2 - 1)s_2{}^2}{n_1 + n_2 - 2}$$

where

$s_1$ and $s_2$ are the standard deviations of the two means

At the $p$ = 0.05 (95%) confidence level a $t$-score must be 1.96 or greater to show a significant difference. For a confidence level of $p$ = 0.01 (99% level), the $t$-score must be 2.58 or greater for a significant difference.

## Summary

In this chapter, I presented the study's research methodology. More specifically, I discussed the research design, the sample, the instrumentation, data collection, and data analysis. The Rate and Potential Test was developed to investigate a relation between how students answer rate of change questions and how they answer electric potential questions. The data for this study comes from a number of students in different introductory physics courses. I explain the specific methodology to be used to relate student answers in rate of change and electric potential as well as the statistical analysis that would be used to show significant results.

[1] http://www.carnegiefoundataion.org
[2] More information on WebAssign can be found at http://webassign.net
[3] Hake, R. R. (1998). Interactive-engagement versus traditional methods: A six-thousand-student survey of mechanics test data for introductory physics courses. American Journal of Physics, 66, 64-74.
[4] Personal conversation with Eric Brewe
[5] Thorton, R. K., and Sokoloff, D. R. (1998) Assessing student learning of Newton's laws: The force and Motion Concept Evaluation and the evaluation of active learning laboratory and lecture curricula. American Journal of Physics, 66, 338-352.
[6] Beichner, R. J. (1994) Testing student interpretation of kinematics graphs. American Journal of Physics, 62, 750-762.
[7] Trowbridge, D. E. and McDermott, L. C. (1980) Investigation of student understanding of the concept of velocity in one dimension. American Journal of Physics 48, 1020-1028
[8] O'Kuma, T. L., Maloney, D. P., & Hieggelke, C. J. (2000) Some results from the Conceptual Survey of Electricity and Magnetism. A paper presented at the winter 2001 meeting of the American Association of Physics Teachers, San Diego, CA.
[9] Doran, R. L. (1980). Basic Measurement and Evaluation of Science Instruction. Washington, DC: National Science Teachers Association.
[10] Hopkins, K. (1998). Educational and Psychological Measurement and Evaluation (Eighth Edition). Boston: Allyn and Bacon.
[11] See reference 9
[12] Hopkins, K. (1998). Educational and psychological measurement and evaluation (8th ed). Boston, MA: Allyn & Bacon.
[13] see reference 9
[14] see reference 9
[15] see reference 9
[16] see reference 12
[17] http://icp.giss.nasa.gov/education/statistics/

# Chapter 4

# Results

The main goal of this study was to investigate the possibility that difficulties with the concept of rate of change are responsible for the difficulties students have with the concept of electric potential. To accomplish this goal, I investigated the relation between student responses on rate of change items and electric potential items. A necessary step to achieve this goal was to create an instrument that contained both types of items. This instrument is the Rate And Potential Test (RAPT). In the first part of this chapter, I present data related to the investigational analysis of the instrument itself. In the second part of the chapter, I present data pertaining to the relationship between student responses on rate of change and electric potential items.

## 4.1 Item Difficulty and Discrimination

The index of item difficulty (*b)* and the index of discrimination *(D)* were calculated with the data from the multiple–choice version of the RAPT. Table 4.1 displays these values and the corresponding description of the item as described in chapter 3. The average difficulty of $b_{mean}$ = 0.60 suggests that the overall instrument is moderately difficult. The difficulty analysis revealed that item V1c to be very difficult with a *b*-value of 0.32. This indicates that students may hold a strong misconception with this item. Only one item, V1b, was found to be very easy. It should not be surprising to find item V1b to be rated as very easy, as this item asks the students to determine the location of zero potential on an equipotential line map.

The average item discrimination for the RAPT was found to be $D_{mean}$ = 0.45. Based on the guidelines outlined in the previous chapter, the instrument, as a whole, is considered to be an excellent discriminator. It should be noted that there were no items of poor discrimination. The only items with low discrimination (fair) were items V1b and V2a. Cross-referencing these items to their corresponding difficulty indices, item V1b is considered to be very easy and item V2a is moderately difficult. It seems clear that V1b has a low discrimination because it is an easy item. If most students answer a item correctly, it is difficult for that item to discriminate between good and poor students. For a typical assessment instrument, an item such as V1b should be discarded. This item has some importance for the RAPT as it was used in this study. Responses to V1b were used to insure that students were correctly reading the equipotential map. If students could not read the equipotential map, it would be impossible to get reliable information about students' ideas of how electric potential relates to electric field lines. For V2a, the discrimination along with the difficulty suggest either a misconception or a possible problem with the question.

Table 4.1 – Index of difficulty and index of discrimination for the items in the RAPT

| **Item** | ***b*** | ***b*-analysis** | ***D*** | ***D*-analysis** |
|---|---|---|---|---|
| **R1a** | 0.50 | mod difficult | 0.81 | excellent |
| **R1b** | 0.44 | mod difficult | 0.81 | excellent |
| **R1c** | 0.53 | mod difficult | 0.73 | excellent |
| **R2** | 0.50 | mod difficult | 0.38 | good |
| **R3** | 0.73 | mod easy | 0.44 | excellent |
| **R4a** | 0.81 | mod easy | 0.34 | good |
| **R4b** | 0.77 | mod easy | 0.42 | excellent |
| **R4c** | 0.78 | mod easy | 0.41 | excellent |
| **R5** | 0.73 | mod easy | 0.52 | excellent |
| **R6a** | 0.70 | mod easy | 0.47 | excellent |
| **R6b** | 0.60 | mod difficult | 0.52 | excellent |
| **R6c** | 0.71 | mod easy | 0.43 | excellent |
| **R6d** | 0.73 | mod easy | 0.43 | excellent |
| **R7** | 0.75 | mod easy | 0.41 | excellent |
| **V1a** | 0.72 | mod easy | 0.42 | excellent |
| **V1b** | 0.89 | very easy | 0.13 | fair |
| **V1c** | 0.32 | very diff | 0.31 | good |
| **V1d** | 0.38 | mod difficult | 0.50 | excellent |
| **V2a** | 0.56 | mod difficult | 0.22 | fair |
| **V2b** | 0.70 | mod easy | 0.42 | excellent |
| **V3** | 0.41 | mod difficult | 0.42 | excellent |
| **V4a** | 0.40 | mod difficult | 0.34 | good |
| **V4b** | 0.37 | mod difficult | 0.46 | excellent |
| **V5** | 0.55 | mod difficult | 0.53 | excellent |
| **V6** | 0.38 | mod difficult | 0.35 | good |
| **Average** | 0.60 | **Average** | 0.45 | |
| | mod diff | | excellent | |

## 4.2 Test Reliablity

One measure of the reliability of an instrument is found with the reliability coefficient. This coefficient was calculated for the RAPT using the Kudder-Richardson formula 20. The KR-20 reported a reliability coefficient of 0.83 with the sample of 340 students, this is considered to be unexpectedly high, especially considering that there are only 25 items.

## 4.3 Item Analysis

In this section, I will give a detailed analysis of each item. For each item, I will explain the difficulties students have as well as problems with the item itself. The sources of these explanations will be interviews and the multiple-choice data. The main goal of this section is to report on findings related to the validity of the items. There are two points of focus for this analysis. First, did the students interpret the question correctly? And second, what major difficulties did students have with this question? The quotations included are from the think-aloud interviews which I described in chapter 3. A later section will focus on how rate of change items relate to electric potential items.

#### 4.3.1 Item R1a

This is the first question from the coin toss set. The question asks about the acceleration of the coin as it is moving upwards. The correct answer is that the acceleration is in the negative direction and constant (G). There were a large number of students that answered (A), the acceleration is in the positive direction and decreasing. This choice is the one that corresponds to the all=all model, that is , the acceleration is decreasing because the velocity is decreasing. It is possible that these students are simply answering the question about velocity, but it should be noted that the terms “velocity” and “speed” are not used in this item. The following is a quote from a student interview in which it is clear that the student is not confusing acceleration and speed/velocity.

> *“I answered B because…the acceleration, the coin is moving in the positive up direction and acceleration, I think…the direction of the acceleration is the direction of the motion of the coin, of the object. It is constant because the only force acting on it is gravity and gravity’s acceleration is constant, it is 9.8* $m/s^2$*”*

This student was one of the few that answered choice (B), the acceleration is in the positive direction and constant. It seems the reason for this student’s choice of answer (B) was due to a conflict between the two models. The student states two things, first that the acceleration is in the direction of motion and second that the acceleration is due to gravity and it is constant. This student combined pieces from the two models to choose answer (B). A similar type of conflict is likely with students that chose (E), the acceleration is in the negative direction and decreasing. There was no evidence of students misreading the item. A student that misread the item would present an explanation that was not related to the situation given.

#### 4.3.2 Item R1b

This item asks the students to describe the acceleration when the coin is at its highest point. From the frequency of responses, it can be seen that almost all students chose either choice (D), the acceleration is zero, or choice (G) the acceleration is negative and constant. Looking at the interview responses, none of the students answered this item correctly, even though some had R1a and R1c correct. One student that answered R1a correctly gave the following explanation for R1b.

> *"Is it zero or negative? If it is up here, it stops. So, velocity equals zero. But, what about the acceleration? If…well…acceleration is the derivative of velocity, but…do we use this? At the top, what happens? It stops…"*

This same student also explained the question with the following quote.

> *"For over here, at its highest point, I just said that…it is zero because it completely stops and v is zero. So…basically…yeah, it is not even going up or down so it must be zero."*

The student clearly knows the relationship between velocity and acceleration as a derivative with respect to time, but it is possible the student does not really believe this definition. Another student that answered R1a correctly also had difficulties. The following quote shows that this student also distinguishes between velocity and acceleration.

> *"When the coin is at its highest point, the acceleration is still in the negative direction. The acceleration is going to be the same in all…just the velocity is going to change. I would say that the acceleration…the acceleration is going to be zero because it is at its highest point."*

For a second explanation from the same student:

> *"At the highest point, it has no acceleration. Oh well, the acceleration is still pointing downward, but the coin itself is not accelerating. I mean the force is pointing downward so the acceleration is going to be zero."*

This student has a mental conflict between the two models. The one model says that acceleration is the derivative of velocity and in this case constant. The other model says that if the velocity is zero, the acceleration must also be zero. The interviews showed no evidence that students were misreading this item.

### 4.3.3 Item R1c

This item is very similar to R1a. The choice that represents the all=all model is choice (F), the acceleration is in the negative direction and increasing. The following quote is from a student that believes the acceleration should change in the same manner as the velocity.

> *"When the coin is moving downward, on number 3, I answered (F) the acceleration is negative direction and increasing because gravity is assisting the way the coin is coming down which will increase the acceleration. It is in a negative direction because it is going down."*

In the last statement, the student states that the acceleration should be in the same direction as the motion (all=all). There was no evidence of students misreading the item.

### 4.3.4 Item R2

This item presents the student with two balls released from rest on a hill. Ball 1 is at a higher elevation, but at a part of the hill that is not very steep. Ball 2 is not as high as ball one, but at a part of the hill that is steeper. The students were asked to determine which ball will have the greater acceleration. The majority of students chose ball 2 as having the greater acceleration.

There were a number of students that chose answer (C) both balls will have the same acceleration. The reason students use for picking choice (C) can be seen in the interviews. One student that answered (C) had the following to say.

> *"I said they were the same because it would be due to gravitational…acceleration. Unless they were pushed, but from the question I assumed they were just…released."*

Two other interview students responded in a manner similar to the above student. These students answered that they would both have the same acceleration since they are both pulled by gravity.

#### 4.3.5 Item R3

This item presents the student with 5 graphs of the gallons of water in a tank as a function of time. The question asks the student to choose the tank that has the water level changing the fastest at some time. The majority of the students answered this item correctly. There did seem to be some confusion from a few of the interview students. One problem several of the students were confused with was the phrase "at some time." They were not sure how to interpret this phrase.

> *"Yeah, changing…the water level changing the fastest with time…I just…you know…I just analyzed that as the greatest slope. This one is…the problem with this one is that I got confused because it didn't specify where exactly the time…so over here, this is …it goes high and this one is pretty much the same. I just went with this one since it is big time over here."*

The interviews showed no evidence of students misreading the item.

#### 4.3.6 Items R4a, R4b, and R4c

Items R4a, R4b, and R4c present the student with a graph of the gallons of water in a tank as time goes on for two separate tanks. The items ask the student which tank is filling faster at a three different times. As can be seen by the large percentages of students answering correctly, greater than 80%, students had little trouble with this item. There was no evidence that students were misreading this item.

#### 4.3.7 Item R5

This item presents the students with a graph of position versus time for an object and asks them to choose the corresponding velocity versus time graph. Over 75% of the students answered this correctly. All of the interview students answered it correctly. There was no evidence of students misreading the question.

#### 4.3.8 Items R6a, R6b, R6c, and R6d

These items present the student with a map of isobars for a portion of the United States. The item also explains that the strongest winds are found in regions where the pressure is changing the most. For items R6a and R6b, the students are asked where the wind is the strongest and the weakest. Items R6c and R6d ask the students where the pressure is the strongest and weakest. The multiple-choice data indicate that few students had difficulties with these items, but it is certain that some students did misinterpret the isobars map. One unexpected interpretation was seen in an interview. The student determined where the greatest change in pressure was by determining which point was the furthest from a labeled isobar line.

> *"The biggest difference…this looks about the biggest difference…between 1010 and 1020, that looks about 8 or 9. D is the biggest and C and A…"*

This student chose D as the location where the pressure was changing the most. The student estimated that it was about 8 or 9 mb from the 1010 line to point D, thus point D represented the region with the greatest change. Another student interpreted the isobar map as though the isobar lines were a series of graphs. For this student the strongest winds were found in regions with the "greatest slope". With this student's model, the "greatest slope" would be the line that has a segment pointing the most North-South.

> *"I found that the stronger wind, is when the pressure is higher..so..basically that takes care of the second part which is finding the wind. Same answer. Wind strongest, that is just the greatest slope again which is C. And then the lowest one, which is…I chose D which is pretty much zero and this is the low one goin down and this is …kinda…yeah its actually a little going down, A. D looks pretty much the same."*

#### 4.3.9 Item R7

This item asks the students about the acceleration of a ball that is rolling at a constant velocity. Most of the multiple-choice students answered this item correctly. All of the interview students answered the item correctly and displayed no evidence that they did not understand the question.

#### 4.3.10 Item V1a and V1b

These items present the students with a map of equipotential lines. They are similar to the isobars questions R6a through R6d. Items V1a and V1b ask the student to choose the points where the electric potential is the greatest and where it is zero. There was no evidence in either the multiple-choice data nor in the interviews that indicated students had difficulties with these items.

#### 4.3.11 Item V1c

This item asks the students to determine the point at which the magnitude of the electric field is the greatest. The correct choice is point C where the electric potential is changing the most, this is also the point that the electric potential is zero. Points A and C were the two most popular choices in the multiple-choice data. Choice A is the point at which the electric potential is the greatest. In the interviews, not one student answered this item correctly. Six interview students answered A, and two students answered B. It was clear that one student was attempting to use the equation for the electric field due to a point charge to determine the answer.

> *"At which of the points is the magnitude of the electric field…electric field is kq/r², it is the same as the electric potential. It depends on distance…oh, wait…the closer you are to the charge, the greater the electric field. That would be …I guess, A. The electric potential is 90 volts, the electric field is the greatest there. It is closest to the charge."*

This student related electric potential and electric field strength to the distance from the charge. Although no charges are shown in the diagram, many students see that there is likely a charge near the 90 volt line. The problem with the above student's reasoning is that his strategy does not work when there is more than one charge present. There was no evidence of students misreading this item.

### 4.3.12 Item V1d

This item is similar to V1c except that it asks the students to choose the point at which the magnitude of the electric field is zero. Since there is no location at which the potential remains constant, there is no point at which the electric field is zero. The correct choice is E, none of the points. Nearly 50% of the students chose point C as having zero electric field. It is at this point that the electric potential is zero. From the interviews, every student answered that at point C the electric field was zero.

> *"The electric potential is zero, then the electric field is zero."*

Another student answered

> *"Ok, equal to zero…obviously B is not equal to zero. If it were the weakest, then I would have trouble between B and C, but since it says zero….I am going to go with C being zero and B being furthest away. The greatest is going to be B."*

This student seems to understand the relation between electric potential and electric field, for he correctly states that point B would be the weakest electric field. He seems to choose point C though because it is at zero electric potential. There was no evidence of students misreading this item.

### 4.3.13 Item V2a

This item presents students with a set of equipotential lines that change only in one dimension. The students are asked to predict the motion of a proton that is released from rest at a point where the electric potential is zero, but the electric field is not zero. One thing that I observed in the interviews was that students often associate the negative potential with a negative charge. This being done, it was a simple matter for the students to say that the proton is attracted to the negative charge.

*"It's a positive charge…a proton moves to negative charges…so it moves to the left."*

This is a case of the student choosing the correct answer, but not using the correct strategy. This reasoning will not work for all cases. Another strategy used by students is to say the proton will want to go to lower potentials. While this is not incorrect, it does not show that the student understands the relationship between electric potential and electric field. Some students did choose the answer that the proton will not move. The common explanation for such a choice was that since the electric potential is zero, the proton can not move.

*"The proton will not move because it is at zero volts. There is no electric potential, there is no force on the charge."*

Another student believed that the proton wants to move to the lowest potential and zero is this lowest potential.

*"The electric potential is zero. If it is zero, then it doesn't really want to move anywhere because that is the lowest potential it can go to. Or, it wants to be at zero potential…it doesn't want to move…inertia. Or there isn't a force acting on it."*

Although students had difficulty with this item, there was no evidence that they were misreading it.

**4.3.14 Item V2b**

This item asks the student to describe the motion of a proton released from rest at a location where the potential is not zero and not constant. Most students answered this one correctly. It is possible that the reason many students answered this correctly is that there are several strategies that will lead to the right answers including incorrect strategies. One such incorrect strategy says that the proton is attracted to negative potential. Another strategy that is not incorrect says that there must be a negative charge in the region of negative potential and the proton must be attracted to this charge.

*"The proton will move to the left because it has to go to zero volts. It has to go to lower potential."*

According to the above student's explanation, a will go to zero volts rather than the lowest potential. All of the interview students answered this one correctly.

**4.3.15 Item V3**

This item is analogous to item R3. The students are presented with five graphs of electric potential versus distance and asked to chose the graph with the greatest electric field at some point. The two most popular choices were choice A, where the potential is the greatest and choice D, the correct choice. It is possible that some students answered this based on their answer to item R3 as the following interview student did.

*"[This item] is basically the same…as the last question. So it has to be the same thing. Change here…is constant…so the graph…I think I did this wrong. This one, even though it is constant, is greater than any of these. Where is electric potential…electric field here is constant, or zero. The electric potential is constant…the electric field is zero because it is the slope. I basically…I did not think much about this one, I just did the same thing as the last one."*

The student was confused about the relation between electric potential and electric field, but decided to go with the choice that was similar to his choice for item R3. There was no evidence that students were misreading this item.

### 4.3.16 Item V4a

This item presents the students with a graph of electric potential versus distance. The students are asked to describe the motion of a proton released from rest at a location where the electric potential is zero, but the electric field is non-zero. Conceptually, this is the same question as V2a but with a different representation. In this item, many interview students demonstrated their belief that a zero potential means that there must be zero force. This item shows that students can be unaware of the relationship between electric field and electric potential.

*"I was kinda guessing. I figured it was at zero potential and would not move. Then … if its…I want to say if it is released, it goes to a lower potential but that would mean that they would move to the left. I can't remember the relationship between electric potential and electric field."*

One student was clear in her explanation that the proton would not move.

*"For number 4, I said A, that the proton will not move because…at x equal 2 the electric potential is zero. If the electric potential is zero, then the charge will not move…any charge will not move. The electric potential must be negative or positive, a non-zero number for the charge to move."*

Another student expressed similar beliefs

*"Potential…it is going to be zero. So, there is no potential, there is no motion. If it had some potential…it would…potential here is zero, so it will not move. Not move, zero."*

There was no evidence that students were misreading this item.

### 4.3.17 Item V4b

This item asks the students to predict the motion of a proton released from rest at a location where the electric potential is not zero, but the electric field is zero. The correct choice is A, the proton will not move. Choice B, the proton will move to the right, and choice A, the proton will move to the left can be considered the same student difficulty. Both of these choices, A and B, basically say that the proton will move. A common reason for students choosing choice A or B is that if the potential is not zero, then the motion can not be zero. Some student believe that the proton would move, but could not figure out which way so they resorted to choice D, that there was not enough information to determine the direction of motion.

> *"If it is at maybe one volt potential…it can't be determined. I think it will go to a lower potential. I don't know this stuff."*

This student also expressed a lack of knowledge of the relationship between electric potential and electric field. The following student is an example of one that believes that if the potential is not zero, then it must move.

> *"x= 0, its negative 1. The potential is negative. What will be the motion of the proton. The proton will move to the left or to the right. B or C. Its negative, it is moving…The proton will move to the left, move to the left. I think the electron wants to go to the lower potential, the lowest potential. It is already at the lowest potential. lowest potential. ahhhh....It will move...move to the left? It will move left I guess. C."*

There was no evidence that students were misreading this item.

### 4.3.18 Item V5

This item is the same as R5 except that the students are given a graph of electric potential versus distance and asked to choose the graph of the magnitude of electric field as a function of distance. It should be noted that the order of the answers is different than for R5 for no particular reason. Compared to R5, fewer students answered this correctly. There was, however, no clear distracter that the students favored. One student clearly answered this question based on his response to item R5.

> *"For number 13, I …I followed the same logic as the last one and I picked the same answer because….it was driving apon the last question…that the slope…even though we have not studied this…I just assumed that the slope is the electric field. Where electric potential is the distance graph."*

Other than having no popular distractor, there was no evidence that students were misreading this item.

### 4.3.19 Item V6

This item asks the students about the electric field in a region described by a constant, non-zero electric potential. The correct response is A, the electric field is zero. The other popular answer is B, the electric field is a constant, non-zero value. It is possible that some students answered this based on their answer to item R7, which is similar. One student shows a conflict between the correct reasoning and the reasoning that if the potential is not zero, the electric field can not be zero.

> *"If the electric potential is a constant non-zero value is the electric field zero? Yes because equipotential lines…is at a constant…there is no…what is the definition of equipotential lines. Lines along which the electric potential remains the same. If the electric potential remains the same, the electric field remains the same. So, electric field is constant, but non-zero in this region. If the electric field were zero, there would be no electric potential. If the field is constant...? It is either A or B, zero or constant. If it is zero, then the potential would have to be zero. If it is constant however, the potential will be constant - so it is B."*

This same student also gave the following explanation.

> *"For 25, I said B - the electric field is constant but non-zero in this region because…I think I should have gone with A for this one. If I am using the same graphs as the distance vs. time for this one then I should be relying the answers for this one. But I picked constant but non-zero for this region because...I thought that you would have to have an electric field in order to have an electric potential."*

There was no evidence that students were misreading this item.

## 4.4 Does WebAssign Make a Difference?

One of the main sources of data from the RAPT was through student submissions on WebAssign. The primary reason I chose to use WebAssign was that this method is very "non-invasive" to the instructors. I could not force any instructor to give the RAPT to his or her students, but with this method it would not take any class time and would not require any extra effort on their part. The use of online testing is becoming more common not only because it is convenient, but also because it can do things that paper tests cannot. It has been shown in a previous study[1] that students perform similarly on web-based testing and paper-based testing. I needed to investigate this for the RAPT data as well to make sure that it could be grouped with the paper-based data.

A perfect comparison between WebAssign and paper-based RAPT can not be made. The students that took the WebAssign version were all from one of three sections from sample 1 in a traditional introductory physics course. The paper-based sample of RAPT includes students from different schools, but not schools that are very similar to sample 1. To address this problem, several comparisons between WebAssign and paper-based RAPT scores were made. For each comparison, the mean score, the rate-score and the potential score were compared using a *t*-test. The mean rate-score is the average number of correct responses on the rate of change questions out of a possible 14. The mean potential-score is the number of correct potential responses out of a possible 11.

In an ideal study, I would randomly assign students to take either the online version or a paper version and then compare the scores.  Since this research question was not the main goal of this study, I did not carry out this experiment.

### 4.4.1 Comparison 1: WebAssign vs. Paper

The first comparison group is between the students that took the RAPT on WebAssign and those that took it on paper.  I left out the students from sample 4 for it was clear that the class was not a “traditional” lecture style and that was likely to affect their scores as has been shown in the case of the *Force Concept Inventory*[2] (FCI).  Looking at the *t*-scores it can be seen that there is no significant difference between the two groups at the 95% confidence level.

Table 4.2 – Summary statistics for the WebAssign Group and the Paper Group

| | n | Mean Score | Mean Rate-score | Mean Potential-score |
|---|---|---|---|---|
| WebAssign | 224 | 12.30 | 6.86 | 5.44 |
| Paper | 74 | 12.37 | 7.45 | 5.28 |

Table 4.3 – Comparison of Scores between the WebAssign Group and the Paper Group

| | ***t*-score** |
|---|---|
| **Mean Score** | -0.74 |
| **Mean Rate-score** | -1.64 |
| **Mean Potential-score** | 0.49 |

### 4.4.2 Comparison 2: Sub-test WebAssign vs. Sub-test Paper

The only data from students from the same institution as sample 1 but on paper was from the sub-test version of RAPT.  The sub-test version is 10 items from the original RAPT where students displayed use of incorrect strategies.  One section of introductory physics from sample 5 took the sub-test version of RAPT.  Since I was not sure if version made a difference, only version A of WebAssign RAPT will be compared since that is the same order as the sub-test version.  Note that now the mean score is out of a possible 10 points.  The mean rate-score is out of a possible 3 points and the mean potential-score is out of 7 points.  It can be seen that on the sub-test comparison with students from similar conditions there is no significant difference between the means at the 95% confidence level which corresponds to a *t*-score of 1.97.

Table 4.4 – Summary statistics for the sub-test scores from the WebAssign Group and the Paper Group

| | n | Mean Score | Mean Rate-score | Mean Potential-score |
|---|---|---|---|---|
| WebAssign | 146 | 4.46 | 1.17 | 3.29 |
| Paper | 55 | 4.33 | 1.33 | 3.00 |

Table 4.5 – Comparison of Scores on the sub-test for the WebAssign Group and the Paper Group

| | *t*-score |
|---|---|
| **Mean Score** | 0.34 |
| **Mean Rate-score** | -0.74 |
| **Mean Potential-score** | 1.16 |

## 4.5 Does Version Make a Difference?

There are two reasons for making different versions of the instrument. The first reason is to determine if students see questions that use the same rate of change reasoning but are in different context as being similar. The second reason I made two versions of the RAPT was to see if students could "learn" during the test. In version B of the instrument, the students see a rate of change question followed by an electric potential question that uses the same rate of change concepts. If the students were in fact 'learning", then there should be a difference between version A and version B.

For the comparisons between the versions, I have used all the data except for the sub-test data and the sample 4 data. The sub-test data was not really version A or version B so it makes sense to leave it out. The sample 4 data, as explained previously, was not a traditional class. Another factor for leaving the sample 4 data out was that they only took version A. If they had taken both versions it would make sense to include their responses as any benefits they would have would balance out any possible effects of the active-engagement aspect of the course.

I compared the two versions in several ways, the first was to simply look at the mean score, potential-score and rate-score. Looking at just these scores may hide some of the features I am looking for. I am not just looking to see which version did better, but also to see if version influenced the way the students responded to the items. I compared the proportions of students choosing each response for each item. The significance of these proportions can then be compared using a *z*-test. For all the items there are 127 different responses, thus there are 127 *z*-tests between proportions. For a confidence level of 95%, there is a 5% chance for a false-positive test. With 125 such tests, an average of 6 of these test will turn out as significant due to random chance. To lower the possibility of these false-positives, I will use a 99% confidence level test ($p$ = 0.01). For the $t$-test to show a significant difference, the $t$-score must be greater than 2.58.

Table 4.6 – Summary statistics for Version A and Version B of the RAPT

| | n | Mean Score | Mean Rate-score | Mean Potential-score |
|---|---|---|---|---|
| Version A | 188 | 15.02 | 9.28 | 5.73 |
| Version B | 111 | 13.77 | 8.95 | 4.83 |

Table 4.7 – Comparison of scores for Version A and Version B of the RAPT

| | ***t*-score** |
|---|---|
| **Mean Score** | 2.11 |
| **Mean Rate-score** | 0.83 |
| **Mean Potential-score** | 3.21 |

The $t$-test reveals that both the mean score and the mean potential-score are significantly different at the 99% confidence level. The rate-score, however, is not significantly different. A difference in rate-scores would likely indicate that the two groups were not similar. The items I am interested in are the electric potential items. I suspect that the rate of change items may have an influence on the electric potential items but the electric potential items should not influence the rate of change items. The mean score is significantly different because of the difference in potential-scores.

The comparison of distributions of responses shows three electric potential items that had significant differences in the proportion of students answering a particular response. There were no rate of change items with significant differences.

Table 4.8 – The number of students giving a particular response to each question. The *z*-score for the difference between the proportion of students answering each response is also included.

| Response | R1a | R1b | R1c | R2 | R3 | R4a | R4b | R4c | R5 |
|---|---|---|---|---|---|---|---|---|---|
| a | 77 | 0 | 0 | 39 | 8 | 159 | 148 | 27 | 12 |
| b | 7 | 2 | 7 | 80 | 31 | 23 | 4 | 155 | 2 |
| c | 1 | 0 | 6 | 52 | 9 | 4 | 35 | 3 | 10 |
| d | 1 | 106 | 0 | 17 | 139 | 1 | 1 | 2 | 146 |
| e | 18 | 1 | 5 | | 1 | | | | 18 |
| f | 3 | 5 | 80 | | | | | | |
| g | 81 | 74 | 90 | | | | | | |
| Total N | 188 | 188 | 188 | 188 | 188 | 187 | 188 | 187 | 188 |
| | | | | | | | | | |
| Response | R1a | R1b | R1c | R2 | R3 | R4a | R4b | R4c | R5 |
| a | 45 | 0 | 2 | 21 | 3 | 94 | 87 | 18 | 7 |
| b | 5 | 0 | 7 | 55 | 10 | 15 | 2 | 88 | 2 |
| c | 2 | 1 | 7 | 31 | 8 | 2 | 19 | 3 | 15 |
| d | 0 | 70 | 0 | 4 | 87 | 0 | 3 | 2 | 80 |
| e | 9 | 0 | 3 | | 3 | | | | 7 |
| f | 4 | 3 | 43 | | | | | | |
| g | 46 | 36 | 48 | | | | | | |
| Total N | 111 | 110 | 110 | 111 | 111 | 111 | 111 | 111 | 111 |
| | | | | | | | | | |
| a | 0.07 | NA | -1.85 | 0.38 | 0.69 | -0.03 | 0.07 | -0.43 | 0.03 |
| b | -0.33 | 1.09 | -1.02 | -1.17 | 1.82 | -0.32 | 0.19 | 0.68 | -0.54 |
| c | -1.66 | -1.30 | -1.28 | -0.05 | -0.87 | 0.19 | 0.33 | -0.66 | -2.47 |
| d | 0.77 | -1.13 | NA | 1.78 | -0.86 | 0.77 | -1.58 | -0.54 | 1.09 |
| e | 0.43 | 0.77 | -0.02 | | -1.58 | | | | 0.99 |
| f | -1.11 | -0.02 | 0.65 | | | | | | |
| g | 0.28 | 1.20 | 0.78 | | | | | | |
| Correct Response | **g** | **g** | **g** | **c** | **d** | **a** | **a** | **b** | **d** |

Note: The gray cells indicate a significant difference ($p < 0.01$) between the two proportions of students answering that particular response.

Table 4.8 - continued

| | Response | R6a | R6b | R6c | R6d | R7 | V1a | V1b | V1c | V1d |
|---|---|---|---|---|---|---|---|---|---|---|
| Version A | a | 139 | 24 | 24 | 149 | 152 | 131 | 3 | 57 | 10 |
| | b | 4 | 16 | 4 | 2 | 23 | 25 | 1 | 29 | 10 |
| | c | 25 | 116 | 10 | 20 | 4 | 22 | 160 | 76 | 75 |
| | d | 14 | 27 | 145 | 14 | 4 | 6 | 5 | 12 | 5 |
| | e | 6 | 4 | 2 | 3 | 5 | 4 | 18 | 10 | 88 |
| | f | | | | | | | | | |
| | g | | | | | | | | | |
| | Total N | 188 | 187 | 185 | 188 | 188 | 188 | 187 | 184 | 188 |
| | | | | | | | | | | |
| Version B | Response | R6a | R6b | R6c | R6d | R7 | V1a | V1b | V1c | V1d |
| | a | 81 | 12 | 17 | 79 | 83 | 76 | 3 | 43 | 10 |
| | b | 5 | 9 | 4 | 3 | 13 | 16 | 4 | 26 | 2 |
| | c | 12 | 73 | 7 | 18 | 3 | 12 | 96 | 24 | 75 |
| | d | 9 | 15 | 79 | 8 | 6 | 6 | 4 | 12 | 6 |
| | e | 4 | 2 | 4 | 3 | 5 | 1 | 4 | 6 | 18 |
| | f | | | | | | | | | |
| | g | | | | | | | | | |
| | Total N | 111 | 111 | 111 | 111 | 110 | 111 | 111 | 111 | 111 |
| | | | | | | | | | | |
| Z-score | a | 0.18 | 0.50 | -0.62 | 1.59 | 1.24 | 0.22 | -0.66 | -1.49 | -1.23 |
| | b | -1.16 | 0.12 | -0.76 | -1.07 | 0.13 | -0.27 | -2.00 | -1.72 | 1.50 |
| | c | 0.63 | -0.70 | -0.36 | -1.40 | -0.32 | 0.23 | -0.33 | 3.33 | -4.62 |
| | d | -0.21 | 0.20 | 1.15 | 0.08 | -1.52 | -0.94 | -0.46 | -1.36 | -1.22 |
| | e | -0.19 | 0.19 | -1.51 | -0.66 | -0.86 | 0.80 | 1.91 | -0.03 | 5.34 |
| | f | | | | | | | | | |
| | g | | | | | | | | | |
| Correct Response | | **a** | **c** | **d** | **a** | **a** | **a** | **c** | **c** | **e** |

Note: The gray cells indicate a significant difference ($p < 0.01$) between the two proportions of students answering that particular response.

Table 4.8 - continued

| | Response | V2a | V2b | V3 | V4a | V4b | V5 | V6 |
|---|---|---|---|---|---|---|---|---|
| Version A | a | 43 | 8 | 70 | 57 | 71 | 12 | 68 |
| | b | 20 | 33 | 18 | 75 | 52 | 9 | 68 |
| | c | 109 | 129 | 29 | 43 | 36 | 24 | 20 |
| | d | 15 | 18 | 67 | 12 | 29 | 38 | 12 |
| | e | | | 4 | | | 104 | 18 |
| | f | | | | | | | |
| | g | | | | | | | |
| | Total N | 187 | 188 | 188 | 187 | 188 | 187 | 186 |
| | | | | | | | | |
| Version B | Response | V2a | V2b | V3 | V4a | V4b | V5 | V6 |
| | a | 46 | 6 | 32 | 44 | 29 | 10 | 41 |
| | b | 15 | 22 | 9 | 32 | 28 | 9 | 46 |
| | c | 44 | 73 | 15 | 25 | 28 | 21 | 9 |
| | d | 6 | 9 | 50 | 10 | 25 | 20 | 5 |
| | e | | | 5 | | | 51 | 7 |
| | f | | | | | | | |
| | g | | | | | | | |
| | Total N | 111 | 110 | 111 | 111 | 110 | 111 | 108 |
| | | | | | | | | |
| Z-score | a | -3.39 | -0.45 | 1.48 | -1.65 | 2.06 | -0.84 | -0.13 |
| | b | -0.75 | -0.49 | 0.43 | 1.93 | 0.46 | -1.17 | -0.91 |
| | c | 3.07 | 0.51 | 0.45 | 0.07 | -1.24 | -1.44 | 0.71 |
| | d | 0.84 | 0.43 | -1.61 | -0.84 | -1.54 | 0.46 | 0.68 |
| | e | | | -1.16 | | | 1.57 | 0.99 |
| | f | | | | | | | |
| | g | | | | | | | |
| Correct Response | | **c** | **c** | **d** | **b** | **a** | **e** | **a** |

Note: The gray cells indicate a significant difference ($p < 0.01$) between the two proportions of students answering that particular response.

The *z*-test shows that there are three items that had significantly different proportions of students choosing a response. The details of these items are found below.

### 4.5.1 Item V1c

This item asks the students to chose the point with the greatest magnitude of electric field given a map of equipotential lines. Choice C is the correct response for the equipotential lines are closest together there. A response that was a common choice for students was choice A. It is at this point that the value of the electric potential is the highest. Comparing the proportion of responses for choice C shows that the version A students chose this significantly more than the version B students.

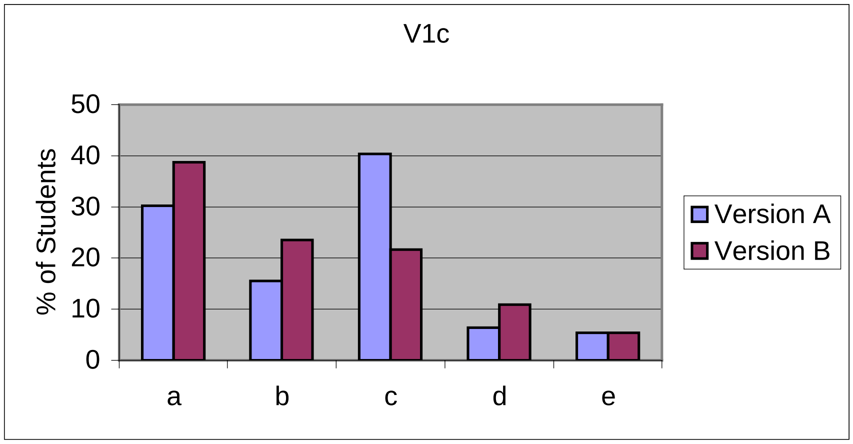



Figure 4.1 – Distribution of student answers for item V1c

**4.5.2 Item V1d**

This item asks the students to choose the point at which the electric field is zero in the given equipotential map. Since there is no location that shows a constant electric potential, there is no location where the electric field is zero. The correct answer is choice E. The choice associated with the zero=zero model is choice C. At this point the electric potential is zero, but the electric field is not. The version A students answered choice E significantly more than the version B students. Also, a significant proportion of version B students chose choice C more than the version A students. So, the version A students were more likely to choose the correct response and the version B students were more likely to choose the response associated with the common misconception.

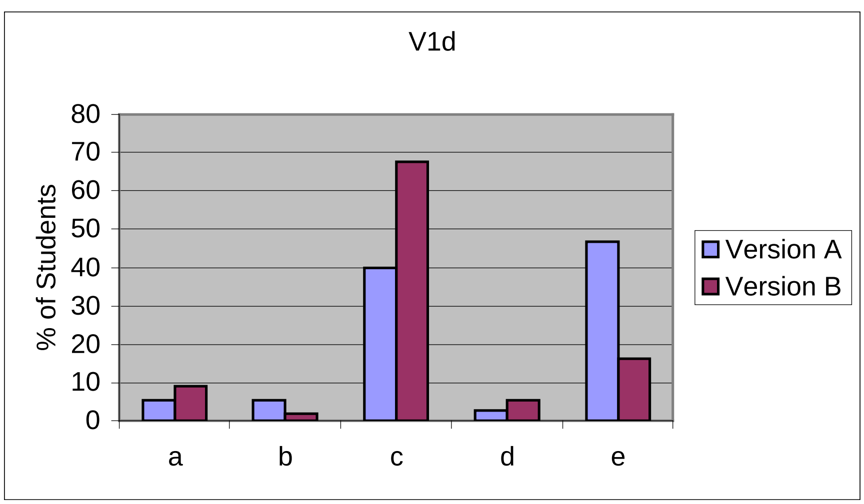



Figure 4.2 – Distribution of student answers for item V1d

**4.5.3 Item V2a**

This item presents the students with a proton released from rest at a location where the electric potential is zero, but the electric field is not zero. The correct answer is choice B, the proton will move to the left. The common incorrect response is choice A, the proton will not move. For this item, the version A students selected choice C significantly more than version B. Version B students picked choice A significantly more than those taking version A.

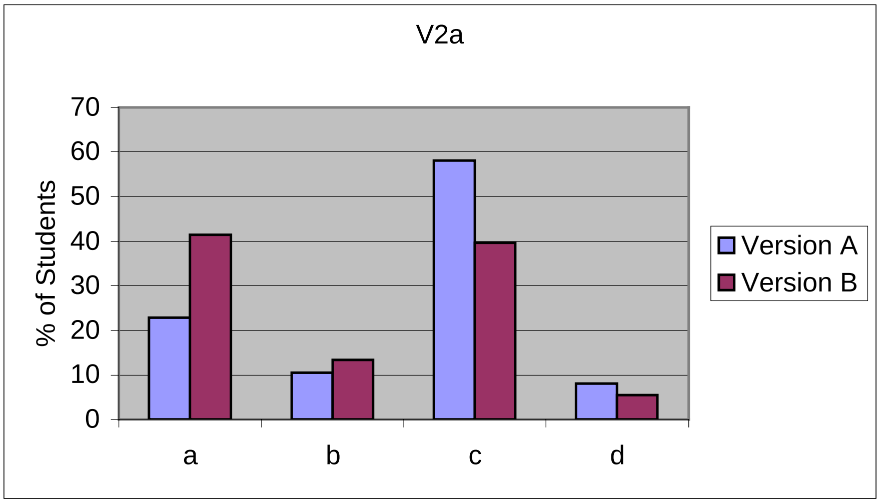



Figure 4.3 – Distribution of student answers for item V2a

## 4.6 Interactive Classes versus Traditional Classes

Sample 4 was the only class that would be considered "non-traditional". Since it has been shown that interactive-engagement methods produce higher gains on the Force Concept Inventory, I was curious to see if this was true also for the RAPT. There is a difference though, Hake's study of interactive-engagement courses compared the gain on the FCI, not the score on the FCI. The gain is calculated with the following formula

$$< h >= \frac{x_f - x_o}{100 - x_o} = \frac{\text{Total Gain}}{\text{Possible Gain}}$$

Where

$x_f$ = the post-test score as a percent

$x_o$ = the pre-test score as a percent

The RAPT, however, is not designed to measure learning over a given interval thus there is no pre/post test. It could be that any difference between the two is due to differences in population and not due to differences in course style.

The first comparison I made was to compare the scores for the active-engagement class with the rest of the data from RAPT. A *t*-test indicated that overall mean, and the rate-score were both significantly different. This led me to further investigate the individual items to see on which items a significant difference could be found. A *z*-test to compare proportions was used to examine the distributions of responses for the active-engagement group and the rest of the students. Seven items have significantly different proportions of responses from the two groups.

Table 4.9 – Summary statistics for the active-engagement classes and the traditional classes

| | **n** | **Mean Score** | **Mean Rate-score** | **Mean Potential-score** |
|---|---|---|---|---|
| Active-engagement | 42 | 16.9 | 11.1 | 5.83 |
| Traditional | 298 | 13.98 | 8.58 | 5.4 |

Table 4.10 – Comparison of scores for active-engagement courses and traditional courses

| | ***t*-score** |
|---|---|
| **Mean Score** | 3.72 |
| **Mean Rate-score** | 4.72 |
| **Mean Potential-score** | 1.11 |

Table 4.11 - The number of students giving a particular response to each question. The *z*-score for the difference between the proportion of students answering each response is also included.

| | Response | R1a | R1b | R1c | V1c | V1d | V4b |
|---|---|---|---|---|---|---|---|
| Active Engagement | a | 5 | 0 | 0 | 32 | 0 | 11 |
| | b | 1 | 1 | 1 | 2 | 2 | 9 |
| | c | 1 | 0 | 0 | 7 | 32 | 6 |
| | d | 0 | 12 | 0 | 1 | 0 | 16 |
| | e | 2 | 0 | 1 | 0 | 8 | |
| | f | 2 | 1 | 10 | | | |
| | g | 31 | 28 | 30 | | | |
| | Total N | 42 | 42 | 42 | 42 | 42 | 42 |
| | | | | | | | |
| Traditional | Response | R1a | R1b | R1c | V1c | V1d | V4b |
| | a | 122 | 0 | 2 | 99 | 20 | 100 |
| | b | 12 | 2 | 13 | 55 | 12 | 80 |
| | c | 3 | 1 | 13 | 100 | 149 | 64 |
| | d | 1 | 175 | 0 | 24 | 11 | 53 |
| | e | 27 | 1 | 8 | 16 | 106 | |
| | f | 7 | 8 | 123 | | | |
| | g | 126 | 110 | 138 | | | |
| | Total N | 298 | 297 | 297 | 294 | 298 | 297 |
| | | | | | | | |
| z-score | a | **-3.64** | NA | -0.53 | **5.36** | -1.73 | -0.95 |
| | b | -0.52 | 1.11 | -0.61 | -2.22 | 0.22 | -0.75 |
| | c | 0.77 | -0.38 | -1.38 | -2.21 | **3.18** | -1.08 |
| | d | -0.38 | **-3.68** | NA | -1.32 | -1.27 | **3.06** |
| | e | -0.93 | -0.38 | -0.11 | -1.54 | -2.12 | 0.99 |
| | f | 0.91 | -0.11 | -2.17 | | | |
| | g | **3.84** | **3.68** | **3.05** | | | |

Note: The gray cells indicate a significant difference ($p < 0.01$) between the two proportions of students answering that particular response.

The *z*-test shows that there are 6 items with significantly different responses for the two groups.

## 4.7 Upper-Level Students

Eighteen students from an advanced undergraduate physics course in sample 6 took the sub-test version of the RAPT. The sub-test version consists of 10 items that consistently showed student difficulties. I was interested to see if students that have been successful in physics had difficulties with rate of change and electric potential. Since there were only 18 students in the upper-level group, I did not do the same comparative statistics as I did with the comparison between different versions of the instrument. Instead, I will present the interesting points.

- Of the 18 students, 11 did well on the instrument. By well, I mean that if they missed an item, it was clear from their explanation that they had made a simple mistake.
- Four students used the model that acceleration changes the same as velocity with the coin toss question (items R1a – R1c).
- On items V4a and V4b, many students used the strategy that the proton will want to go to a lower potential. This is not incorrect, but it is not an explanation in terms of the changing electric potential.
- Only two students explicitly indicated that the proton moved because there was a changing electric potential that created an electric field for item V4a.
- Three students explicitly explained that if the electric potential is zero, there can be no motion (item V4a).
- For item V6, all but one student answered correctly. It may not be surprising to find out that they all explicitly stated that $\mathbf{E} = -\nabla V$ in their explanation.
- On items V1c and V1d, four students answered in a manner that indicated they believe that the electric field and the electric potential must have similar values. One student wrote *"high potential implies high E field"* for his explanation. This same student also wrote *"the potential over this region clearly has a non-zero gradient"* as an explanation for item V1d.

## 4.8 Correlation Between Rate and Potential Scores

One of the primary goals of the RAPT is to investigate the relation between students' rate of change responses and their electric potential responses. As stated previously, other scores were calculated besides the total score. The rate-score is the number of rate of change questions a student answered correctly. The potential-score is the number of electric potential questions the student answered correctly. If there is indeed a connection between rate of change responses and electric potential responses, then there should be a correlation between a students' rate-score and the students' potential-score. The rate and potential correlation is not the only thing to look at, for this is only a measure of how often students get the correct answer. The other side of this is to look at how often students use the incorrect model. The rate-model score is the number of times a student uses the strategy that treats a quantity and its rate of change as similar. The potential-model score is the number of times that a student uses the strategy that electric potential is proportional to the electric field. It should be noted that the term "model score" refers to the use of an incorrect model.

The correlation between rate-score and potential score was found to be 0.45. This is significantly different from 0.0 at the 95% confidence level indicating that there is some correlation between the two variables. The correlation between rate-model score and potential model score was found to be 0.38 and it is also significantly different than 0.0.

## 4.9 Correlation Between Rate and Electric Potential Sub-Groups

In section 4.8, I examined the correlation between rate of change sub-scores and electric potential sub-scores. While this comparison does show a connection between student understanding of rate of change and electric potential, it does not show much detail in how they relate the two concepts. To further investigate this connection, I used the rate of change questions to break students into sub-groups and then compared how these groups answered on electric potential questions. This comparison is then similar to the comparison between the two versions of the RAPT in that I will compare the distributions of responses and not just the percentages of students that answered it correctly. I created sub-groups for two of the cases stated previously, the all=all case and the zero=zero case. The other cases did not contain items that could be used to discriminate between sub-groups. The great=great case did not contain a great=great question that would define the sub-groups very well. All the rate of change questions that fell in the great=great categories had lower difficulties. It is difficult to distinguish between students that use one model or the other with items that most students answer correctly. The no-zero=no-zero case items have been combined with the zero=zero items. I combined these two cases because a preliminary analysis indicated that the best zero=zero item, R1b, correlated well with the no-zero=no-zero electric potential items.

### 4.9.1 Sub-Group for Case: All=All

Students that fall into the all=all case use the model that the rate of change of a quantity changes the same as the quantity itself. An example of this case in kinematics can be seen in item R1a. The question asks about the acceleration of a coin as it is moving upwards after it has been thrown. The student that is in the all=all sub-group would describe the acceleration as being in the positive direction and decreasing because this is how the velocity is changing. Students in this group should not include those that think that the acceleration and the velocity are the same thing, just that they change the same way. The interviews gave no evidence that students confuse the terms velocity and acceleration. None of the 8 students that were interviewed treated velocity and acceleration as the same thing. The same can be said for confusing electric potential and electric field. Again, none of the interview students indicated that they believed these two to be the same quantity.

Table 4.12 – Items in the All = All case including the correct response and the response that corresponds to the All = All model.

| Case: All=All items | Correct response | R-model response |
|---|---|---|
| R1a | G | A |
| R1b | G | D |
| R1c | G | F |
| R4a | A | B |
| R4b | A | C |
| R4c | B | A |
| R5 | D | E |
| V5 | E | D |
| V6 | A | B |

I broke students into sub-groups based on their responses to 7 rate of change questions. Note that I did not include all of the rate of change questions, only the items that included the all=all responses. There were four possible sub-groups a student could be put into.

**r-correct group:** This group of students generally use the correct model relating the rate of change to its quantity. Students in this group answered 5 of the 7 rate of change questions correct and did not answer choice D for R1b, nor C for R4b, nor E for R5. Note that a student does not have to score perfectly to be in this group, this accounts for students that make mistakes.

**r-model group:** This group of students use the all=all model to answer the items. Specifically, a student is put in this sub-group if he/she had a r-model score of 2 or more and answer D for R1b. The r-model score is the number of times the student used the all=all model answer.

**mixed group:** The students in this group sometimes use the correct model and sometimes the r-model. To be in this group, the student had to have a rate-score greater than 4 and answer D for R1b

**null group:** These students do not fall in any of the other three groups. The students in this group were likely either guessing or completely confused.

Table 4.13 – A count of students that were classified as using one of the mental models.

| Group | Number of students |
|---|---|
| R-correct | 120 |
| R-model | 118 |
| Mixed | 30 |
| Null | 72 |

**4.9.2 Comparing the R-correct and R-model Sub-Groups.**

I compared the responses on the electric potential questions for the students in the r-correct and r-model sub-groups. The comparison techniques were the same as those used to compare responses from the version A and version B students. Below, I will discuss how the r-correct and r-model sub-groups compare on responses to the electric potential items in the all=all case.

*Item V5*

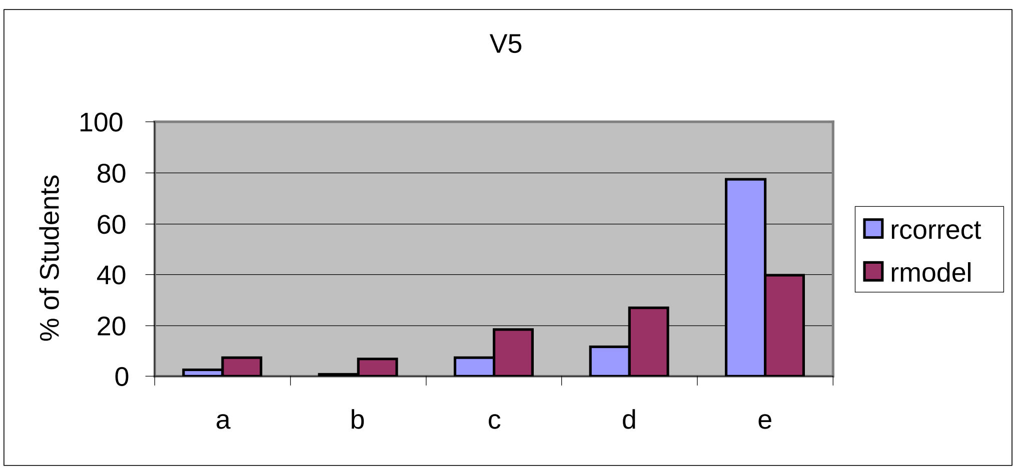



Figure 4.4 – Distribution of student answers for item V5

Table 4.14 - The *z*-score for the difference in proportions of the responses on item V5 between the r-correct students and the r-model students

| Response | Z-score |
|---|---|
| A | -1.81 |
| B | -2.4 |
| C | -2.55 |
| D | -3.02 |
| E | 5.90 |

For a response to have a significant difference between proportions at the $p = 0.01$ confidence level, the *z*-score must be greater than 2.58. A comparison between r-correct and r-model groups shows that two responses are significantly different. The r-correct group chose the correct response significantly more than the r-model group. The r-model group chose the all=all choice, D, more than the r-correct group.

*Item V6*

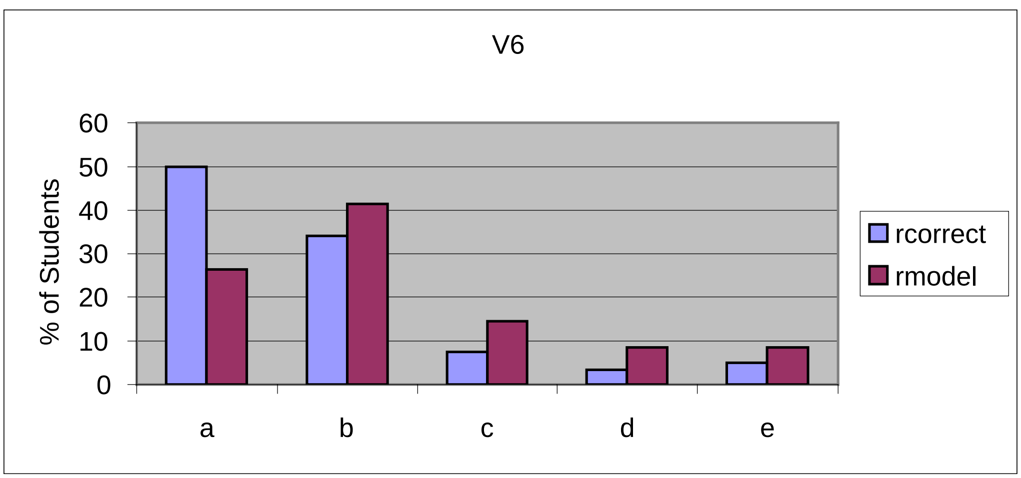



Figure 4.5 – Distribution of student answers for item V6

Table 4.15 - The *z*-score for the difference in proportions of the responses on item V6 between the r-correct students and the r-model students

| **Response** | **Z-score** |
|---|---|
| A | 3.77 |
| B | -1.17 |
| C | -1.71 |
| D | -1.69 |
| E | -1.07 |

For item V6, there was only one response with a significant difference between the proportions for the two sub-groups. The r-correct group choose the correct response, A, more than the r-model group.

I did not compare any other sub-groups. The mixed group was too small with only 30 students. The null group was not just students that were guessing or not understanding the material, but also those students that could not really be measured well. The r-correct and the r-model were the only sub-groups where I was confident of the groupings.

### 4.9.3 Sub-Group for Case: Zero=Zero

As I stated above, the zero=zero case and the no-zero=no-zero case have been combined into one case. I will use the term zero=zero to refer to the case that includes the no-zero=no-zero items. Students that use the zero=zero model believe that if a quantity is zero, then that quantity's rate of change must also be zero. A student that uses the zero=zero model will say that a coin thrown into the air will have zero acceleration at its highest point because its velocity is zero at its highest point. As was true for the all=all case, this does not include students that think that velocity and acceleration are the same thing.

Table 4.16 – Items in the Zero = Zero case including the correct response and the response that corresponds to the Zero = Zero model.

| Case: Zero=Zero items | Correct response | R-model response |
|---|---|---|
| R1b | G | D |
| R5 | D | B |
| R7 | A | B |
| V1d | E | C |
| V2a | C | A |
| V4a | B | A |
| V4b | A | B,C |
| V5 | E | D |
| V6 | A | B |

Sorting into sub-groups was somewhat different for this case than it was for the all=all case. There are only 3 rate of change items that address the zero=zero case. Items R5 and R7 had most students answer them correctly and the interviews showed no sign of students having any great difficulties. Item R1b, on the other hand was very interesting. For the most part, students either answered this correctly or answered it with the zero=zero model. The students that answered either choice D or G comprise more than 95% of the students. I chose the students' responses to item R1b as the means by which to separate the students into the sub-groups. As there is only one question to filter the students, there can only be three groups, those that answered R1b correct, those that chose the zero=zero response and those that did neither. As I already stated, the number of students that did not fall into one of the first two groups was extremely small. The two groups that I compared will be the r-correct group and the r-model group. I compared the responses for the two sub-groups on the electric potential zero=zero items.

*Item V1d*

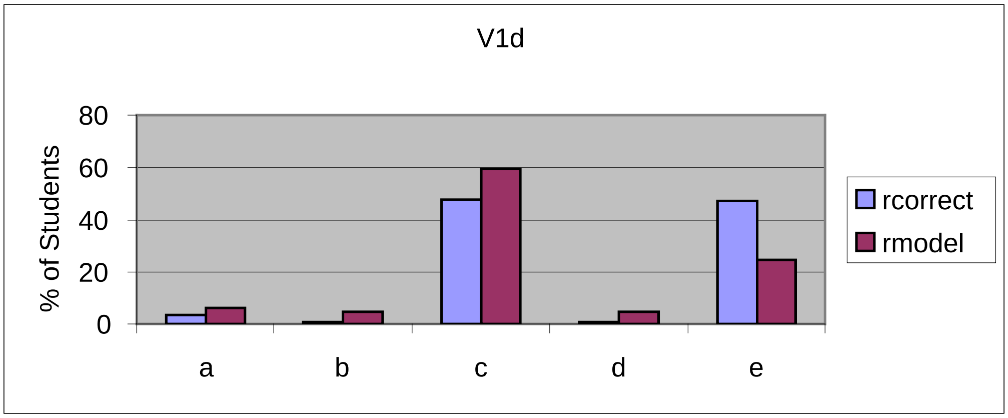



Figure 4.5 – Distribution of student answers for item V6

Table 4.17 - The *z*-score for the difference in proportions of the responses on item V1d between the r-correct students and the r-model students

| **Response** | **Z-score** |
|---|---|
| A | -1.12 |
| B | -2.11 |
| C | -2.06 |
| D | -2.11 |
| E | 4.23 |

There was a significant difference between the two groups for response E, which is the correct response. The r-correct group chose the correct response more often than the r-model group.

*Item V2a*

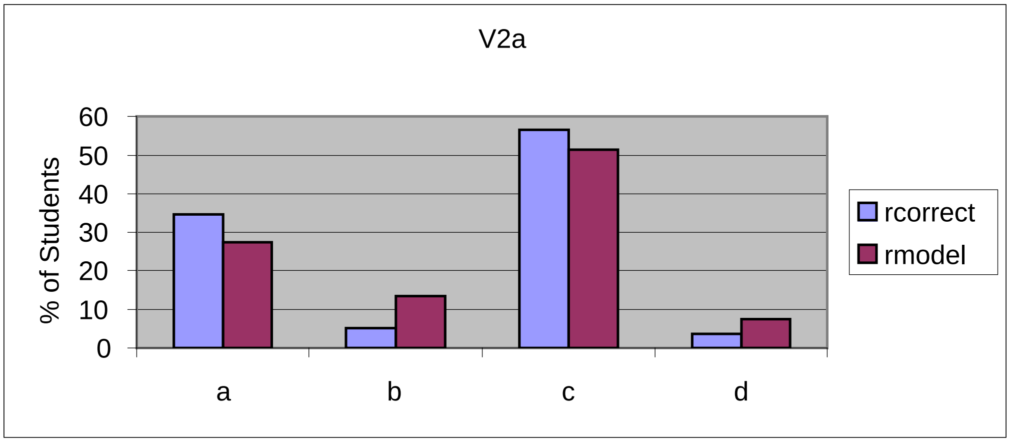



Figure 4.6 – Distribution of student answers for item V2a

Table 4.18 - The *z*-score for the difference in proportions of the responses on item V2a between the r-correct students and the r-model students

| Response | Z-score |
|---|---|
| A | 1.45 |
| B | -2.48 |
| C | 0.93 |
| D | -1.47 |

There were no responses with significant differences on this item.

*Item V4a*

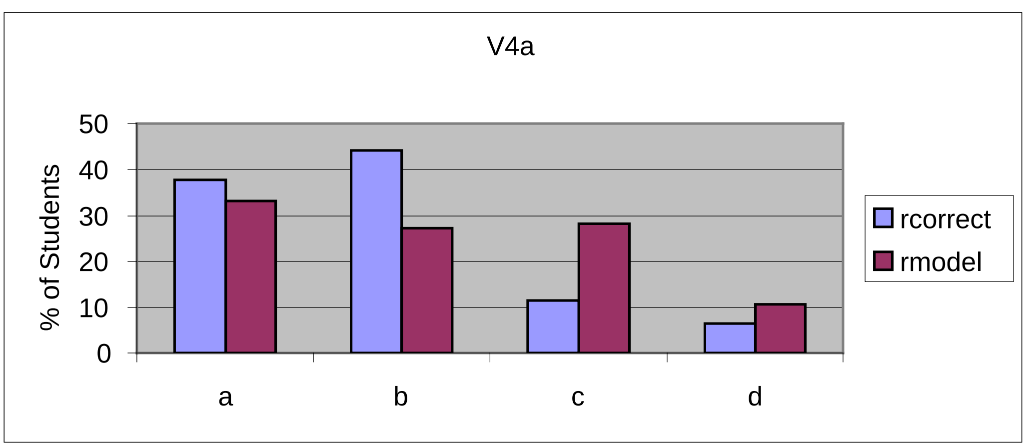



Figure 4.7 – Distribution of student answers for item V4a

Table 4.19 - The *z*-score for the difference in proportions of the responses on item V4a between the r-correct students and the r-model students

| Response | Z-score |
|---|---|
| A | 0.85 |
| B | 3.17 |
| C | -3.65 |
| D | -1.3 |

It is interesting that there is no significant difference for the zero=zero answer, A. The r-correct group did answer the correct choice, B, significantly more than the r-model group. The r-model group answered choice C significantly more than the r-correct group. Item V4a asks the students to describe the motion of a proton released from rest at a location where the potential is zero, but the electric field is not. According to the graph, the correct response is that it will move to the left, choice B. Choice C is similar in that it says the proton will move, but in the opposite direction. I consider choice C to be not entirely incorrect, the important thing is that these students say that the proton will move. My instrument was not designed to investigate if students understand the direction of the electric field due to a changing electric potential, just that there is a field.

*Item V4b*

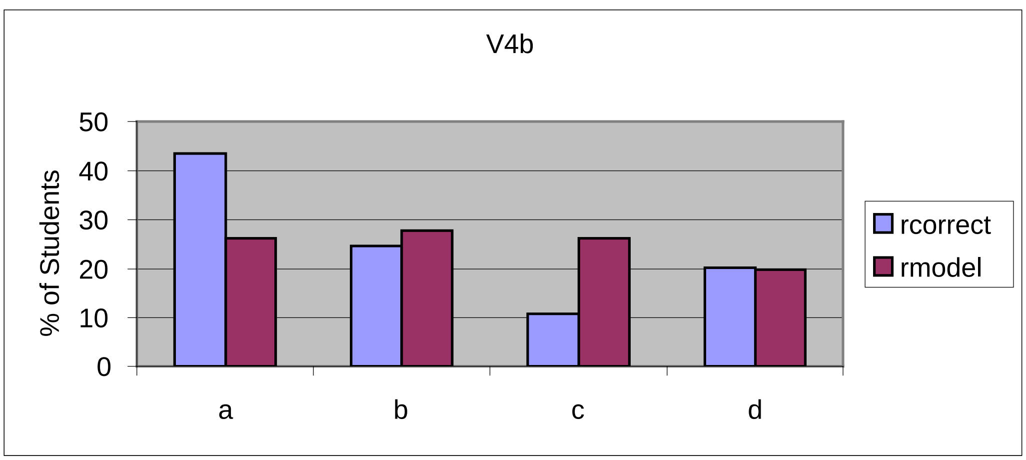



Figure 4.8 – Distribution of student answers for item V4b

Table 4.20 - The *z*-score for the difference in proportions of the responses on item V4b between the r-correct students and the r-model students

| Response | Z-score |
|---|---|
| A | 3.26 |
| B | -0.64 |
| C | -3.44 |
| D | 0.11 |

The two sub-groups answered significantly different on responses A and C. The item asks for a description of the motion of a proton released from rest at a location where the potential is non-zero, but the electric field is zero. The correct choice is A, and the r-correct group answered this more than the r-model group. I considered choice B and C to be equally incorrect as they both describe the proton as moving.

*Item V5*

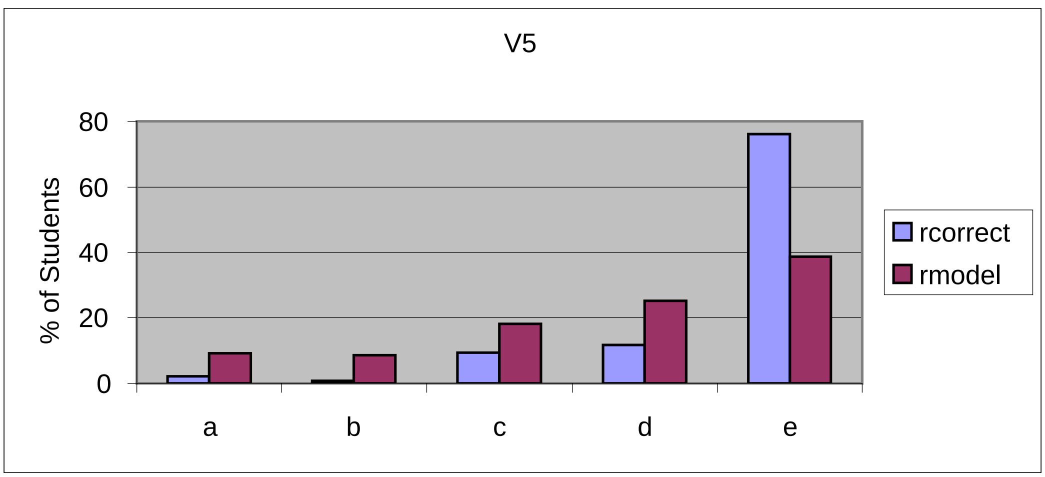



Figure 4.9 – Distribution of student answers for item V5

Table 4.21 - The *z*-score for the difference in proportions of the responses on item V5 between the r-correct students and the r-model students

| Response | Z-score |
|---|---|
| A | -2.56 |
| B | -3.13 |
| C | -2.22 |
| D | -3.05 |
| E | 6.72 |

This item asks the students to choose the electric field versus distance graph that corresponds to a given graph of electric potential versus distance. The correct answer is choice E and the r-correct group chose this more often than the r-model group. The D response corresponds to the graph that looks the same as the electric potential graph and the r-model group chose this more often than the r-correct group.

*Item V6*

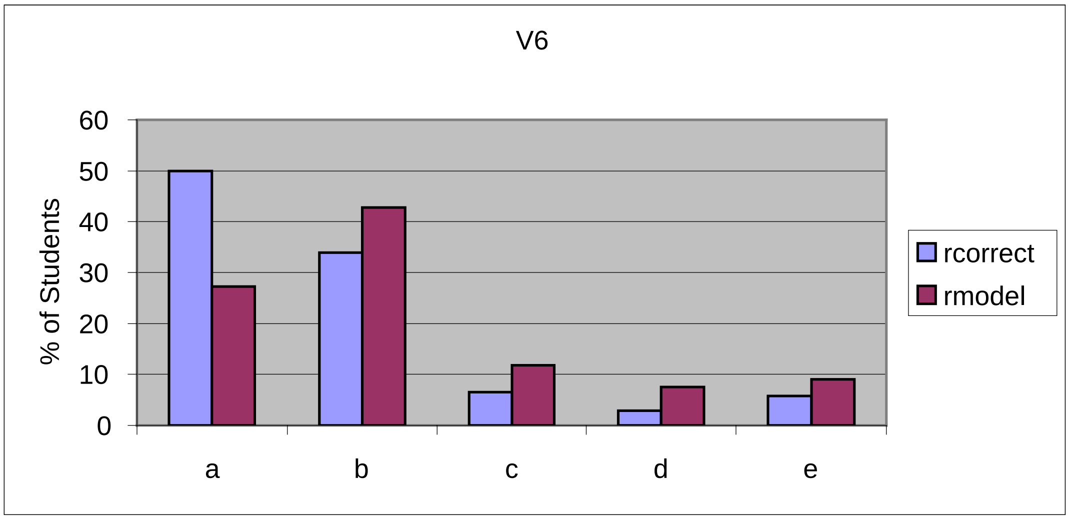



Figure 4.10 – Distribution of student answers for item V6

Table 4.22 - The *z*-score for the difference in proportions of the responses on item V6 between the r-correct students and the r-model students

| Response | Z-score |
|---|---|
| A | 4.20 |
| B | -1.59 |
| C | -1.59 |
| D | -1.79 |
| E | -1.1 |

Item V6 asks the students about the electric field in a region where there is a constant, non-zero electric potential. The correct answer is choice A, that the electric field is zero. The r-correct group chose this response significantly more than the r-model group.

**4.9.4 Correlations in the Sub-Test Version of RAPT**

As described previously, I created a sub-test version of the RAPT. This sub-test version contains the items that students had more difficulty with. This includes the “coin toss” items (R1a, R1b, R1c), the equipotential lines items (V1a, V1b, V1c, V1d), the electric potential graph items (V4a, V4b), and the constant potential item (V6). One advantage of looking at the sub-test version is that I administered this to another section of introductory physics class from sample 5 which increased the total sample of students ($n$ = 394). The only rate of change questions that the sub-test contained were R1a, R1b, and R1c, so these were used to separate students into sub-groups as I did with the full version of the RAPT.

To separate the students into a r-correct and r-model group, I first looked at their responses to item R1b. The interviews showed this to be a very discriminating item. It turns out that only 11 of the 394 students answered G, the correct response, for item R1b and did not answer G for items R1a and R1c. That is to say, if a student answered R1b correctly, that student was very likely to answer the other coin toss questions correctly. On the other hand, there were many more students (n = 60) that answered either R1a or R1c correct, but answered D, the answer corresponding to the r-model, for item R1b. For this comparison, the r-correct group are those students that answered R1a, R1b, and R1c correctly and the r-model group contains the students that answered D for R1b. By using this method to create the two sub-groups, the r-correct group contained 145 students and the r-model group contained 221 students. After a description of the item and the differences between the groups, I have included student quotes followed by that students response from the paper version of the sub-test RAPT.

*Item V1a*

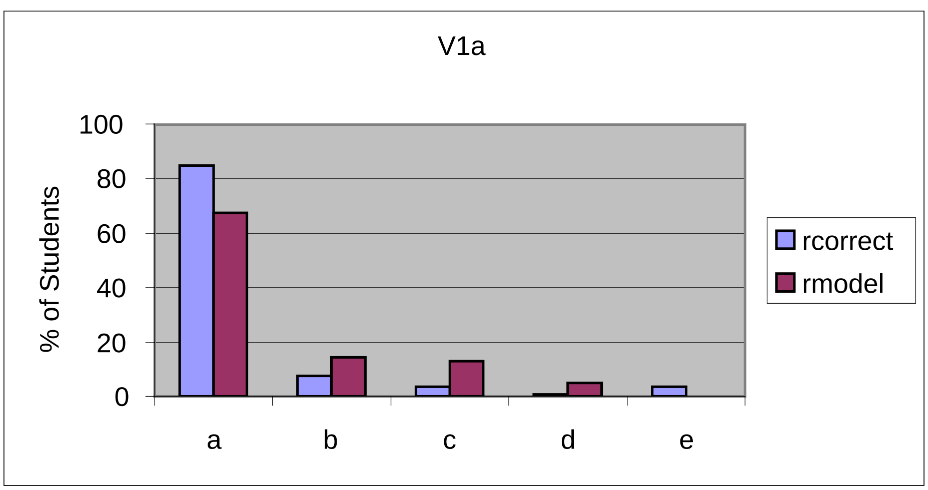



Figure 4.11 – Distribution of student answers for item V1a

Table 4.23 - The *z*-score for the difference in proportions of the responses on item V1a between the r-correct students and the r-model students

| Response | Z-score |
|---|---|
| A | 3.73 |
| B | -2.00 |
| C | -3.12 |
| D | -2.25 |
| E | 2.78 |

This item does not really deal with rate of change, but there is a significant difference on proportions of students choosing responses for this item at the $p$ = 0.01 level ($z$ = 2.58). I included this item to determine if students were distinguishing between the terms "electric potential" and "electric field." The fact that there is a significant difference between the two groups suggests that rate of change knowledge is not the only difference between the two groups. It is apparent that the r-correct group can interpret graphs and read questions better as they answered this item correctly more often.

- "it is closest to the center" (C)
- "not on a line w/ 80 V around" (B)

*Item V1b*

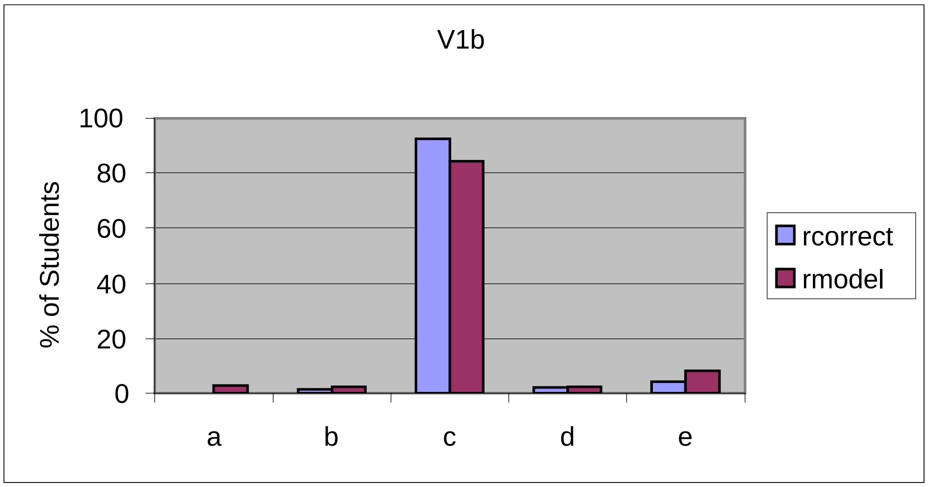



Figure 4.12 – Distribution of student answers for item V1b

Table 4.24 - The *z*-score for the difference in proportions of the responses on item V1b between the r-correct students and the r-model students

| Response | Z-score |
|---|---|
| A | -2.00 |
| B | -0.60 |
| C | 2.33 |
| D | -0.12 |
| E | -1.51 |

This item also does not deal with rate of change but rather with the ability to read equipotential lines. There is no significant difference in responses between the two groups at the $p = 0.01$ confidence level.

- "would only be equal to zero inside the source of EMF" (E)
- "its between 10 V and -10 V and potential is a vector so it is zero (C)
- "every point has potential" (C)

*Item V1c*

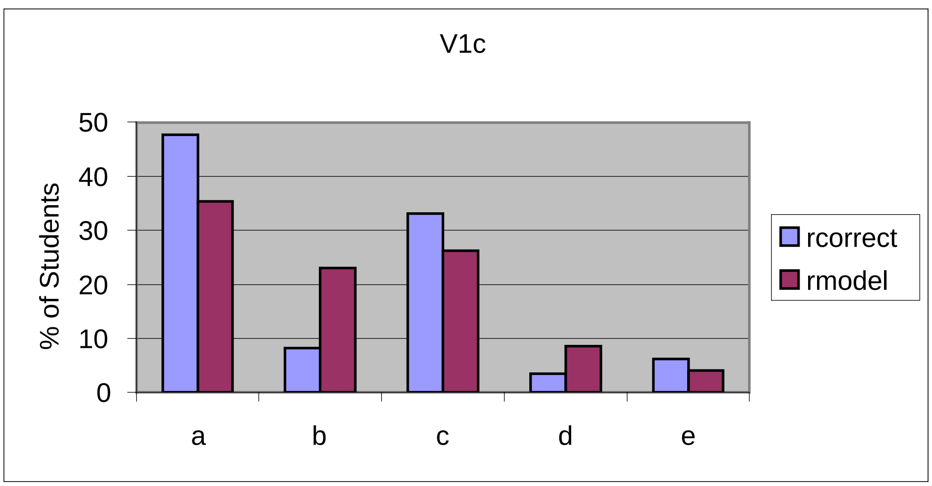



Figure 4.13 – Distribution of student answers for item V1c

Table 4.25 - The *z*-score for the difference in proportions of the responses on item V1c between the r-correct students and the r-model students

| Response | Z-score |
|---|---|
| A | 2.35 |
| B | -3.67 |
| C | 1.41 |
| D | -1.95 |
| E | 0.92 |

The correct choice for this item is C, that the electric field is greatest in the region where the electric potential is changing the most. There was no significant difference on either this response nor on the r-model response of A, the electric field is greatest where the potential is the greatest. The response B did have significantly different responses with the r-model group choosing this more than the r-correct group. The students that chose this response were not using the r-model strategy. Some students on the paper version with room for explanations wrote that they chose B because "it is farthest from the center."

- "E = -dV/dx either B or C b/c they are not at a constant potential like others so dV/dx not 0" (no answer)
- "closest to the center" (C)
- "voltage does not change at a point" (E)

*Item V1d*

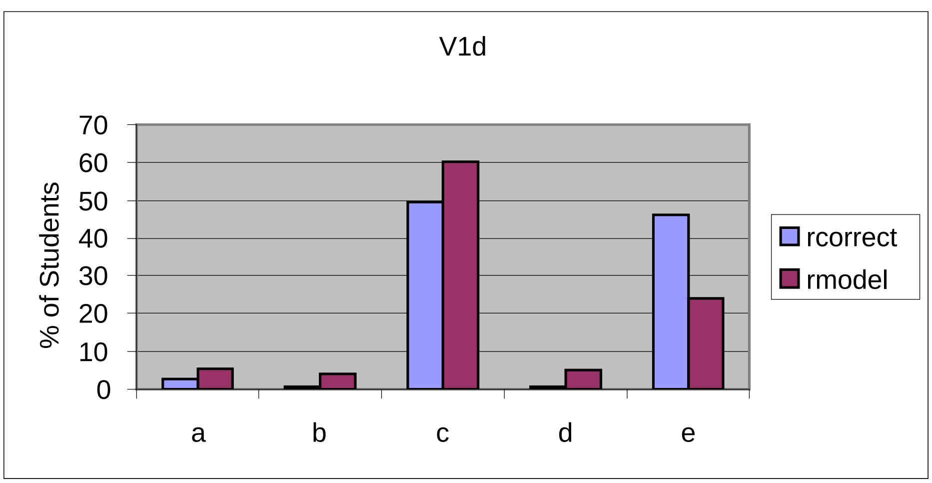



Figure 4.14 – Distribution of student answers for item V1d

Table 4.26 - The *z*-score for the difference in proportions of the responses on item V1d between the r-correct students and the r-model students

| Response | Z-score |
|---|---|
| A | -1.22 |
| B | -1.94 |
| C | -1.98 |
| D | -2.25 |
| E | 4.43 |

There were significantly more students from the r-correct group that chose response E, the correct response.

- "the field cancels each other out" (C)
- "C has 0 potential and 0 field strength" (C)
- "because the potential there is zero" (C)

*Item V4a*

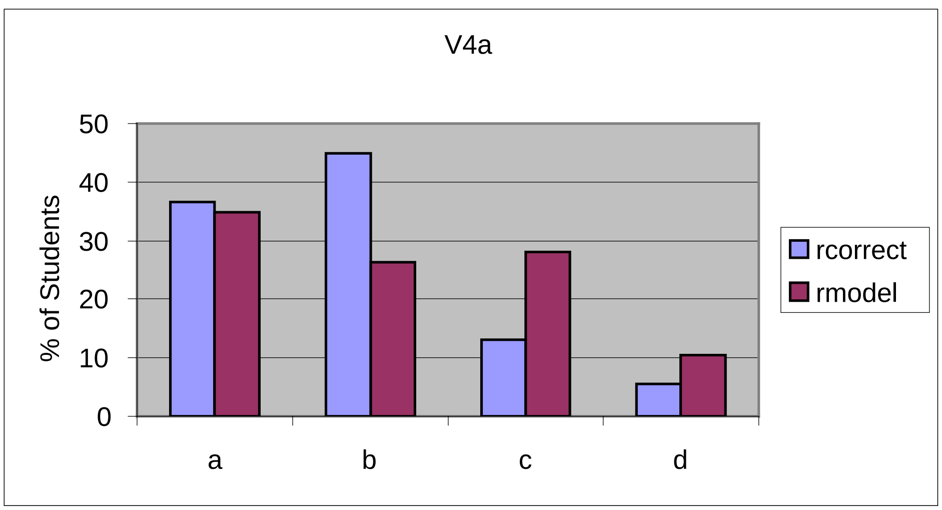



Figure 4.15 – Distribution of student answers for item V4a

Table 4.27 - The *z*-score for the difference in proportions of the responses on item V4a between the r-correct students and the r-model students

| Response | Z-score |
|---|---|
| A | 0.33 |
| B | 3.68 |
| C | -3.37 |
| D | -1.64 |

The r-correct group choose the correct response, B, than the r-model group.

- "positive voltage repels positive charge" (B)
- "electricity makes positive charges move" (C)

*Item V4b*

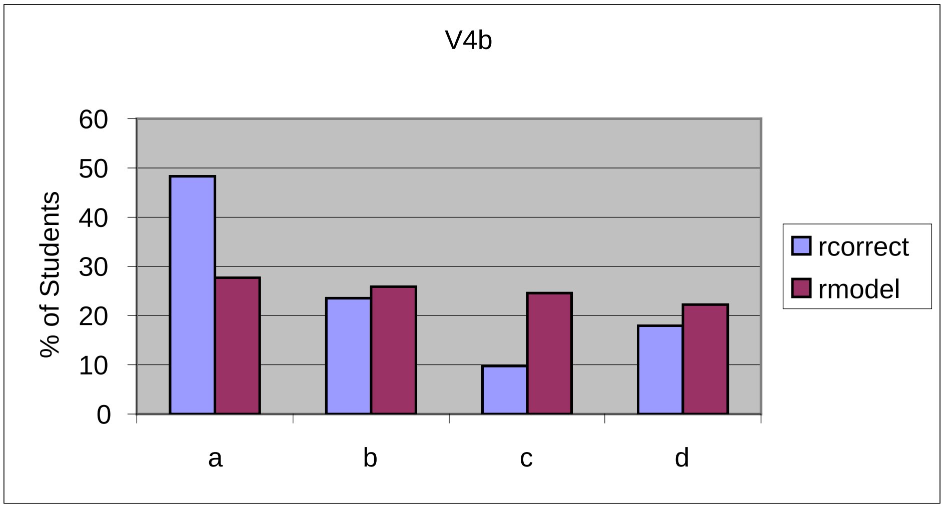



Figure 4.16 – Distribution of student answers for item V4b

Table 4.27 - The *z*-score for the difference in proportions of the responses on item V4b between the r-correct students and the r-model students

| Response | Z-score |
|---|---|
| A | 4.04 |
| B | -0.51 |
| C | -3.56 |
| D | -0.98 |

The r-correct group chose A, the correct answer, more and r-model group chose C, r-model answer, significantly more. The item asks about the motion of a proton released from rest at a location where the potential is zero but the electric field is not. The correct response (A) is that it will not move. The students that answered (B) or (C) were likely thinking along the same lines , “there is a potential so it will move.” I was not too concerned with the direction they chose. On this question, the distribution for the r-model group is very close to that of guessing.

- “the potential is increasing, so it will move to the left” (B)
- “negative potential = move left” (B)

*Item V6*

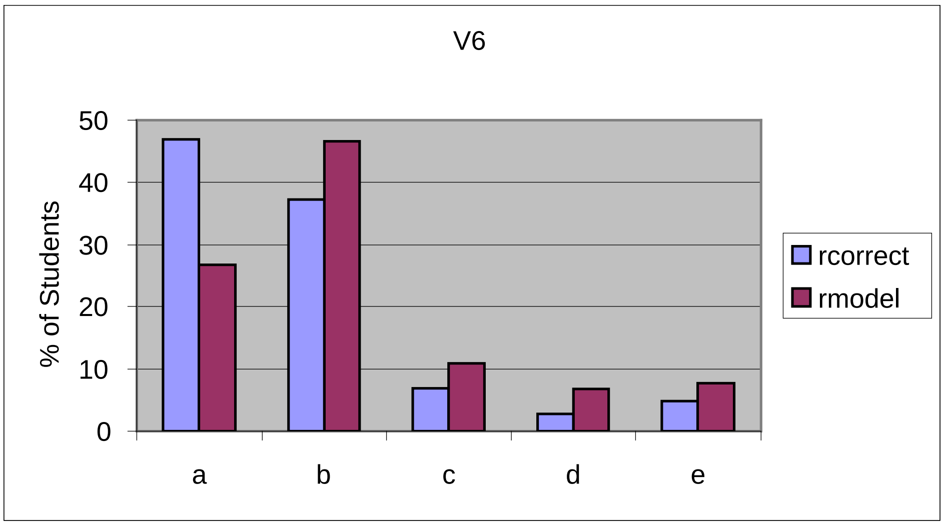



Figure 4.17 – Distribution of student answers for item V6

Table 4.28 - The *z*-score for the difference in proportions of the responses on item V6 between the r-correct students and the r-model students

| Response | Z-score |
|---|---|
| A | 3.97 |
| B | -1.77 |
| C | -1.28 |
| D | -1.70 |
| E | -1.08 |

The r-correct group answered A, the correct answer, significantly more than the r-model group.

- “E = dV/dx = constant volts/change in distance or zero, V constant? dV = 12 – 12 = 0?” (D)
- “E potential and E fields have a linear relationship” (B)
- “the region has electric potential so the electric field is non-zero” (B)

## 4.10 A Detailed Look at Two Students

The think-aloud interviews provide qualitative data on the relationship between student answers on rate of change and electric potential items. In this section, I will discuss two students that were interviewed. These two students were chosen because they represent two different uses of mental models. One student uses a mixture of models, sometimes using the correct reasoning and sometimes incorrect. The other student consistently uses the incorrect model. I will provide evidence that shows how their rate of change reasoning and electric potential reasoning relate. Both of these students are from the interview sample as explained previously. The real students' names have been replaced with false ones. A discussion of how the students explanations on rate of change items relate to their electric potential explanations will be done in chapter 5.

### 4.10.1 Joe

Joe has interesting responses for the coin toss items, R1a, R1b, and R1c. When asked to describe the acceleration of the coin as it is on its way up, Joe answers correctly that it has a constant acceleration in the negative direction (G). He gives the following as his explanation.

> *"I chose g because the acceleration …is going to be negative in the direction while the coin is going upwards and the acceleration is going downward because the acceleration doesn't change, it is always constant."*

The second part of this item asks about the acceleration when the coin is at its highest point. Joe chose answer (D) the acceleration is zero. While answering the question, Joe has the following thoughts.

> *"When the coin is at its highest point, the acceleration is still in the negative direction. The acceleration is going to be the same in all…just the velocity is going to change. I would say that the acceleration…the acceleration is going to be zero because it is at its highest point. Its not going to be positive. It is not going to be...I think it is zero, it is at its highest point."*

After the think-aloud portion of the interview, I asked the students to explain the reason they chose the answer they did. Joe gave the following explanation.

> *"At the highest point, it has no acceleration. Oh, well, the acceleration is still pointing downward, but the coin itself is not accelerating. I mean the force is pointing downward, so the acceleration is going to be zero."*

The last part of this item asks about the acceleration of the coin as it is moving back down. Joe appears to use the correct reasoning for this item but answers incorrectly. He answers choice B that the acceleration is constant and in the positive direction. It may be that he simply was confused by the definition of positive.

> *"While the coin is moving downward…the acceleration is positive in the direction it is moving since it is moving downward…it is not negative, negative, negative, or zero…it is going to be positive and constant."*

For the coin toss items, Joe seems to have a conflict between two models. One model he uses is the correct one that says an object has a constant negative acceleration due to gravity. The other model Joe attempts to use says that if the velocity is zero, the acceleration must also be zero. When the coin is at its highest point, Joe's conflict between models can be seen in his explanation. He starts off with the correct explanation, but in the end he says that since the velocity is zero, the acceleration must also be zero. Joe knows that the acceleration due to gravity is constant, but how he knows this is unclear. He may know it because that is what was told to him in lecture class and in reading the textbook. If this is the case, his conflict may be between what he believes and what he has been told.

The rest of the rate of change items contain nothing especially interesting. One thing to note is that Joe answers the rest of these rate of change items correctly with a correct explanation. He also uses the explanation that the slope of a volume versus time graph describes how fast the volume is changing.

Items V1a-d present the student with a map of equipotential lines. V1a and V1b ask the student about the value of the electric potential. Joe answers these correctly demonstrating that he can interpret the map. The next item asks the student to chose the location at which the electric field is greatest. Joe chooses point B which is neither the location of the largest electric field nor the location of the largest electric potential. He gives the following thoughts while answering this item.

> *"At which of the points is the magnitude of the electric field the greatest? The electric field is …lines that are…making 90 degree angles like that [draws some field lines perpendicular to the equipotential lines] So, the electric field is going to be farthest away at point B. At which of the points is the magnitude of the electric field the greatest? Electric field...is going to be highest...the electric field is going to be highest when they are closer together or farther apart?"*

To further explain his answer, he said the following.

> *"For 17 I chose B because I drew the electric field and it was the farthest one going out."*

Joe correctly states that electric field lines are perpendicular to equipotential lines. He also indicates that the greatest electric field is at the location where the lines are closest together or farthest apart. Item V1d asks to chose the point at which the electric field is zero.

> *"Ok, equal to zero…obviously B is not equal to zero. If it were the weakest, then I would have trouble between B and C, but since it says zero…I am going to go with C being zero and B being farthest away. The greatest is going to be B."*

He later changes his answer.

> *"For 18 I chose C because it was the neutral one, it was in the center so it was zero."*

It is strange that he considers point B as a candidate for the location where the electric field is zero for he also chose this point as the location of the greatest electric field.

Item V3 presents the students with 5 graphs of electric potential versus distance and asks which one has the greatest electric field at some point. Joe gives the following thoughts while answering the item.

> *"ok…The electric potential to displacement. Electric field…ok…from the last question, we saw that …E was…magnitude of the electric it was B…if it was farther away. Ok, this is kinda tricky. There is obviously some relation here with…I am going to have to go with slope because they are not giving me any values. It said that some distance, it doesn't say...seems like D is going to be...something potential increasing. Ok I will have to go with D because they are not giving me any numbers. I am going to have to go with the slope."*

It seems that he is unsure of the answer, but resorts to the correct answer perhaps through the use of test-taking skills. That is he does not really know the answer, but he is able to arrive at a logical choice that happens to be correct.

Items V4a and V4b present the student with a graph of electric potential as a function of time. V4a asks the student to describe the motion of a proton released from rest at a point where the electric potential is zero, but the electric field is non-zero. Joe chooses the answer that says the proton will not move.

> *"Potential…it is going to be zero. So, there is no potential, there is no motion. If it had some potential…it would. ..potential here is zero, so it is not move. Not move. Zero."*

V4b asks the same question for the case where the proton is released at a location where the electric potential is non-zero, but the electric field is zero. Joe gives the following explanation.

> *"At 1, it has negative potential, so it will move. To the left or to the right? It is* not *saying…proton. At x=1 it is going to be moving towards…it does not say to the left or to the right because they are not saying where the positive or the negative...ok, it could move anyway. If it wants to reach like zero potential, it can move to 2 or it can move...ok this is closer. If I had to guess, it is not going to be to the right. It would be between B and D. They are not going to give something can not be determined. So, since...the potential being zero is closer to the left...I have to go to the left. To the left."*

Joe says that it will move, but he debates on the direction it should move. Since he can not resolve the issue of the direction of motion, he chooses choice D, that the motion of the proton can not be determined.

Item V6 asks the students about the electric field in a region where the electric potential is constant but non-zero. For this item, Joe was confused at first, but then began using the correct reasoning.

> *"If I had a formula, at least I could look it up. Ok…if it is constant…it means, it has got to be a horizontal line. It is not changing. They are not saying anything about distance. Umm…do they mean constant like over some distance or just at a point, in a certain region of the distance. I am going to have to go with this. It is going to be zero. In a certain region...it is not saying just a point...it is not saying...oh this is a region. From 2 to 4 is a region, so I am going to with zero, because it is going to be the slope. Is that correct."*

It seems that he is correctly thinking about the electric potential in terms of a graph. He correctly states that if this were a graph, a constant potential would be a region where the slope is zero and thus the electric field would be zero. This is the correct answer. Later, as Joe was giving an explanation for his response, he became confused.

> *"Where it is constant, I assume there is going to be a graph with a straight horizontal line which is…oh, that doesn't make much sense. It could be constant, just linear. I am going to choose E and change my answer. It doesn't say, it just says constant. Magnitude of electric field is constant, non-zero. It could be constant decreasing or constant increasing, so there is not enough information."*

In this second explanation, Joe becomes confused about what constant means. He is not sure if it means the quantity is constant or the rate of change of that quantity is constant.

### 4.10.2 Jan

Jan responds correctly for the first coin toss item. She gives the following thoughts while answering this item.

*"Well, according to me…(read choices) …Its, well its going up…it is in the positive direction but it is slowing down, so it is A. Second try: The acceleration would be in the positive direction…but…its velocity is in the positive direction and its velocity is decreasing.  So the acceleration is constant – so g"*

She initially starts to answer with the choice that corresponds to the model that says the rate of change and its quantity change in the same manner.  That is, the velocity is slowing down, so the acceleration must also be slowing down.  Jan then states that the acceleration is constant and negative.  The second part of this item asks about the acceleration of the coin when it is at its highest point.  Jan gives the following thoughts.

*"The acceleration … the velocity is zero…I don't remember this stuff.  The acceleration…well according the meaning of downward – the acceleration is in the negative direction and it would be increasing.   Wait a minute…this is due to gravity.  If it is due to gravitiy, it is going to be 9.8.  So acceleration is always the same.  OK.  It would be in the negative direction.*

Later Jan gave the following explanation for this item.

*The velocity is zero – so it should be zero?"*

Jan begins to explain this item, but then seems to come to the realization that gravity is constant. She also correctly states that acceleration is always the same.  The second time she looked at the item she thought that since the velocity is zero, the acceleration must also be zero.  For the last part of the coin toss question, she seems to be confused.

*"I really don't know what it is up here.  I don't think it is zero.  I answered that it was in the negative direction, but now I think it is in the positive direction.  I really don't know why."*

The other rate of change items reveal nothing interesting.  Jan answers them correctly with correct explanations.

For item V1, Jan answers the first two parts correct which indicated that she correctly interprets the equipotential map.  Item V1c asks at which point is the electric field the greatest.  Jan chooses the point at which the electric potential is the greatest (point A), rather than the location of the greatest electric field (point C).  She gives the following thoughts while answering this item.

*"I chose A, I really don't know why  - I know the electric field gets…no I don't really know how it relates to this"*

For item V1d, she choose the point at which the electric potential is zero when asked where the electric field is zero.  She gives the following reason.

*"I chose C because its potential was zero"*

In items V4a and V4b, Jan shows that a conflict between the use of two models.  One model is a correct one that says a proton will move to a lower potential.  This is not incorrect, but it is also not the best explanation for why a proton will move.  The appropriate model says that the proton will move if there is a change in electric potential. The other model Jan uses says that if the electric potential is zero, there can be no motion. For item V4a, a proton is released at a location where the electric potential is zero, but the electric field is non-zero. Jan gives the following thoughts.

> *"I was kinda guessing. I figured it was at zero potential and would not move. Then…if its …I want to say if it is released, it goes to a lower potential. But that would mean that they would move to the left. I can't remember the relationship between electric potential and electric field."*

It seems that the conflict between these two models leads her to say that she does not know and she does not know the relationship between the electric field and electric potential. Jan attempts to apply the model that says the proton will move to a lower potential for item V4b also.

> *"If it is at maybe one volt potential…it can't be determined. Second try: At x = 1 is here. I think it will go to a lower potential – I don't know this stuff."*

Jan says that it will go to a lower potential, but the proton is released at a point that is the lowest potential. She also states that the proton is at one volt potential indicating that this may be important for her.

## Summary

In this chapter, I presented the results of the reliability and item analysis. I also presented results for comparing student responses on rate of change and electric potential items. The mean score for the 340 students was 15.05 out of 25 with a standard deviation of 4.97. The average item mean difficulty was 0.60. The reliability coefficient was 0.83, which is sufficient for individual measurement.

[1] Macisaac, D., Cole, D., & Cole, R. (1999, March). Standardized testing in physics via the World Wide Web. Paper presented at the meeting of the National Association of Research in Science Teaching, Boston, MA.
[2] Hake, R. R. (1998). Interactive-engagement versus traditional methods: A six-thousand-student survey of mechanics test data for introductory physics courses. American Journal of Physics, 66, 64-74.

# Chapter 5

# Conclusions

The purpose of this study was to investigate the possibility that student difficulties with the concept of rate of change contribute to their difficulties with electric potential. In this chapter, I will discuss the conclusions that can be made from the results presented in chapter 4.

## 5.1 Research Question 1

*Is the Rate And Potential Test (RAPT) an appropriate instrument to investigate the relationship between student models in rate of change and electric potential?*

Since I used a multiple-choice instrument to investigate the relationship between rate of change and electric potential, I must make sure that the instrument itself is suitable to the task.

### 5.1.1 Is the RAPT a Reliable Instrument?

Yes. According the reliability coefficient, as measured by the KR-20 = 0.83, the RAPT is a reliable instrument suitable for measurement of individual students. The average item difficulty of $b_{mean}$ = 0.60 suggests that the overall instrument is moderately difficult. The average item discrimination is $D_{mean} = 0.45$, so the test is considered to be an excellent discriminator. The think-aloud interviews gave no indication of students misreading or misinterpreting the items.

### 5.1.2 Does WebAssign Make a Difference?

No. Even though the instrument was administered in both a paper format and an online format, there was no difference in the student scores between the two. There were no significant differences on mean score, the rate-score, or the potential score between the students that took the RAPT on paper and those that took it on WebAssign. There also was no difference on sub-test scores between the WebAssign group and the paper group from the same university. Because no significant differences were found, I did not distinguish whether data came from WebAssign or the paper version of the RAPT. This result is important not only for this study but others as well. If online tests are to be used more frequently in physics education research, there must be evidence that it does not significantly hinder student performance. The data I collected provides some of this evidence. I think that online testing will become more common, especially with the increased use of the World Wide Web as a research and teaching tool. Online testing has the ability to do much more than conventional paper testing.  New tests can be created that use animations as well as adaptive tests that are impossible to do with a paper format.

#### 5.1.3 Does Order of the Items Make a Difference?

Maybe.  One important reason to investigate the effect of order is to determine is if students see a connection between similar rate of change items and electric potential items.  The RAPT contains several items that are identical in a mathematic perspective, that is they relate the quantity and its rate of chang ein the same fashion.  One example of an "identical" set of items are items R7 and V6.  Item R7 asks students about the acceleration of a ball rolling at constant speed.  Item V6 asks about the electric field in a region where the electric potential is constant.  These two items are "mathematically identical" in that the acceleration is the time rate of change of the velocity and the electric field is the gradient (distance rate of change) of the electric potential.

If students do see a connection between the rate of change items and the electric potential items, then the version B students should perform better on the electric potential items.  If students do not see the connection, there should be little difference between the two versions.  Whichever the case, I did not think I should see a difference on the rate of change items because I doubted that students would know more about electric potential than they do about rate of change items.  That is to say, if I student understands the electric potential items so well that he or she can use those answers to help with the rate of change items, that student likely can do well on the rate of change items anyway.  If I did find a difference between the rate of change items for the two versions, that would likely indicate that the students are from different populations.

Comparing the mean score, rate-score, and potential-score for the two versions , A and B, of the RAPT shows there is a significant difference between both the mean score and the potential-score.  I did not find a significant difference between the rate-scores for the two versions.  RAPT version A gave the students all the rate of change questions followed by all the electric potential questions and version B gave the students a rate of change question followed by a similar electric potential question.  Since the two groups performed similarly on the rate of change questions it can be assumed that they are composed of equivalent samples of students.  In examining the difference in potential-scores between the two groups shows that the version A students performed better than the version B students.  I found this to be a surprising result as the purpose of version B was to help the students see the connection between the kinematics items they may be more familiar with and the electric potential items that they may see as more abstract.

The difference in potential-score between the two groups is due to three specific electric potential items: V1c, V1d, and V2a.  For items V1c and V1d, the difference between version A and version B is very small.  These items present the students with a map of equipotential lines and ask about the electric field. The corresponding rate of change items are R6a and R6b which show the students a map of isobars and ask them about the wind. The wind is strongest in locations where the pressure is changing the most, that is where the isobar lines are closest together. The electric field is the strongest where the electric potential is changing the most, that is where the equipotential lines are closest together. For version A of the instrument, there is only one item between R6 and V1. For version B, items R6 and V1 are right next to each other. The item between R6 and V1 in version A is item R7 which asks the students about the acceleration of an object moving at constant velocity.

There are several possible explanations for why version A students did better on these electric potential items.

*1. The order of items had an effect*

I think that this is not the case, for if the order did have an effect it should been in favor of the version B students. Also, the only evidence of effect of order in the interviews was slanted towards the version B students. Some of the version B interview students answered the same on electric potential items as they did on the rate of change items with the explanation "I am not sure, so I will just answer the same as I did before". If order had an effect, it would also likely be seen in other items besides these three.

*2. A difference in population for the two groups*

I think this also is not the cause for the difference between the version A and version B students. I examined these three items for just data from students that took the RAPT on WebAssign. All of these students were from the same university taking different sections of the same course. The only difference for this WebAssign comparison was found for item V1d. The version A WebAssign students answered it correctly more than the version B WebAssign students. To do a similar comparison for the paper students is more difficult due to the smaller numbers for the two groups. I included only schools that took both versions of the RAPT on paper. There were 36 paper version A students and 38 version B students. Comparing these same items between the groups shows that version A did significantly better on both V1c and V1d. I can attach little weight to this finding since the numbers are rather small.

*3. Random occurrence*

It is possible that these items turn up as significantly different as a random occurrence. I used a 99% confidence level for the significance tests, so it is possible that these items are not significantly different. There were 123 different choices on the RAPT. Comparing all of these choices for the two groups means that 123 *z*-tests were performed. At the 99% confidence level, one would expect an average of 1% of the results to be false-positives. So, it is possible that these tests are merely false-positives.

### 5.1.4 Are the Items in the RAPT Appropriate?

The interesting thing about student responses to the rate of change items is that many students answered the coin toss questions incorrectly, but answered the rest of the rate of change items correctly. It is possible that the coin toss items and the other rate of change items investigate different aspects of students understanding of rate of change. Another possible explanation is that the students use some alternative strategy when answering the easier rate of change items that "bypasses" their true beliefs about rate of change. I believe that students' true models of the relationship between a quantity and its rate of change become visible when the context is abstract. For the case of relating velocity to position, this is not likely to be considered by the students as abstract. For this context, the student will not need to use their model for the relationship between position and velocity. When dealing with position and velocity, students "just know" that velocity is the slope of a position versus time graph. The following are one student's think-aloud explanations for some of the rate of change items.

- **R1b:** *"is it zero or is it negative? If it is up here, it stops. So, v=0. But, what about the acceleration? If…well…acceleration is the derivative of v, but….do we use this? At the top, what happens? For over here, at its highest point, I just said that…it is zero because it is completely stops and v is zero. So…basically…yeah, it is not even going up or down so it must be zero"*
- **R3:** *"Yeah, changing…the water level changing the fastest with time…I just …you know…I just analyzed that as…the greatest slope, so…"*
- **R4a:** *"This one is pretty easy…just put the slope for each…for this case"*
- **R4c:** *"For time = 8, its B - pretty simple"*
- **R7:** *"Well, we know that its like driving a car when you are going at 70 or 60, or whatever and you are constant, you are not changing. So, basically, your acceleration is the same."*

From the student's response to R1b, it seems clear that he uses the model that if the quantity is zero, its rate of change is zero also. It is also clear that he understands the difference between velocity and acceleration for he defines acceleration as the derivative of velocity. For the other rate of change items shown above, the student displays no use of the zero=zero model. He seems to be able to answer these items without addressing the relationship between a quantity and its rate of change. For item R7, he relates the ball rolling at a constant speed to driving a car at constant speed. For this item he uses a simple analogy of a context in which he has much experience. Thus, it seems that students can answer most of the rate of change items correctly and still hold onto incorrect models. The items that seem to consistently bring out students' true beliefs on rate of change are the coin toss items (R1a, R1b, and R1c). The point is that even though students answer the other rate of change items correctly does not mean that they have a full understanding of the relationship between a quantity and its rate of change.

### 5.1.5 Summary

For the most part the order of items seems to have little effect on student performance on the RAPT. The fact that the version B students do not do significantly better on any of the electric potential questions indicates that students do not see a connection between similar rate of change items and electric potential items.

## 5.2 Research Question 2

*Is there a relationship between the way students answer rate of change questions and the way they answer electric potential questions?*

It is important to realize that these questions looks for a relationship between rate of change answers and electric potential answers. The results can not show causality, only a relationship.

### 5.2.1 Is There a Correlation Between Rate of Change Scores and Electric Potential Scores?

Yes. There is a correlation between the rate-scores and the potential-scores for the students that took the RAPT. The correlation coefficient for these two scores is 0.45. This coefficient is significant at the 95% confidence level. There is a correlation coefficient of 0.38 between the rate-model score and the potential-model score that is also significant at the 95% confidence level.

### 5.2.2 Is There a Correlation Between Rate of Change Models and Electric Potential Model?

*Comparison for All=All sub-set*

Yes. There is a significant difference in responses to electric potential items from students in the r-correct and r-model groups. The r-correct students chose the correct electric potential responses significantly more than the r-model group. Also, the r-model group chose the response that corresponded to the all=all model significantly more than the r-correct group. (Section 4.8.2)

*Comparison for Zero=Zero sub-set*

Yes. There is a significant difference in the responses to electric potential items from students in the r-correct and r-model groups. The r-correct students chose the correct electric potential responses significantly more than the r-model group. Also, the r-model group chose the response that corresponded to the all=all model significantly more than the r-correct group. (Section 4.8.3)

### 5.2.3 Do the Interview Students Show a Connection Between Rate of Change and Electric Potential Models?

Yes. Both of the students described in chapter 4, Jan and Joe, showed some connection between their rate of change responses and their electric potential responses.

Joe demonstrated that he understood the difference between velocity and acceleration in items R1a and R1b, but he still responded that the coin will have a zero acceleration when its velocity is zero. In this instance, he is using the zero=zero model described previously. There is no other evidence of him using the zero=zero model on the other rate of change items, this may be because the items are simple enough that he can answer them with other strategies as discussed in section 5.1.4. Joe also demonstrated his use of the zero=zero model for electric potential item V1d. The item asks for the point at which the electric field is zero and he chose the point at which the electric potential was zero. Joe used the zero=zero model on item V4a as well. This item asks for the motion of a proton released at a location where the electric potential is zero but the electric field is non-zero. Joe explained that since the potential is zero, there can be no motion. For item V4b, where the potential is non-zero but the electric is zero, Joe stated that the proton will move because the potential is not zero.

For the case of Joe, it seems clear that he answered the coin toss question using the zero=zero model. He also used this model when he answered some of the electric potential items.

### 5.2.4 Summary

The data suggests that there is indeed a correlation between the answers students give for rate of change items and their answers for electric potential items. This is not to say that there is only a correlation between correct responses. There is also a connection between the responses that are associated with the common incorrect student model. The correlation does not address causality. The goal of this study was to examine a possible reason why students have difficulties with electric potential. The results clearly indicate that student difficulties with rate of change could be part of the problem. (The results do not show that rate of change *is* the problem, but that there is a correlation between problems students have with rate of change and their problems with electric potential. It is possible that both of these problems are related to overall student performance.) This is an important result as it shows a possible curricular path for improving student understanding of electric potential.

## 5.3 Research Question 3

*Do students in active-engagement course perform differently on the RAPT than students in traditional courses?*

Probably. This study was not designed to specifically address this question. To properly address this issue, a more complete sample would be needed. I only collected data from one active-engagement class, so the analysis is somewhat incomplete. However, I can comment on the performance of this one class. The active-engagement class did significantly better on the rate of change items. They also chose the incorrect model answers significantly less for the rate of change items. This class did not perform significantly different than the traditional classes for the electric potential items. An explanation for these differences can be found in the teaching assistant's description for the class:

- There are no entrance requirements for the students, and the TA said that they have found the students to be representative of the rest of the students at that university.
- The course integrates physics, calculus, engineering and English. The students take these courses together and the curriculum is integrated by the instructors.
- This group of students was also in this same class and arrangement for the first semester of introductory physics.
- The course tends to focus on an energy approach to solving problems.
- The course focuses on modeling as a means for conceptual understanding as well as problem solving.

It seems likely that the kinematics activities this class did in a previous semester were effective. The electric potential activities did not seem to be as effective. In order to further investigate this research question, more data from active-engagement courses would need to be gathered.

## 5.4 Research Question 4

*Do upper-level physics undergraduate students use similar models as introductory physics students?*

Upper-level physics students have demonstrated that they are able to succeed in the academic realm of physics. They have shown this by passing the courses that are pre-requisites for the advance course they are in now. Are these students able to succeed because they have a solid understanding of the concepts, or have they merely learned how work with the system that is currently in place. This study is not a complete investigation of this question, but does offer some evidence. The investigation of upper-level students is also important because if these students have difficulties with rate of change and electric potential, it is likely these difficulties exist also with the introductory students.

The student responses from the sub-set of the RAPT show that some of the students in the upper-level course do use the same models relating a quantity and its rate of change as the introductory students. It is interesting to see that all of the 18 upper-level students that took the sub-test RAPT wrote the following equation in their explanation for item V6.

$$\mathbf{E} = -\nabla V$$

This is the correct relationship between the electric field and the electric potential. It says the electric field vector is the negative of the gradient of the electric potential. Item R6 asks the students about the electric field in a region where the electric potential is constant. The use of this equation shows that these students recognize that the electric field and the electric potential as different quantities. This does not prevent some of them from using the incorrect model in later electric potential items.

By looking at the explanations the students gave, I classified 6 of the 18 students as being users of the incorrect rate of change and electric potential models. Thus it is possible that there are more students that pass the introductory courses, but still use incorrect rate of change models.

## 5.5 Recommendations for Future Research

The physics education research community continues to investigate student difficulties as well as develop new curricula to address these difficulties. In the course of this study, I have contributed to this endeavor as well as opened the doors to future investigations. The following questions are those which I feel are both interesting and related to this study.

*Are student difficulties in other areas related to difficulties with rate of change?*
Since there seems to be a connection between rate of change difficulties and electric potential difficulties, it seems plausible that this connection could exist for other student difficulties. One area that would be particularly interesting is that of magnetic induction.

*Are there any activities on rate of change that affect students' performance on electric potential questions?*
This is a natural follow up question to this study. Since it is likely that students have trouble with rate of change because of their difficulties with rate of change, the logical next step to create or use some rate of change activities in a class and see if it has an effect on students

performance on electric potential questions.  An electric potential diagnostic could be created based on the RAPT since its items have been shown be reliable.

*What are the models upper-level undergraduate physics students use for other areas of physics.*

As I have discussed previously, some upper-level physics students demonstrate the use of models similar to introductory students.  An interesting study would be to take an inventory of the models these upper-level students use.  The interesting thing about the upper-level students is that they have succeeded in traditional physics courses.  With a more complete investigation, it might show that it is possible to perform sufficiently in traditional classes while still using incorrect models.

*Can an instrument be created to survey students' use of models in electric fields and electric potential?*

In my analysis, I have shown that the RAPT is very reliable.  This could be used as the basis for a different instrument, one that can be used to assess student understanding of electric fields and electric potential.  Such an instrument could be used to evaluate the effectiveness of new curricula or to compare active-engagement courses to traditional courses.

## 5.6 Recommendations for Instructors

This study did not attempt to determine the most effective curriculum in the area of electric potential.  I can make recommendations for curriculum that may help students understand electric potential concepts.  These recommendations come from my experience in examining what students have problems with.  These recommendations address the relationship between electric potential and electric field.  I will present some activities that focus on the issues discussed in this study, namely finding the electric field from information about the electric potential.

### 5.6.1 Introduction to Topographical Maps

The following activity introduces students to topographical maps.  Students can work on these questions in groups during a class activity.

Below is a topographical map of a certain region.  Each topographical line represents points that are at the same height above sea level.  The vertical distance between adjacent lines is 10 feet.

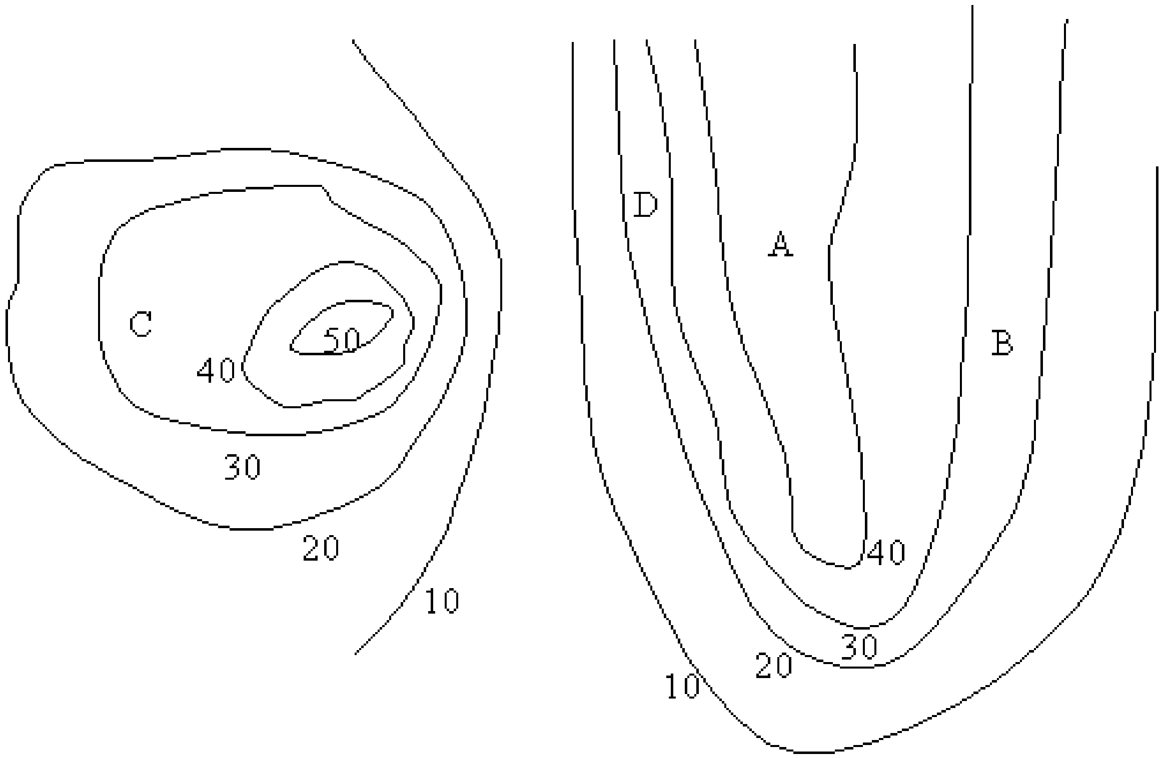



Figure 5.1 – A topographical map used to illustrate important features that relate slope and height of the hill.

1. Describe the region represented by this map
2. Describe the directions a ball would roll if placed at positions A – D.
3. If a ball were placed at location D and another ball were placed at location C and both were released, which would have the greater acceleration? Which has the greater potential energy when released? Which will have the greater speed at the bottom of the hill?
4. What factors does the speed of ball at the bottom of the hill depend on? What factors does the acceleration of the ball depend on?
5. Is it possible to have a zero acceleration, but a non-zero height? Is it possible to have a zero height, but a non-zero acceleration?

*Rationale*

The relationship between a ball's acceleration on a hill and the height of the hill is the same as the relationship between the electric potential and the electric field. The electric field is the gradient of the electric potential just as the acceleration of a ball on a hill depends on the gradient of the hill at that location. The main advantage of the topographical map is that students have a better connection with it than they do with electric potential. All students have seen a hill at some time in their life.

This activity focuses on two main concepts. The acceleration of the ball is related to the slope of the hill, not the height of the hill. The speed of the ball at the bottom of the hill is related to the height of the location the ball was released, not the slope of the hill. Item 5 of the activity address two difficulties that students have with electric potential questions. The first part asks if it is possible to have a zero acceleration but a non-zero height. This is true at location A on the topographical map. The students should be able to see that height and acceleration are not the same thing and are not directly related. This question is similar to the RAPT item V4b which asks about the motion of a proton released at a location where the electric potential is non-zero but the electric field is zero. This is also similar to RAPT item V6 in which the students are asked about the electric field in a region with constant electric potential.

The second part of item 5 asks if it is possible to have zero height but a non-zero acceleration. The answer to this question is yes, if the ball were at sea level but in a region that went below sea level such as Death Valley. Another important point students should get from this is that the location of zero height is actually arbitrary. This question addresses the same concept as the zero=zero case which says that if the height is zero, the acceleration should be zero also. This is similar to the RAPT item V4a in which students are asked about the motion of a proton released at a location where the electric potential is zero, but the electric field is not zero.

### 5.6.2 Equipotential Maps

The following activity is analogous to the topographical activity except that it deals with the relationship between electric potential and electric field instead of height and acceleration.

Below are equipotential lines representing the electric potential for a certain region of space. Each equipotential line consists of point that have the same value of electric potential.

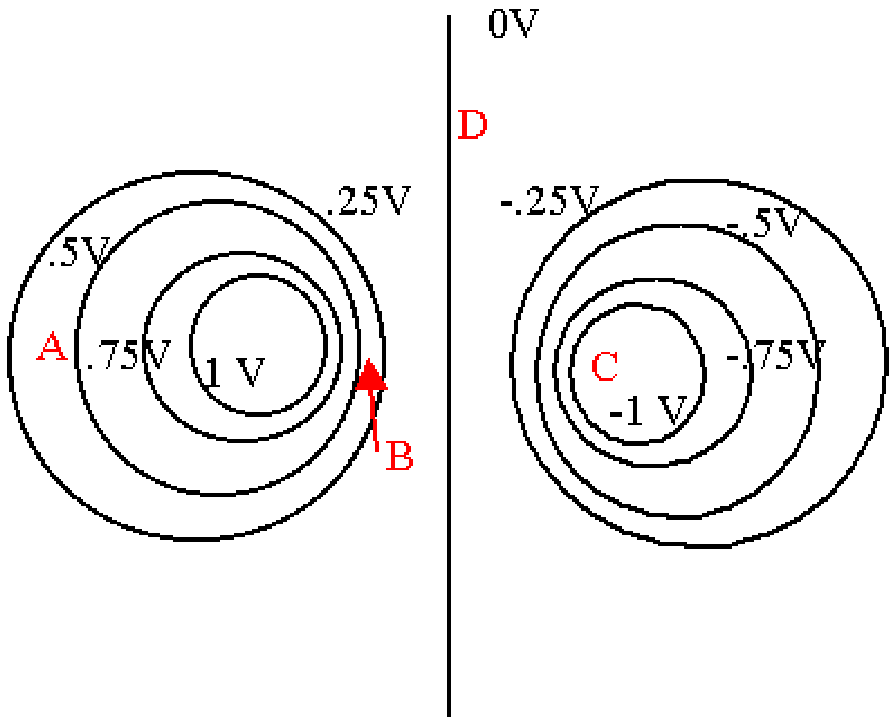



Figure 5.2 – An equipotential map used to address student difficulties with electric potential and electric field.

1. Describe the charges that could create equipotential lines such as shown above.
2. Describe the force a proton would feel at locations A and B.
3. Where could a proton be placed so that there is no net force on it?
4. At which point is the magnitude of the electric field the greatest?
5. Is it possible to have a zero electric field, but a non-zero electric potential?
6. Is it possible to have a zero electric potential, but a non-zero electric field?

*Rationale*

This activity addresses the same issues as the topographical map activity. These two activities should be used together, that is if students have problems answering a question from the equipotential map, have them first discuss the same question with the topographical map.

### 5.6.3 Three-Dimensional Plot of the Electric Potential

The similarities between electric potential and hills can be seen more easily with a three-dimensional plot of a two-dimensional equipotential. In these plots, the height of the graph is related to the electric potential. Below is an activity that uses the three-dimensional electric potential plot.

Below is a representation of the electric potential in a region of space. The height of the surface represents the value of the electric potential at that point.

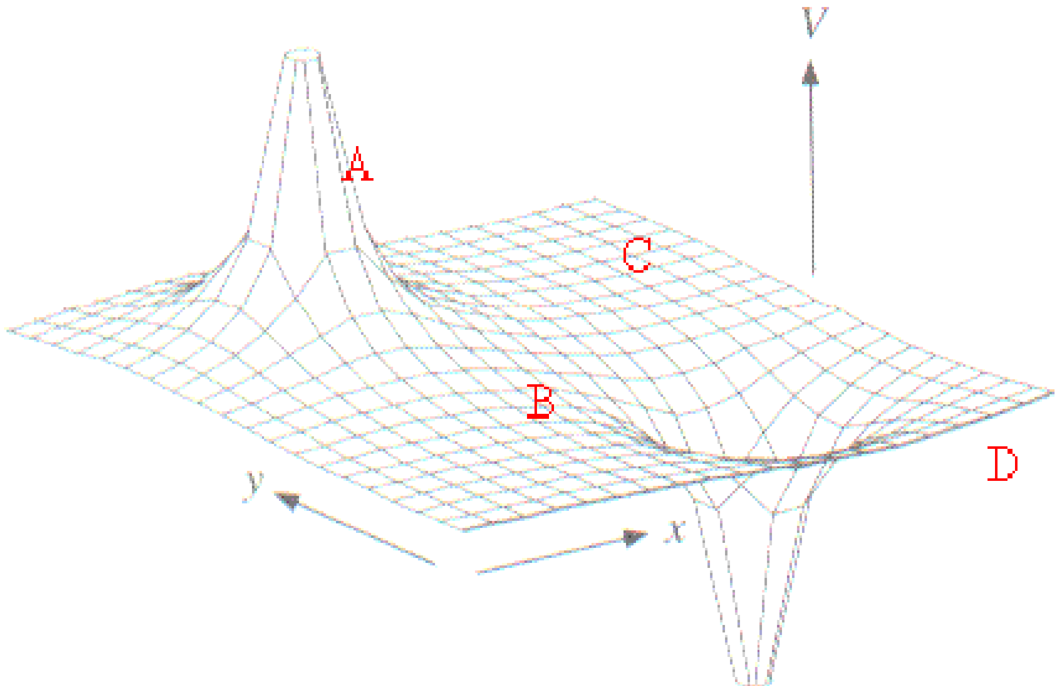



Figure 5.3 – 3-D plot of the electric potential for a region of space

1. Describe the charges that would produce this electric potential.
2. Where (if anywhere) is the value of the electric potential approximately zero?
3. Where is the magnitude of the electric field approximately zero?
4. At what point is the magnitude of the electric field the greatest?
5. What direction would a proton move if released at point B?
6. What direction would a proton move if released at point A?

*Rationale*

If students have difficulty with this activity, they can answer questions regarding a ball placed on a hill of the similar shape. The electric field at point C is nearly zero and a ball place at point C on a similar hill would have no acceleration. The electric field is greatest at point A and this would also be the location of the greatest acceleration if this were a hill.

Although many students are unfamiliar with topographical maps, they can be a useful analogy to electric potential. It can be productive to review topographical maps before discussing electric potential. Topographical maps have many of the key features that can help students see the difference between electric potential and electric field. For the topographical map, the height of a hill is similar to the electric potential and the acceleration of a ball released from rest is like the electric field. When students are confused about the electric field and the electric potential, it can be helpful to point to a similar situation with topographical maps.

## 5.7 Concluding Statement

From the findings presented in this study, I believe that one of the major hindrances to students' understanding electric potential is their difficulties with the concept of rate of change. I suggest that future curriculum in the area of electric potential incorporate activities that assist students in obtaining a solid model for the relation between a quantity and its rate of change.

**Appendix A – The Rate and Potential Test Version A**

# Physics Education Research Instrument

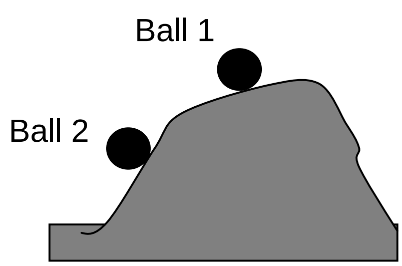
**Version A**

**The following questions are designed to probe your understanding of physics for research purposes. You will not be penalized for incorrect answers. As the data from these questions will be used to improve physics curricula, please answer the questions thoughtfully.**

Questions 1 – 3 refer to the following passage. A coin is tossed straight up into the air. After it is released it moves upward, reaches its highest point and falls back down again. Choose the correct description of the acceleration of the coin for the three phases. Take up to be the positive direction.

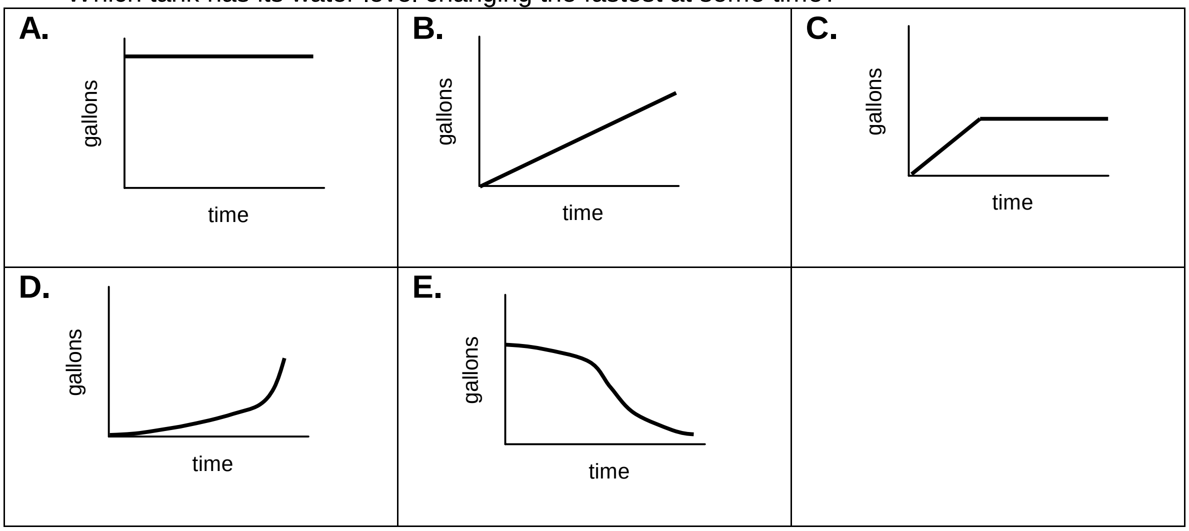
1. While the coin is moving upward after it is released.
    a. The acceleration is in the positive direction and decreasing
    b. The acceleration is in the positive direction and constant
    c. The acceleration is in the positive direction and increasing
    d. The acceleration is zero
    e. The acceleration is in the negative direction and decreasing
    f. The acceleration is in the negative direction and increasing
    g. The acceleration is in the negative direction and constant
2. When the coin is at its highest point.
    a. The acceleration is in the positive direction and decreasing
    b. The acceleration is in the positive direction and constant
    c. The acceleration is in the positive direction and increasing
    d. The acceleration is zero
    e. The acceleration is in the negative direction and decreasing
    f. The acceleration is in the negative direction and increasing
    g. The acceleration is in the negative direction and constant
3. While the coin is moving downward.
    a. The acceleration is in the positive direction and decreasing
    b. The acceleration is in the positive direction and constant
    c. The acceleration is in the positive direction and increasing
    d. The acceleration is zero
    e. The acceleration is in the negative direction and decreasing
    f. The acceleration is in the negative direction and increasing
    g. The acceleration is in the negative direction and constant

4. Two identical balls are placed at the two locations on a hill as shown below.

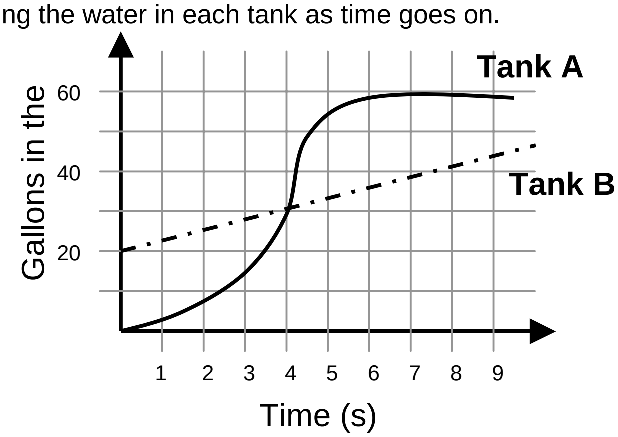



Both balls are released from rest. At the moment they are released, which ball will have the greater acceleration?
    a. Ball 1 will have the greater acceleration
    b. Ball 2 will have the greater acceleration
    c. Both balls will have the same acceleration
    d. Not enough information is given to compare the accelerations of the balls

5. Several tanks are being filled by hoses with adjustable nozzles (so that the amount of water coming out of the hose can be changed). The tanks also have plugs that allow them to be drained. Below are graphs of the amount of water (in gallons) in the tanks as time goes on. Which tank has its water level changing the fastest at some time?

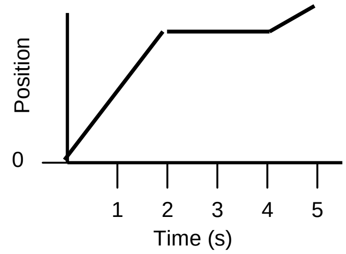



Questions 6 – 8 refer to two tanks that are being filled by separate water hoses. Below is a graph representing the water in each tank as time goes on.



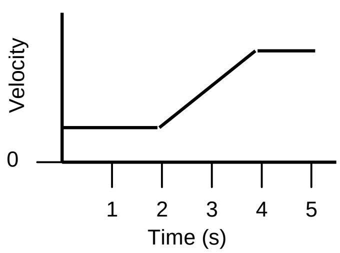

6. Which tank is filling up faster at $t = 3$ seconds?
   a. Tank A is filling up faster
   b. Tank B is filling up faster
   c. Neither tank is filling up faster
   d. There is not enough information to determine the answer
7. Which tank is filling up faster at $t = 4$ seconds?
   a. Tank A is filling up faster
   b. Tank B is filling up faster
   c. Neither tank is filling up faster
   d. There is not enough information to determine the answer
8. Which tank is filling up faster at $t = 8$ seconds?
   a. Tank A is filling up faster
   b. Tank B is filling up faster
   c. Neither tank is filling up faster
   d. There is not enough information to determine the answer

9. The following is a position-time graph for an object during a 5 second time interval.

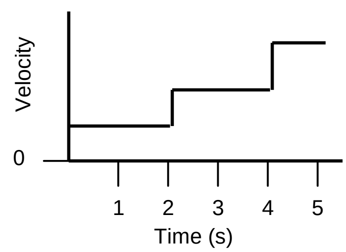



Which of the following graphs of velocity versus time would best represent the object's motion during the same time interval?

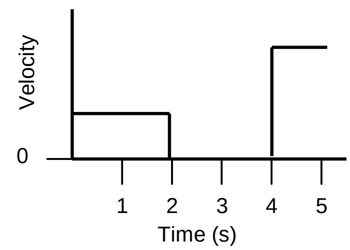
**A.**



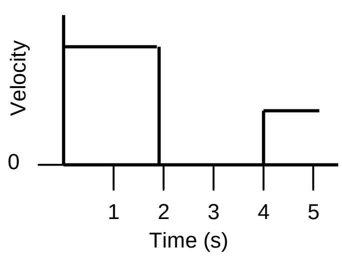

**B.**



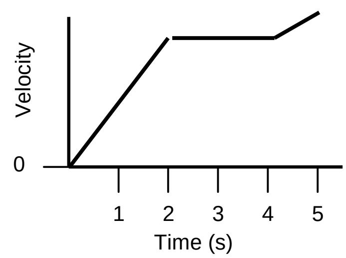

**C.**



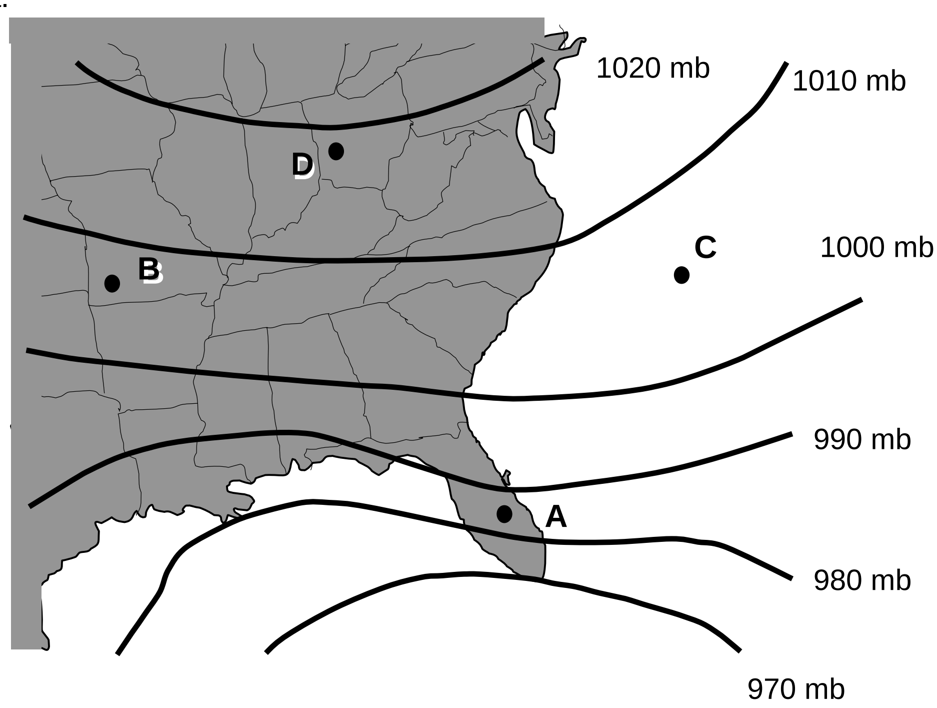

**D.**




**E.**



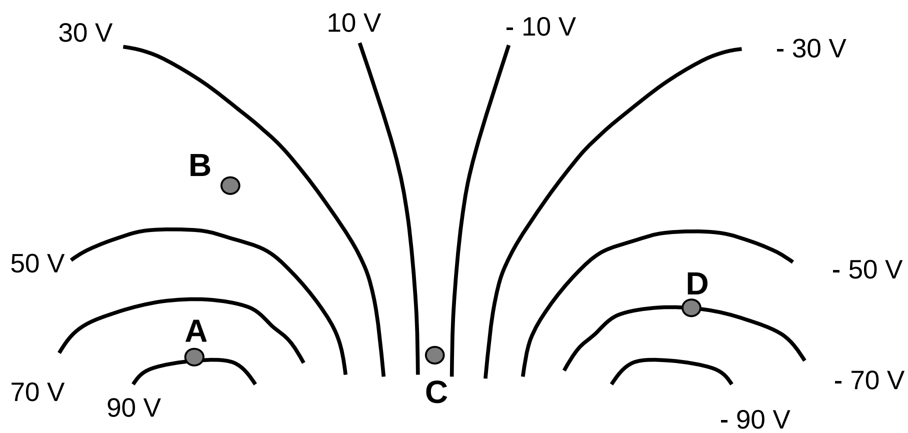

Questions 10 – 13 refer to the weather map of part of the U. S. The map displays isobars. Isobars are lines along which the barometric pressure is the same. A map of isobars is useful for locating areas of strong winds. The strongest winds are found in regions where the pressure is changing the most.



10. At which point (A, B, C, D, or E for none) is the wind the strongest?
11. At which point (A, B, C, D, or E for none) is the wind the weakest?
12. At which point (A, B, C, D, or E for none) is the pressure the greatest?
13. At which point (A, B, C, D, or E for none) is the pressure the lowest?

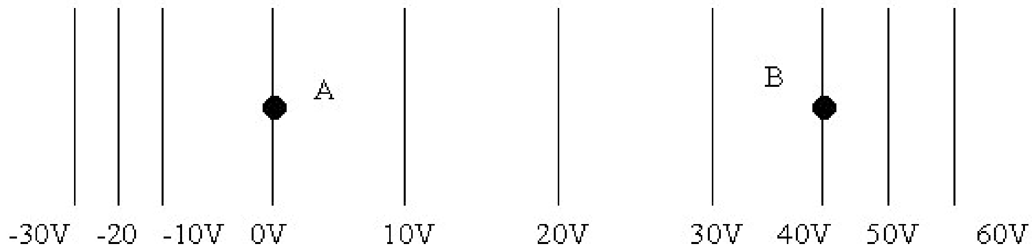
14. A ball is rolling on a flat surface with a constant, non-zero velocity. Which statement describes the magnitude of the acceleration of the ball?
    a. The acceleration is zero
    b. The acceleration is constant, but non-zero
    c. The acceleration is increasing
    d. The acceleration is decreasing
    e. There is not enough information to describe the acceleration

Questions 15 – 18 refer to the region of space with an electric field represented by the equipotential lines as shown below. (An equipotential line is a line along which the value of the electric potential remains the same).




15. At which of the points (A, B, C, D, or E for none) is the value of the electric potential the greatest?
16. At which of the points (A, B, C, D, or E for none) is the value of the electric potential equal to zero?
17. At which of the points (A, B, C, D, or E for none) is the magnitude of the electric field the greatest?
18. At which of the points (A, B, C, D, or E for none) is the magnitude of the electric field equal to zero?

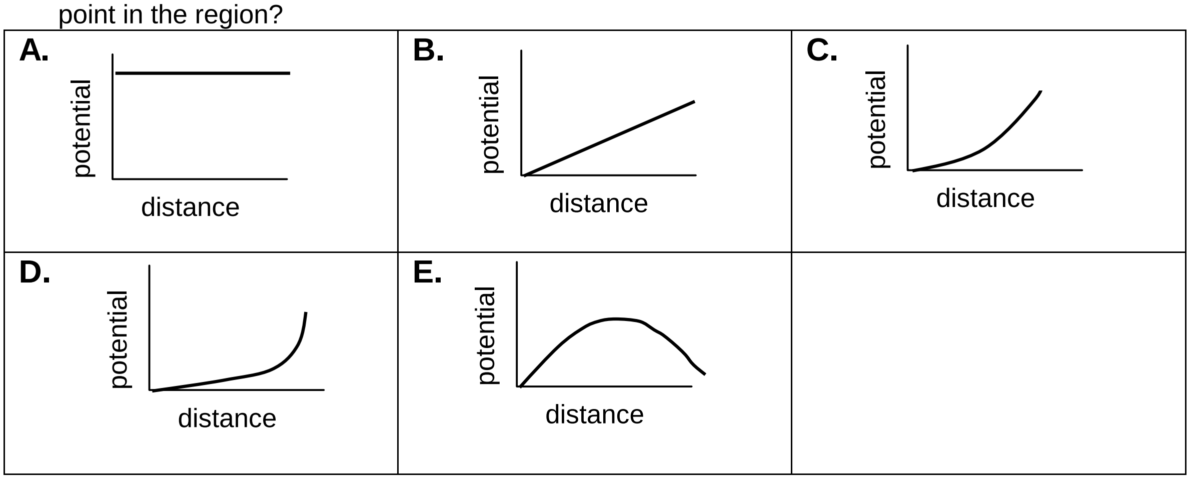
Question 19 and 20 refer to a region of space that has an electric potential described by the equipotential lines as shown below. An equipotential line is a line containing all the points with the same value of the electric potential.



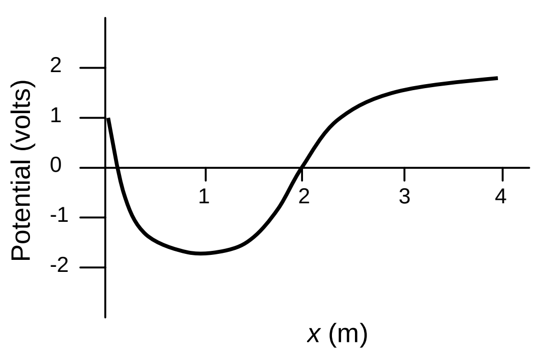

19. A proton is placed at point A. If released from rest, what will its motion be?
    a. The proton will not move
    b. The proton will move to the right
    c. The proton will move to the left
    d. The motion can not be determined

20. The proton is now moved to point B and released from rest, what will its motion be?
    a. The proton will not move
    b. The proton will move to the right
    c. The proton will move to the left
    d. The motion of the proton can not be determined

21. Five regions with different electric fields are represented below by the graphs of electric potential (V) vs. displacement (x). Which graph represents the region with the largest electric field at some point in the region?



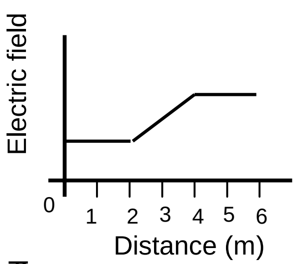

Question 22 and 23 refer to the region of space with an electric potential described by the following graph of electric potential (V) vs. distance (x).

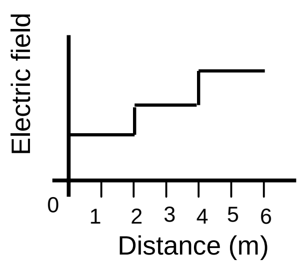


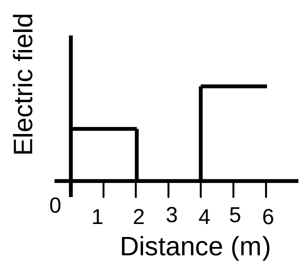
22. A proton is released from rest at $x = 2$ meters in a region as represented by the above graph. What will be the motion of the proton?
    a. The proton will not move
    b. The proton will move to the left
    c. The proton will move to the right
    d. The motion of the proton can not be determined
23. The proton is now released at $x = 1$ meters in the region represented by the above graph. What will be the motion of the proton?
    a. The proton will not move
    b. The proton will move to the left
    c. The proton will move to the right
    d. The motion of the proton can not be determined

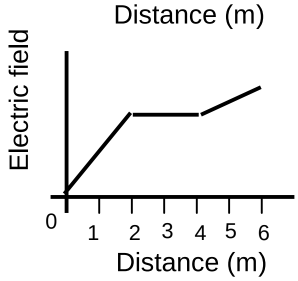
24. The following is a graph of electric potential ($V$) vs. distance ($x$) for a certain region of space.

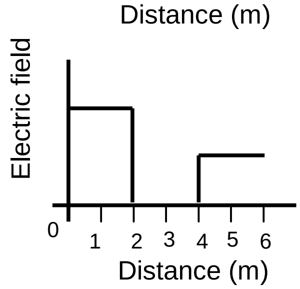

Which of the following graphs of the **magnitude** of the electric field ($E$) vs. distance ($x$) corresponds to the same region?

**A.**



**B.**



**C.**



**D.**



**E.**



25. The electric potential in a certain region is at a constant, non-zero value. Which statement describes the magnitude of the electric field in this region?
    a. The electric field is zero in this region
    b. The electric field is constant, but non-zero in this region
    c. The electric field is increasing in this region
    d. The electric field is decreasing in this region
    e. There is not enough information to describe the electric field in this region

**Appendix B – The Rate and Potential Test Version B**

# Physics Education Research Instrument

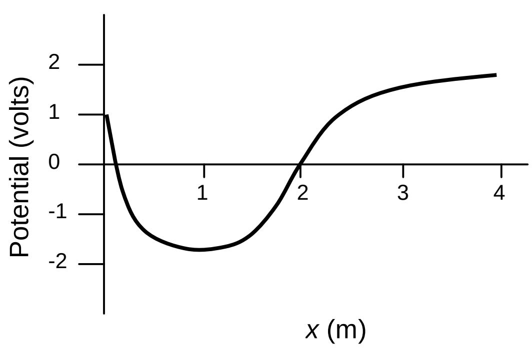
**Version B**

**The following questions are designed to probe your understanding of physics for research purposes. You will not be penalized for incorrect answers. As the data from these questions will be used to improve physics curricula, please answer the following questions thoughtfully.**

Questions 1 – 3 refer to the following passage. A coin is tossed straight up into the air. After it is released it moves upward, reaches its highest point and falls back down again. Choose the correct description of the acceleration of the coin for the three phases. Take up to be the positive direction.

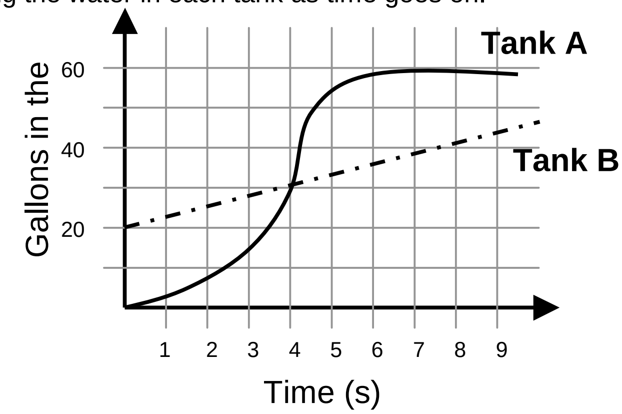
1. While the coin is moving upward after it is released.
    a. The acceleration is in the positive direction and decreasing
    b. The acceleration is in the positive direction and constant
    c. The acceleration is in the positive direction and increasing
    d. The acceleration is zero
    e. The acceleration is in the negative direction and decreasing
    f. The acceleration is in the negative direction and increasing
    g. The acceleration is in the negative direction and constant
2. When the coin is at its highest point.
    a. The acceleration is in the positive direction and decreasing
    b. The acceleration is in the positive direction and constant
    c. The acceleration is in the positive direction and increasing
    d. The acceleration is zero
    e. The acceleration is in the negative direction and decreasing
    f. The acceleration is in the negative direction and increasing
    g. The acceleration is in the negative direction and constant
3. While the coin is moving downward.
    a. The acceleration is in the positive direction and decreasing
    b. The acceleration is in the positive direction and constant
    c. The acceleration is in the positive direction and increasing
    d. The acceleration is zero
    e. The acceleration is in the negative direction and decreasing
    f. The acceleration is in the negative direction and increasing
    g. The acceleration is in the negative direction and constant

Question 4 and 5 refer to the region of space with an electric potential described by the following graph of electric potential (*V*) vs. distance (*x*).



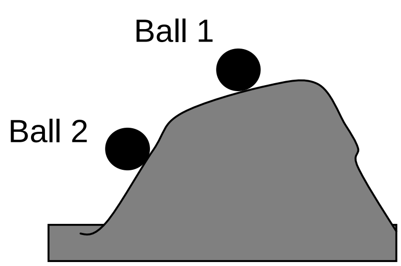

4. A proton is released from rest at $x$ = 2 meters in a region as represented by the above graph. What will be the motion of the proton?
    a. The proton will not move
    b. The proton will move to the left
    c. The proton will move to the right
    d. The motion of the proton can not be determined
5. The proton is now released at $x$ = 1 meters in the region represented by the above graph. What will be the motion of the proton?
    a. The proton will not move
    b. The proton will move to the left
    c. The proton will move to the right
    d. The motion of the proton can not be determined

Questions 6 – 8 refer to two tanks that are being filled by separate water hoses. Below is a graph representing the water in each tank as time goes on.



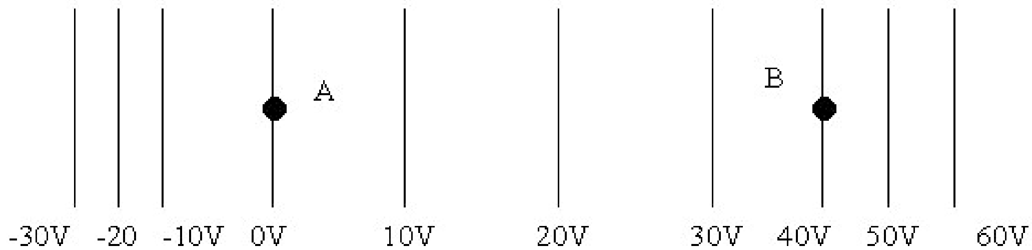

6. Which tank is filling up faster at 3 seconds?
   a. Tank A is filling up faster
   b. Tank B is filling up faster
   c. Neither tank is filling up faster
   d. There is not enough information to determine the answer
7. Which tank is filling up faster at 4 seconds?
   a. Tank A is filling up faster
   b. Tank B is filling up faster
   c. Neither tank is filling up faster
   d. There is not enough information to determine the answer
8. Which tank is filling up faster at 8 seconds?
   a. Tank A is filling up faster
   b. Tank b is filling up faster
   c. Neither tank is filling up faster
   d. There is not enough information to determine the answer

9. Two identical balls are placed at the two locations on a hill as shown below.



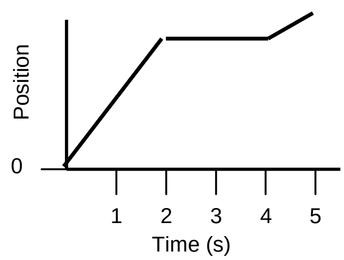

Both balls are released from rest. At the moment they are released, which ball will have the greater acceleration?
   a. Ball 1 will have the greater acceleration
   b. Ball 2 will have the greater acceleration
   c. Both balls will have the same acceleration
   d. Not enough information is given to compare the accelerations of the two balls

Question 10 and 11 refer to the region of space that has an electric potential described by the equipotential lines shown below. An equipotential line is a line containing all the points with the same value of electric potential.

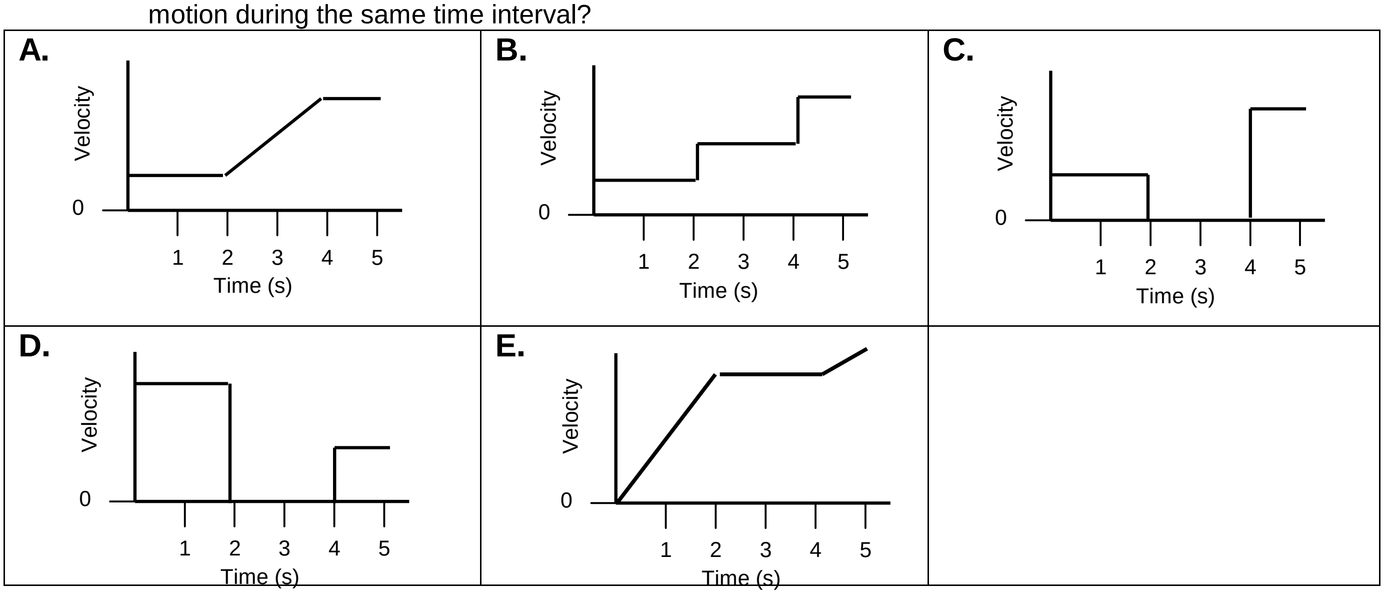



10. A proton is placed at point A. If released from rest, what will its motion be?
    a. The proton will not move
    b. The proton will move to the right
    c. The proton will move to the left
    d. The motion of the proton can not be determined
11. The proton is now moved to point B and released from rest, what will its motion be?
    a. The proton will not move
    b. The proton will move to the right
    c. The proton will move to the left
    d. The motion of the proton can not be determined

12. The following is a position-time graph for an object during a 5 second time interval.

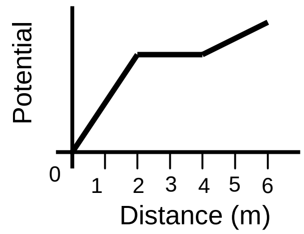



Which of the following graphs of velocity versus time would best represent the object's motion during the same time interval?



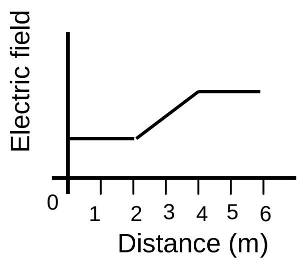
| | | |
|---|---|---|
| | | |

13. The following is a graph of electric potential ($V$) vs. distance ($x$) for a certain region of space.

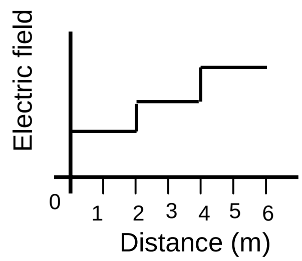



Which of the following graphs of the **magnitude** of the electric field ($E$) vs. distance ($x$) corresponds to the same region?

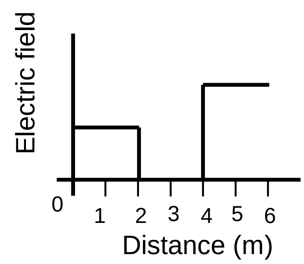
**A.**



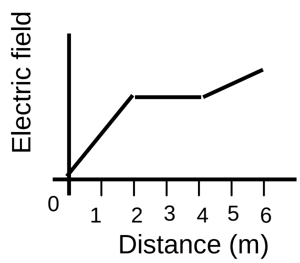

**B.**



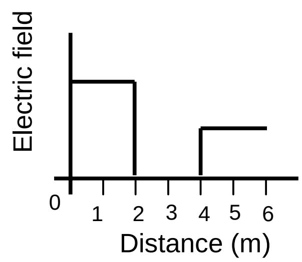

**C.**



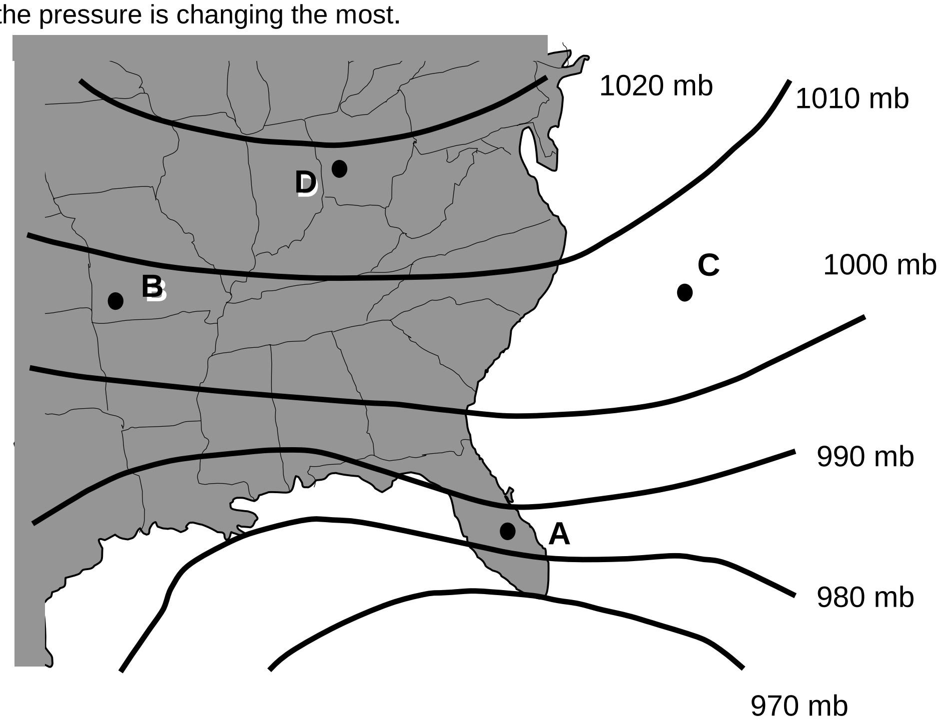

**D.**




**E.**

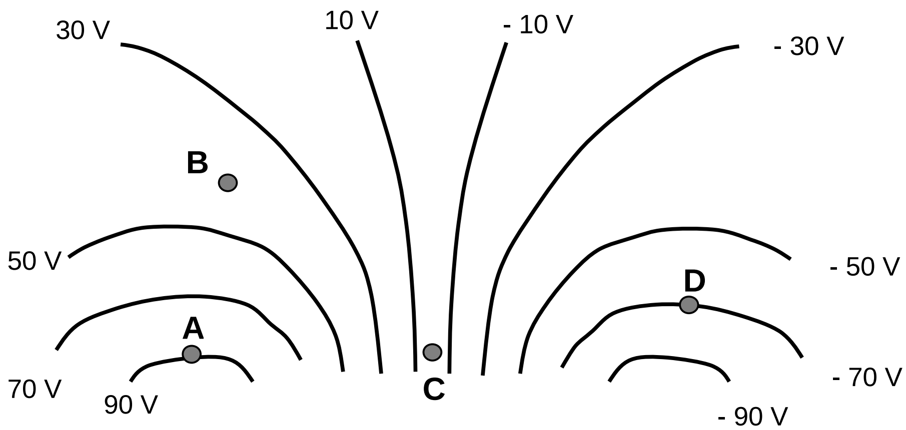



Questions 14 – 17 refer to the weather map of part of the U.S. The map displays isobars. Isobars are lines along which the barometric pressure is the same. A map of isobars is useful for locating areas of strong winds. The strongest winds are found in regions where the pressure is changing the most.



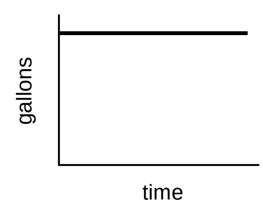
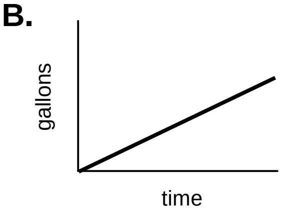
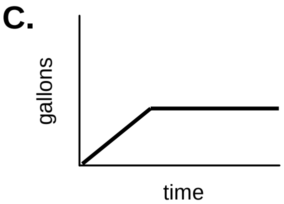
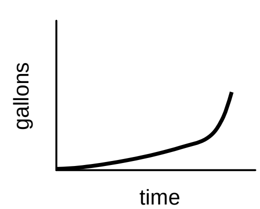
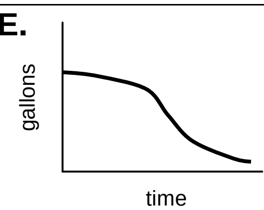
14. At which point (A, B, C, D, or E for none) is the wind the strongest?
15. At which point (A, B, C, D, or E for none) is the wind the weakest?
16. At which point (A, B, C, D, or E for none) is the pressure the greatest?
17. At which point (A, B, C, D, or E for none) is the pressure the lowest?

Questions 18 – 21 refer to the region of space with an electric field represented by the equipotential lines as shown below. (An equipotential line is a line along which the value of the electric potential remains the same).

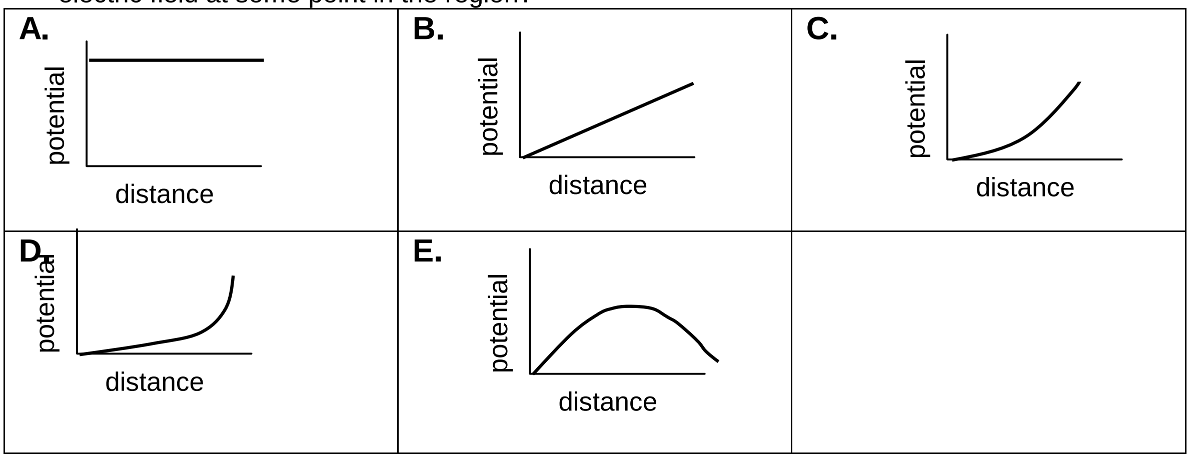



18. At which of the points (A, B, C, D, or E for none) is the value of the electric potential the greatest?
19. At which of the points (A, B, C, D, or E for none) is the value of the electric potential equal to zero?
20. At which of the points (A, B, C, D, or E for none) is the magnitude of the electric field the greatest?
21. At which of the points (A, B, C, D, or E for none) is the magnitude of the electric field equal to zero?

22. Several tanks are being filled by hoses with adjustable nozzles (so that the amount of water coming out of the hose can be changed). The tanks also have plugs that allow them to be drained. Below are graphs of the amount of water (in gallons) in the tanks as time goes on. Which tank has its water level changing the fastest at some time?

| | | |
|---|---|---|
| **A.**   | **B.**   | **C.**   |
| **D.**   | **E.**   | |

23. Five regions with different electric fields are represented below by the graphs of electric potential (V) vs. displacement (x). Which graph represents the region with the largest electric field at some point in the region?




24. A ball is rolling on a flat surface with a constant, non-zero velocity. Which statement describes the magnitude of the acceleration of the ball?
    a. The acceleration is zero
    b. The acceleration is constant, but non-zero
    c. The acceleration is increasing
    d. The acceleration is decreasing
    e. There is not enough information to describe the acceleration

25. The electric potential in a certain region is at a constant, non-zero value. Which statement describes the magnitude of the electric field in this region?
    a. The electric field is zero in this region
    b. The electric field is constant, but non-zero in this region
    c. The electric field is increasing in this region
    d. The electric field is decreasing in this region
    e. There is not enough information to describe the electric field in this region

## Appendix C – Sub-Test Version of RAPT

**Name:**________________________________

**The following questions are designed to probe your understanding of physics for research purposes. You will not be penalized for incorrect answers. As the data from these questions will be used to improve physics curricula, please answers the questions thoughtfully.**

Questions 1 – 3 refer to the following passage. A coin is tossed straight up into the air. After it is released it moves upward, reaches its highest point and falls back down again. Choose the correct description of the acceleration of the coin for the three phases. Take up to be the positive direction.

1. While the coin is moving upward after it is released.
    a. The acceleration is in the positive direction and decreasing
    b. The acceleration is in the positive direction and constant
    c. The acceleration is in the positive direction and increasing
    d. The acceleration is zero
    e. The acceleration is in the negative direction and decreasing
    f. The acceleration is in the negative direction and increasing
    g. The acceleration is in the negative direction and constant

**Explain the reason for your answer:**

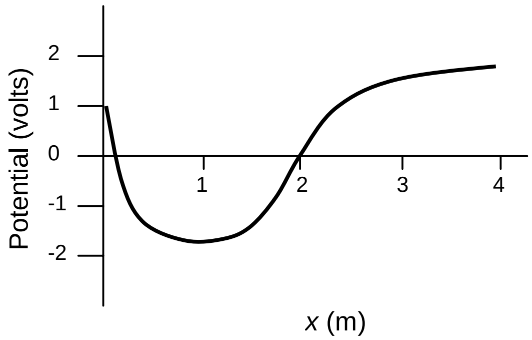
2. When the coin is at its highest point.
    a. The acceleration is in the positive direction and decreasing
    b. The acceleration is in the positive direction and constant
    c. The acceleration is in the positive direction and increasing
    d. The acceleration is zero
    e. The acceleration is in the negative direction and decreasing
    f. The acceleration is in the negative direction and increasing
    g. The acceleration is in the negative direction and constant

**Explain the reason for your answer:**

3. While the coin is moving downward.
    a. The acceleration is in the positive direction and decreasing
    b. The acceleration is in the positive direction and constant
    c. The acceleration is in the positive direction and increasing
    d. The acceleration is zero
    e. The acceleration is in the negative direction and decreasing
    f. The acceleration is in the negative direction and increasing
    g. The acceleration is in the negative direction and constant

**Explain the reason for your answer:**

Question 4 and 5 refer to the region of space with an electric potential described by the following graph of electric potential (V) vs. distance (x).




4. A proton is released from rest at $x = 2$ meters in a region as represented by the above graph. What will be the motion of the proton?
   a. The proton will not move
   b. The proton will move to the left
   c. The proton will move to the right
   d. The motion of the proton can not be determined

**Explain the reason for your answer:**

5. The proton is now released at $x = 1$ meters in the region represented by the above graph. What will be the motion of the proton?
   a. The proton will not move
   b. The proton will move to the left
   c. The proton will move to the right
   d. The motion of the proton can not be determined

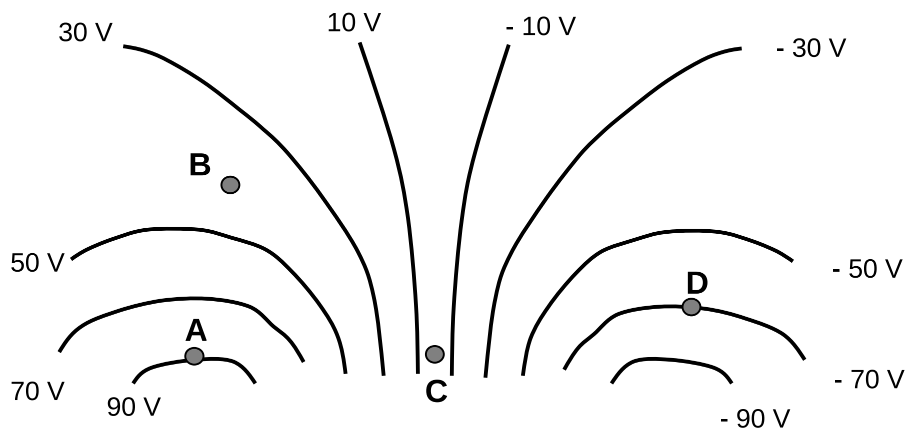
**Explain the reason for your answer:**

6. The electric potential in a certain region is at a constant value of 12 volts. Which statement describes the magnitude of the electric field in this region?
   a. The electric field is zero in this region
   b. The electric field is constant, but non-zero in this region
   c. The electric field is increasing in this region
   d. The electric field is decreasing in this region

**Explain the reason for your answer:**

There is not enough information to describe the electric field in this region

Questions 7 - 10 refer to the region of space with an electric field represented by the equipotential lines as shown below. (An equipotential line is a line along which the value of the electric potential remains the same).




7. At which of the points (A, B, C, D, or E for none) is the value of the electric potential the greatest?

**Explain the reason for your answer:**

8. At which of the points (A, B, C, D, or E for none) is the value of the electric potential equal to zero?

**Explain the reason for your answer:**

9. At which of the points (A, B, C, D, or E for none) is the magnitude of the electric field the greatest?

**Explain the reason for your answer:**

10. At which of the points (A, B, C, D, or E for none) is the magnitude of the electric field equal to zero?

**Explain the reason for your answer:**

**Appendix D – Transcript of Think-Aloud Interview with Joe**

The structure of the think-aloud interviews was described in chapter 3. The format was such that the student first went through the items using the think-aloud protocol and then went back and gave an explanation for his or her answers. Since the student responded to each item twice, I will present both of the student's responses for a particular item together. That is the transcript below is not in chronological order. For each question, the student's think-aloud response will be followed by that student's explanation for the same item.

**Student:** Joe
**Date:** 2/21/01
**RAPT Version A**

**R1a**
**Think-Aloud:** While the coin is moving upward, after it is released - it is going to be a negative acceleration because gravity is going to be pointing downward. …B the acceleration is positive, no…D it is zero..no…it is negative and decreasing…the acceleration is always going to be constant because acceleration does not vary. The speed varies, so it is going to be choice G.

**Explanation:** While the coin is moving upward, after it is released - it is going to be a negative acceleration because gravity is going to be pointing downward. …B the acceleration is positive, no…D it is zero..no…it is negative and decreasing…the acceleration is always going to be constant because acceleration does not vary. The speed varies, so it is going to be choice G.

**R1b**
**Think-Aloud:** When the coin is at its highest point, the acceleration is still in the negative direction. The acceleration is going to be the same in all…just the velocity is going to change. I would say that the acceleration…the acceleration is going to be zero because it is at its highest point. Its not going to be positive. It is not going to be...I think it is zero, it is at its highest point.

**Explanation:** At the highest point, it has no acceleration. Oh, well, the acceleration is still pointing downward, but the coin itself is not accelerating. I mean the force is pointing downward, so the acceleration is going to be zero.

**R1c**

**Think-Aloud:** While the coin is moving downward…the acceleration is positive in the direction it is moving since it is moving downward…it is not negative, negative, negative, or zero…it is going to be positive and constant.

**Explanation:** When the coin is moving downward, the acceleration is going to be in the same way. It is still going to be constant.

**R2**

**Think-Aloud:** Ok…I say ball 2 because it is going to be …steeper slope. Ball 1 will travel more in the x direction. Ball 2 would be my guess.

**Explanation:** For number 4, I chose B because ball 2 has the highest slope so it is going to accelerate faster and ball 1, which is going to go slower. Ball 2 is going to go faster.

**R3**

**Think-Aloud:** ok…gallons per time….ok, it wants the steepest slope I would say. A…ok it looks like D at the end that has the steepest slope at a time…steeper than this one. Maybe it could be E or D. Well…the picture is not too clear, I should say that the greatest change is...it is not A, B, or C. I would have to go D looks the steepest.

**Explanation:** I chose D because the rate of change from at some time is the slope of the line. And I chose D - looks like the highest slope. The steepest slope.

**R4a**

**Think-Aloud:** Oh…wait, this is the same question no? Oh, no…the fastest tank A, it has a steeper slope.

**Explanation:** ok same thing. Tank A has the higher slope change in t 3 and t 4.

**R4b**

**Think-Aloud:** At t 4 seconds…it is still tank A …it is just the slope.

**Explanation:** Even though they have the same gallons at t 4, the tank A has a higher slope at that point

**R4c**

**Think-Aloud:** T 8 seconds…ok this is tank B because the other one has ….

**Explanation:** At t 8, tank B has a higher slope so I chose that.

**R5**

**Think-Aloud:** The velocity is going to be slope…the slope is constant at 2….it is going to be slope…2 is going to be zero…2 to 4…so …2 to 4 is going to be zero so that eliminates A, B, and E…it is either C or D. The slope has to be…This slope is going to be smaller than this one. It is going to be D.

**Explanation:** For number 9, I chose D because from 2 to 4 is going to be zero, the slope going be zero. That cancels out A, B, E.. And then I know that the slope from 0 to 2 is gonna be steeper than the slope from 4 to 5, that makes D.

**R6a**

**Think-Aloud:** Stronger winds, pressure is changing the most. The pressure changes the most as lines get closer together right? Yes…at point A, the lines are closest together, the winds change more. The winds are going to be strongest at point A.

**Explanation:** For 10, I chose A because the winds are going to be the strongest when the lines are closest together

**R6b**

**Think-Aloud:** Which point is the wind the weakest? That's C. The lines are further apart. Yeah, I would say C.

**Explanation:** The weakest when the lines are farther apart ©

**R6c**

**Think-Aloud:** At which point is the pressure the greatest? Pressure the greatest at point D. That is the one with more pressure. D.

**Explanation:** For 12, I chose point D because the pressure is going to be greatest where it says that the pressure is going to be greatest. The higher you go up on the map, the pressure goes up.

**R6d**

**Think-Aloud:** Pressure the lowest…point A. It is 980.

**Explanation:** At point A because, it is the opposite of the last one.

**R7**

**Think-Aloud:** Ball is rolling on a flat surface…ok…constant non-zero velocity…ok it has v. It would be a constant …the acceleration is zero. It says the ball is rolling at constant velocity…then the acceleration is zero. If it were decreasing the velocity, there would be an acceleration, but there is non.

**Explanation:** ok for 14, I chose A because it says that it has a constant, non-zero velocity. That means that it is not accelerating, so the acceleration is zero.

**V1a**

**Think-Aloud:** Ok..10 Volts…Potential, value the greatest…electric potential…that is just V. 90…yeah. Oh, wait…at which points is the value of the electric potential the greatest? Yeah…the electric field is farther away the potential is just where they are. Ok, so it is going to be point A.

**Explanation:** For 15 I chose…the value…point A because that is what it says. It says point A is lying on the line that is 90 volts.

**V1b**

**Think-Aloud:** At which point is the value…zero? At point C it is at the middle.

**Explanation:** It is going to be equal to zero at point C. Where it is going to be between 10 and minus 10.

**V1c**

**Think-Aloud:** At which of the points is the magnitude of the electric field the greatest? The electric field is …lines that are…making 90 degree angles like that [draws some field lines perpendicular to the equipotential lines] So, the electric field is going to be farthest away at point B. At which of the points is the magnitude of the electric field the greatest? Electric field...is going to be highest...the electric field is going to be highest when they are closer together or farther apart?

**Explanation:** For 17 I chose B because I drew the electric field and it was the farthest one going out.

**V1d**

**Think-Aloud:** Ok, equal to zero…obviously B is not equal to zero. If it were the weakest, then I would have trouble between B and C, but since it says zero…I am going to go with C being zero and B being farthest away. The greatest is going to be B.

**Explanation:** For 18 I chose C because it was the neutral one, it was in the center so it was zero.

**V2a**

**Think-Aloud:** ok…proton will want to go to the negative side. It is at zero volts and this is the negative side and this the positive side. The proton will want to move from poistive to negatve - to the left.

**Explanation:** For 19 I chose c the proton…since it has a positive charge is going to want to the negative charges.

**V2b**

**Think-Aloud:** It is still going to be to the left. It is still to the more negative side. Let me think. Yeah, because you…I remember…change, you could have negative part of this side and positive part on this side and having something like - 40 volts and positive 40 volts, so that means that you can have a zero spot somewhere in the middle, but it is still going to move to the same side. So it is going to be to the left.

**Explanation:** The same for 20, the proton will move to the left because it is still the negative …its moving towards the negative side and away from the positive.

**V3**

**Think-Aloud:** ok…The electric potential to displacement. Electric field…ok…from the last question, we saw that …E was…magnitude of the electric it was B…if it was farther away. Ok, this is kinda tricky. There is obviously some relation here with…I am going to have to go with slope because they are not giving me any values. It said that some distance, it doesn't say...seems like D is going to be...something potential increasing. Ok I will have to go with D because they are not giving me any numbers. I am going to have to go with the slope.

**Explanation:** For 21, I chose D because I wasn't too sure. I wasn't too sure with the electric potential and displacement. Since they didn't give me any values, I figured this is going to be the slope. So, if I went for the slope in 21, I took my chances I went from 21 to 25 assuming that the electric potential versus the displacement is ....the electric field is going to be the slope of the potential versus distance time graph so...I went from there to 22

**V4a**

**Think-Aloud:** Potential…it is going to be zero. So, there is no potential, there is no motion. If it had some potential…it would. ..potential here is zero, so it is not move. Not move. Zero.

**Explanation:** 22 I answered A because at x=2, it is already at zero potential and when it is at zero potential it is no motion.

**V4b**

**Think-Aloud:** At 1, it has negative potential, so it will move. To the left or to the right? It is not saying…proton. At x=1 it is going to be moving towards…it does not say to the left or to the right because they are not saying where the positive or the negative...ok, it could move anyway. If it wants to reach like zero potential, it can move to 2 or it can move...ok this is closer. If I had to guess, it is not going to be to the right. It would be between B and D. They are not going to give something can not be determined. So, since...the potential being zero is closer to the left...I have to go to the left. To the left.

**Explanation:** In 23 I chose B because if it is …at 1, then wanting to get to zero potential, it is closer to move to the left towards the line at which …potential is going to be zero at 0. Something meters and not at 2 the difference is left. It will move to the left.

**V5**

**Think-Aloud:** ok..so, it would be a read on slope. From 2 to 4 it is zero…it is the same graph as the one before. Is it going to be the integral? I could get an area from that ….but the derivative. So from 2 to 4 is zero…so A, B, D, ok - the slope is going to be highest on the first part than on the second part. It is going to be E. It is the same shape as before.

**Explanation:** With the same slope theory in 24, so I chose E. The slope from 2 to 4 is going to be zero leaving C and E. The slope is going to be highest from o 0 to 2.

**V6**

**Think-Aloud:** If I had a formula, at least I could look it up. Ok…if it is constant…it means, it has got to be a horizontal line. It is not changing. They are not saying anything about distance. Umm…do they mean constant like over some distance or just at a point, in a certain region of the distance. I am going to have to go with this. It is going to be zero. In a certain region...it is not saying just a point...it is not saying...oh this is a region. From 2 to 4 is a region, so I am going to with zero, because it is going to be the slope. Is that correct.

**Explanation:** Where it is constant, I assume there is going to be a graph with a straight horizontal line which is…oh, that doesn't make much sense. It could be constant, just linear. I am going to choose E and change my answer. It doesn't say, it just says constant. Magnitude of electric field is constant, non-zero. It could be constant decreasing or constant increasing, so there is not enough information.

**Appendix E – Transcript of Think-Aloud interview with Jan**

The structure of the think-aloud interviews was described in chapter 3. The format was such that the student first went through the items using the think-aloud protocol and then went back and gave an explanation for his or her answers. Since the student responded to each item twice, I will present both of the student's responses for a particular item together. That is the transcript below is not in chronological order. For each question, the student's think-aloud response will be followed by that student's explanation for the same item.

**Student:** Jan
**Date:** 2/16/01
RAPT Version B

**R1a**
**Think-Aloud:** Well, according to me…(read choices) …Its, well its going up…it is in the positive direction but it is slowing down, so it is A. Second try: The acceleration would be in the positive direction…but…its velocity is in the positive direction and its velocity is decreasing. So the acceleration is constant – so g

**Explanation:** I stated that its velocity is zero at the top. It said that up was the positive direction, down was negative direction. I said the acceleration was constant because the acceleration is due to gravity which is negative 9.8. And…it is moving in the positive direction here and here it is moving down.

**R1b**
**Think-Aloud:** The acceleration … the velocity is zero…I don't remember this stuff. The acceleration…well according the meaning of downward – the acceleration is in the negative direction and it would be increasing. Wait a minute…this is due to gravity. If it is due to gravitiy, it is going to be 9.8. So acceleration is always the same. OK. It would be in the negative direction. Second try: The velocity is zero – so it should be zero?

**Explanation:** I really don't know what it is up here. I don't think it is zero. While the coin is moving downward.

**R1c**
**Think-Aloud:** [no response]

**Explanation:** I really don't know what it is up here. I don't think it is zero. I answered that it was in the negative direction, but now I think it is in the positive direction. I really don't know why.

**R2**

**Think-Aloud:** I don't think you can tell. Second Try: The acceleration is going to be due to gravity, so they would be the same. Unless they are…

**Explanation:** I said they were the same because it would be due to gravitational…acceleration. Unless they were pushed, but from the question I assumed that they were just…released.

**R3**

**Think-Aloud:** I would say D because the slope is greater.

**Explanation:** So, I took that to be the slope would be greatest…so I chose D because at the end, the slope is…seems to be steepest.

**R4a**

**Think-Aloud:** Tank A

**Explanation:** I based these on… the slope of the lines at each times.

**R4b**

**Think-Aloud:** it is still A – because the slope is doing

**Explanation:** [no response]

**R4c**

**Think-Aloud:** That would be tank B

**Explanation:** [no response]

**R5**

**Think-Aloud:** D – no, wait, yes D. The slope is zero here.

**Explanation:** [no response]

**R6a**

**Think-Aloud:** [reread] – here because they are closer together

**Explanation:** It said the strongest winds are found in the region where the pressure is changing the most….I figured A, because the lines are closer together, it would be changing the 10 mb faster than say C.

**R6b**

**Think-Aloud:** I say C

**Explanation:** …and that is why I chose C …because the lines were further apart there

**R6c**

**Think-Aloud:** I guess D would be the highest pressure

**Explanation:** And the pressure the greatest, I chose D because it was at 1025

**R6d**

**Think-Aloud:** [no response]

**Explanation:** I chose A because it was at 980

**R7**

**Think-Aloud:** Rolling on a flat surface. Constant velocity…

**Explanation:** If its velocity is constant, it would not be accelerating – because if it were accelerating, it would be changing its velocity. I chose the acceleration would be zero.

**V1a**

**Think-Aloud:** Electric potential would be greatest…potential…

**Explanation:** I chose A, it was at 90 V

**V1b**

**Think-Aloud:** C – it is between 10 and –10

**Explanation:** C, because it is in between 10 and –10, so it would be the closest to zero

**V1c**

**Think-Aloud:** The magnitude of the electric field.

**Explanation:** I chose A, I really don't know why - I know the electric field gets…no I don't really know how it relates to this

**V1d**

**Think-Aloud:** The electric potential, then the electric field is zero.

**Explanation:** I chose C because its potential was zero

**V2a**

**Think-Aloud:** its zero, so…

**Explanation:** [these questions] it was a matter of …A was at zero potential,

**V2b**

**Think-Aloud:** [no response]

**Explanation:** B was …wanted to move to a lower potential so it would move to the left.

**V3**

**Think-Aloud:** What is the equation for the electric field? Would be the area under the graph or the slope. The area under the graph would be A.

**Explanation:** Similar for 13 – I choose E. It is zero between 2 and 4. Its greater slope is at the first part – so I chose E over C.

**V4a**

**Think-Aloud:** The proton is released at x=2. pause…mumble…well since it is at zero potential, it won't move. Second try: Proton is positive. Its at zero potential…zero potential… I don't think it will move

**Explanation:** I was kinda guessing. I figured it was at zero potential and would not move. Then…if its …I want to say if it is released, it goes to a lower potential. But that would mean that they would move to the left.  I can't remember the relationship between electric potential and electric field.

**V4b**

**Think-Aloud:** If it is at maybe one volt potential…it can't be determined.  Second try: At x = 1 is here.  I think it will go to a lower potential – I don't know this stuff.

**Explanation:** [no response]

**V5**

**Think-Aloud:** [unintelligible mumble]

**Explanation:** Well, if it is some point in the region, it might be D.  I was thinking it might be the area under the graph.  No, I don't really know.

**V6**

**Think-Aloud:** I can't remember – I am used to having the equations.  It is not C,D or …I have no idea.

**Explanation:** I did not know the answer to this question