MINILAB: SPRING-LIKE FORCES
(Record data and results on accompanying worksheet)

1. INTRODUCTION

This experiment will give you experience with spring-like forces, forces that are proportional to the amount of stretch of a spring or wire. You will experiment with a straight wire and with a coiled spring.

2. STRAIGHT WIRE: Measuring the stretch

> ***If an apparatus with a straight wire is not available at this time, go to Section 3 on a coiled spring and come back to this section when you have finished studying the coiled spring.***

When you apply a force whose magnitude is F to a long straight wire of length L_0, the length of the wire increases a little, to a new length $L_0 + \Delta L$. The extra length ΔL is also called the "stretch" s. As long as the stretch isn't too large to disrupt the atomic structure of the wire, it is observed experimentally that the stretch s is proportional to the magnitude F of the applied force: $F = k_s s$, where k_s is called the spring "stiffness" and has units of N/m.

2.1) Get the kinks out of the wire. Add 6 kg to the platform hanging from your wire. Remove the 6 kg. Repeat. This helps get small kinks out of the wire, so the "straight" wire doesn't act like a coiled spring.

2.2) Measure the position with 0 added mass. Remove the 6 kg so the added mass is 0 (that is, the only weight is that of the hanging pan you put weights on). Look carefully at the vertical scale. You will see that the smallest division is one millimeter (1 mm = 1E-3 m). You can easily distinguish between being closest to a line, being closest to halfway between lines, or being closest to one-quarter of a division from a line, so it is possible to measure to the nearest 0.1 mm or 0.2 mm. ***Record the position in the 0 kg row of the first column of the table on your minilab worksheet.***

2.3) Measure the position with 2 kg added mass, 4 kg of added mass, and 6 kg of added mass. Record the positions in the corresponding rows in the first column of the table on your minilab worksheet.

2.4) Make a second set of measurements with 0, 2, 4, and 6 kg of added mass.

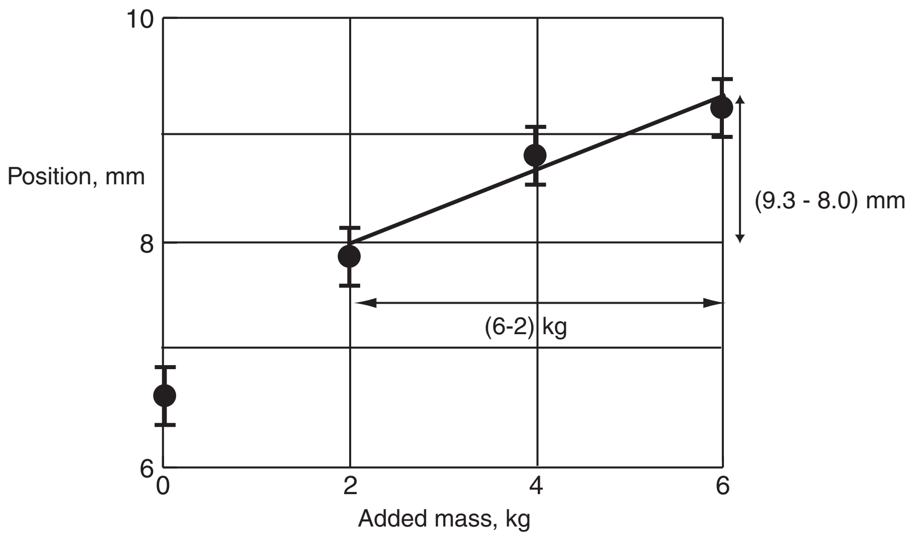
2.5) Make a third set of measurements with 0, 2, 4, and 6 kg of added mass. If your sets of measurements differ by more than about 0.3 mm, repeat the measurements until they repeat well, and discard the early measurements, because you were probably getting more kinks out of the wire.

2.6) Reproducibility. The variation in the measurements (the "reproducibility") gives you an indication of how meaningful the individual measurements are. For example, suppose that you had some measurements that are 5.5, 5.7, 5.4, 5.6, 5.8, and 5.6 mm. The average is (5.5+5.7+5.4+5.6+5.8+5.6)/6 = 5.6, with a range from a minimum of 5.4 to a maximum of 5.8. A compact way to report this average with the approximate variation is 5.6± 0.2 mm. Record your estimate of the variation on your worksheet.

> ***If you have not yet obtained data on the coiled spring, go to Section 3 and return here when you have finished studying the coiled spring. Be sure that you get data for both the straight wire and the coiled spring, which you can analyze outside of class if necessary.***

2.7) Graph the data. ***On your minilab worksheet, make a graph of your data for the straight wire that looks like the one below.*** Plot added mass along the horizontal (x) axis and position along the vertical (y) axis. Because the stretch is expected to be proportional to the force *mg*, the data are expected to fall near a straight line. We can't take data for truly zero force, since some force is required to keep the wire straight.

We're only interested in the slope, so just look at a portion of the line. Here is an example of what your graph might look like. Note that the y axis has been labeled in such a way as to use most of the available graph area. For drawing the line, exclude the data for 0 kg of added mass because it is the point most likely to be affected by remaining kinks in the wire.



Shown on the graph are "error bars" for each data point, based on an observed variation in individual measurements of about $\pm$ 0.25 mm. Show appropriate error bars on your graph, consistent with the variation you observed in repeated measurements. The straight line is drawn in such a way as to take account of the accuracy of the measurements. (There are elaborate statistical techniques for dealing with variation in data, but all that is needed in this experiment is to draw a line by eye.) ***Use a straight edge to draw a reasonable straight line through your data for 2, 4, and 6 kg of added mass.*** You may assume that the error bars you determined in your repeated measurements apply to all of your measurements.

2.8) Determine the stiffness. For our model for the stretching of a wire, we have $F = k_s s$, and you can use your measured data to determine the stiffness k_s of the wire. The graph above shows the analysis scheme. As you can visualize on the graph above, the stiffness k_s is the change in the force (4 kg times g, where g = +9.8 N/kg) divided by the corresponding change in position of the bottom of the wire ((9.3-8.0) mm)$\times$0.001 m/mm:

$$k_s = \frac{(4\,\text{kg})(9.8\,\text{N/kg})}{(9.3-8.0)\times 0.001\,\text{m}} = 3.0\times 10^4\ \text{N/m}$$

On your minilab worksheet, determine the stiffness k_s of the wire; show the details of your calculations. You can get an idea of the accuracy of your determination of the stiffness by noting how differently you could draw the straight line on your graph and still stay pretty well within the error bars. However, in this experiment the biggest source of error is not the reproducibility or accuracy of your measurements but the fact that the long wire isn't perfectly straight and probably still has small kinks in it that alter the effective stiffness. For that reason we don't ask you to estimate the accuracy of your determination of k_s.

2.9) Determine Young's modulus. The value of k_s which you obtain depends on the length and thickness of the wire:

If the cross-sectional area *A* of the wire were twice as large, it would act like two parallel wires, and it would take twice as much force to stretch the wire 1 mm, so the stiffness would be twice as large.

If the length *L* of the wire were twice as long, it would act like two of your original wires connected end to end, and the same force that stretched the original wire 1 mm would stretch the longer wire 2 mm, so the stiffness would be half as large.

"Young's modulus" Y is a stiffness measure that doesn't depend on the shape of the wire, by compensating for the effects of thickness (cross-sectional area) and length:

$$Y = \frac{(F/A)}{(\Delta L/L_0)} = \frac{(k_s s/A)}{(s/L_0)} = \frac{k_s L_0}{A},$$

where the area A of a cross section of wire with circular cross section is $A = \pi r^2$ (r is the radius of the wire; $2r$ is its diameter); if a wire or rod had a rectangular cross section h by w the cross-sectional area would of course be hw.

The ratio F/A is called "stress" (measured in $\mathrm{N/m^2}$) and the ratio $\Delta L/L_0$, the fractional change in length, is called "strain" (dimensionless; m/m). Because Young's modulus doesn't depend on the length or diameter of the wire, it describes an important physical property of the material the wire is made of. For example, $Y = 6.2\times10^{10}\ \mathrm{N/m^2}$ for aluminum; $Y = 1.6\times10^{10}\ \mathrm{N/m^2}$ for lead. The higher value for aluminum reflects the fact that aluminum is much stiffer than lead.

Your instructor will give you the length of the wire; you can check this value with a meter stick.

Determine the wire's cross-sectional area $A = \pi r^2$ from the radius r of the wire given to you by your instructor. ***Use these data to determine Young's modulus for the material of which your wire is made; show your calculations on your minilab worksheet.***

3. COILED SPRING: Measuring the stiffness and the period of oscillations

If a straight wire is coiled around into a helical spring, the coiled spring can be described by $F = k_s s$, and the stiffness k_s is much smaller than for a straight wire, due to the ease of distorting the coils.

You have a long coiled spring and two or more small masses that can be hung from the spring. You will determine the stiffness k_s of your spring, and the period (round-trip time) for oscillations of the spring-mass system. You will use these data later in a computer program you will write to model the motion of the system.

Procedure

3.1) Measure the length of the spring with 0, 1, and 2 masses hanging motionless from the vertical spring. ***Record your measurements of lengths in the table on your minilab worksheet.***

3.2) Calculate the stretch corresponding to 1 and 2 masses: $s = L - L_0$. The stretch is the length L minus the relaxed length L_0, which is the length with 0 masses. ***Record these stretches in the table on your minilab worksheet.***

3.3) Calculate the spring stiffness twice (once for each of the two stretches), using the relation $F = k_s s$ in each case. With the mass hanging motionless, its momentum isn't changing, so according to the momentum principle the net force on the mass must be zero:

Since

$$\frac{d\vec{p}}{dt} = \vec{F}_{\text{net}} = \vec{F}_{\text{spring}} + \vec{F}_{\text{grav}},$$

we have

$$\langle 0,0,0 \rangle = \left\langle 0, \left|\vec{F}_{\text{spring}}\right| - \left|\vec{F}_{\text{grav}}\right|, 0 \right\rangle,$$

and we conclude that the magnitude of the spring force is equal to the magnitude of the gravitational force, so

$$\left|\vec{F}_{\text{spring}}\right| = \left|\vec{F}_{\text{grav}}\right| = mg \text{ , where } g = +9.8 \text{ N/kg.}$$

Record your results for k_s on your minilab worksheet. The two values for k_s should be very similar. If they are not, repeat your measurements.

3.4) Pull down and release the mass and observe its oscillations. The distance you pull down is called the "amplitude". The "period" is the time it takes for one "cycle" (one round trip from bottom to top and back to bottom).

There are unavoidable fluctuations in starting and stopping the timing, but you can minimize the error this contributes by timing 10 complete cycles so that the starting and stopping fluctuations are a small fraction of the total time measured.

Start counting from zero, counting out loud for each complete cycle: "0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10." If you start counting with "1" you'll only have 9 cycles! ***Report the total time, the period, and the amplitude of the oscillation on your minilab worksheet.*** Amplitude is the maximum displacement, plus or minus, from the equilibrium position (the position where the mass can hang motionless).

3.5) Repeat your measurements of the period. If the two measurements differ by a lot, start over.

3.6) Pull the mass down twice as far as you did in Part 3.4 and measure the period again. With twice the amplitude the mass has to move twice as far, so one might expect the period to lengthen. Make two measurements and average the two. ***Record on your minilab worksheet what period you observe with twice the amplitude.***

3.7) Repeat Part 3.4 with twice the mass but the same amplitude as you used in Part 3.4. Make two measurements and average the two. ***Report on your minilab worksheet what period you observe with twice the mass.***

If you have not completed Section 2 on a straight wire, do so now.