Nuclear Fission

Applying the energy principle to a complex phenomenon

See discussion and diagram on p. 147 of the textbook. For some isotopes of some very heavy nuclei, including nuclei of thorium, uranium, and plutonium, the nucleus will fission (split apart) when it absorbs a slow-moving neutron. Uranium-237, with 94 protons and 145 neutrons, can fission when it absorbs a neutron and becomes Uranium-238. The two fission fragments can be almost any two nuclei whose charges Q_1 and Q_2 add up to $92e$ (where e is the charge on a proton, e = 1.6e-19 coulomb), and whose nucleons add up to 238 protons and neutrons (U-238, formed from U-237 plus a neutron). One of the possible fission modes involves nearly equal fragments, palladium nuclei (Pd) each with electric charge $Q_1 = Q_2 = 46e$. The rest masses of the two palladium nuclei add up to less than the rest mass of the original nucleus. (In addition to the two main fission fragments there are typically one or more free neutrons in the final state; in your analysis make the simplifying assumption that there are no free neutrons, just two palladium nuclei.) We will use the GOAL scheme: Gather, Organize, Analyze, Learn.

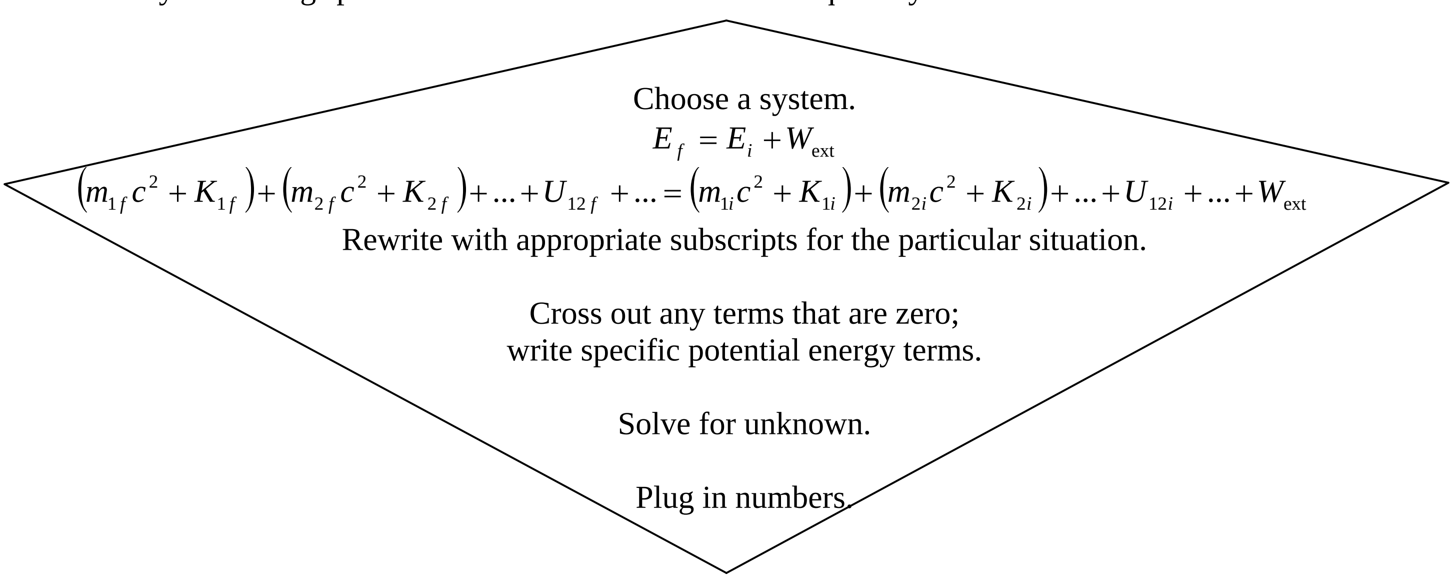
GATHER: The rest mass of the U-238 nucleus (formed from U-237 plus a neutron) is 238.000 u (unified atomic mass units), and the rest mass of each of the two Pd-119 nuclei is 118.898 u, where 1 u = 1.66e-27 kg (approximately the mass of one nucleon). **In your calculations, keep at least 6 significant figures, because the calculations involve subtracting large numbers from each other, leaving a small difference.**

ORGANIZE: As an important part of planning an approach, we need to choose a system and choose appropriate states of that system to use in the energy principle. We'll take all the particles as the system (initially a U-238 nucleus, later two palladium nuclei), so that there is no external work done on the system. There are three states you should consider in your analysis:

1) The initial state of the U-238 nucleus, before it fissions.
2) The state just after fission, when the two palladium nuclei are close together, and momentarily at rest.
3) The state when the palladium nuclei are very far away from each other, traveling at high speed.

ANALYZE: We will guide you in detail through the application of the energy principle to the process whose initial state is 1) and final state is 3). Then you will be asked to use the same *method* (but not the same final equation!) to analyze the process whose initial state is 1) and final state is 2). Finally, for additional practice, you will be asked to analyze the process whose initial state is 2) and final state is 3). You will be asked to turn in similar analyses to WebAssign.

The analysis can be thought of as a diamond. Choose a system, and write a compact statement of the energy principle. Expand to include all the possible energy terms. Rewrite with appropriate subscripts for the situation. Contract by evaluating specific terms. Solve for the unknown quantity of interest.



(a) Calculate the final speed *v*, when the palladium nuclei have moved very far apart due to their mutual electric repulsion.

GATHER: We collected information about the various nuclei. In your analysis it is all right to use the nonrelativistic formulas, but you then must check that the calculated *v* is indeed small compared to *c*. (The large kinetic energies of these palladium nuclei are eventually dissipated into thermal energy of the surrounding material. In a nuclear reactor this hot material boils water and drives an electric generator.)

ORGANIZE: Choose the initial state of the system: __The U-238 nucleus at rest____

Draw and label a sketch of the initial state:

Choose the final state of the system: __The two palladium nuclei far apart from each other___

Draw and label a sketch of the final state:

ANALYZE: Start with a general statement of the energy principle, then expand to include all possible energy terms, rewrite with appropriate subscripts, then contract by discarding terms that are zero or that cancel:

$$E_f = E_i + W_{\text{ext}}$$

Expand:

$$\left(m_{1f}c^2 + K_{1f}\right) + \left(m_{2f}c^2 + K_{2f}\right) + \ldots + U_{12f} + \ldots = \left(m_{1i}c^2 + K_{1i}\right) + \left(m_{2i}c^2 + K_{2i}\right) + \ldots + U_{12i} + \ldots + W_{\text{ext}}$$

Rewrite this equation using subscripts identifying the particles, such as $m_{\text{Pd}}c^2 + K_{\text{Pd},f}$, etc.

Include only as many terms as necessary for each state. ***Use symbols; don't plug in numbers***.

Cross out terms above that are zero; briefly state why.

Write remaining terms below. ***Don't plug in numbers***.

Solve for the final speed *v*.

Assume that $v << c$ (to be checked later).

Don't plug in numbers.

Now plug in numbers to determine the final speed *v*.

LEARN: Do the units make sense? Did the speed of each palladium nucleus turn out to be small enough that $\frac{1}{2}mv^2$ or $\frac{p^2}{2m}$ was an adequate approximation for the kinetic energy of one of the palladium nuclei? Is the final speed a high speed? (High speed goes with lots of heating of the metal, which can run electric generators.)

(b) Using energy considerations, calculate the distance between centers of the palladium nuclei just after fission, when they are momentarily at rest.

Use the same *method* you used in part (a). Don't skip steps!
Don't plug in numbers until the end.

ORGANIZE: Choose the initial state of the system: __The U-238 nucleus at rest____

Draw and label a sketch of the initial state:

Choose the final state of the system: __The two palladium nuclei at rest near each other___

Draw and label a sketch of the final state:

ANALYZE: Start with a general statement of the energy principle, then expand to include all possible energy terms, rewrite with appropriate subscripts, then contract by discarding terms that are zero or that cancel, **just as you did in part (a). Don't skip steps!**

Now plug in numbers to determine the distance between centers of the palladium nuclei just after fission.

LEARN: Let's investigate whether our results make physical sense in the context of our model for fission. A proton or neutron has a radius r of roughly 1e-15 m, and a nucleus is a tightly packed collection of nucleons. Therefore the volume of the nucleus, $\frac{4}{3}\pi R^3$, is approximately equal to the volume of one nucleon, $\frac{4}{3}\pi r^3$, times the number N of nucleons in the nucleus: $\frac{4}{3}\pi R^3 = N\frac{4}{3}\pi r^3$. So the radius R of a nucleus is about $N^{1/3}$ times the radius r of one nucleon. More precisely, experiments show that the radius of a nucleus containing N nucleons is (1.3e-15 m)$N^{1/3}$. What is the radius of a palladium nucleus?

Make a careful scale drawing of the two palladium nuclei in part (b), just after fission, and label the drawing with the distance that you calculated in parts (b) and the radii of the palladium nuclei. If the two palladium nuclei are nearly touching, this would be consistent with our model of fission, in which the U-238 nucleus fissions into two pieces that are initially nearly at rest. How big is the gap between the surfaces of the two nuclei? (If you have done the calculations correctly, you will indeed find that the gap is a rather small fraction of the center-to-center distance, which means that our model for the fission process is a pretty good model.)

As a check, and for further practice, find the distance between centers of the palladium nuclei just after fission, when they are momentarily at rest by considering the two states specified below.

Use the same *method* you used in part (a). Don't skip steps!
Don't plug in numbers until the end.

ORGANIZE: Choose the initial state of the system: __The two palladium nuclei at rest near each other___

Draw and label a sketch of the initial state:

Choose the final state of the system: __The two palladium nuclei far apart from each other___

Draw and label a sketch of the final state:

ANALYZE: Start with a general statement of the energy principle, then expand to include all possible energy terms, rewrite with appropriate subscripts, then contract by discarding terms that are zero or that cancel, **just as you did in part (a). Don't skip steps!**

LEARN: Now plug in numbers to determine the distance between centers of the palladium nuclei just after fission. This should agree with your results in part (b).

Ask an instructor to check your work.