Name: ______________________________________

After you have finished this worksheet, check your answers (see last page). Make sure you understand how to do all of the exercises, which cover operations we will use frequently.

I. Vector components and equality of vectors

(a) What are the components of the vector $\vec{a}$? (Note that since the vector lies in the xy plane, its z component is zero.)

$\vec{a}$ = < ____ , ____ , ____ >

(b) What are the components of the vector $\vec{b}$?

$\vec{b}$ = < ____ , ____ , ____ >

(c) Is this statement true or false? $\vec{a} = \vec{b}$ ________

(d) What are the components of the vector $\vec{c}$?

$\vec{c}$ = < ____ , ____ , ____ >

(e) Is this statement true or false? $\vec{c} = -\vec{a}$ __________

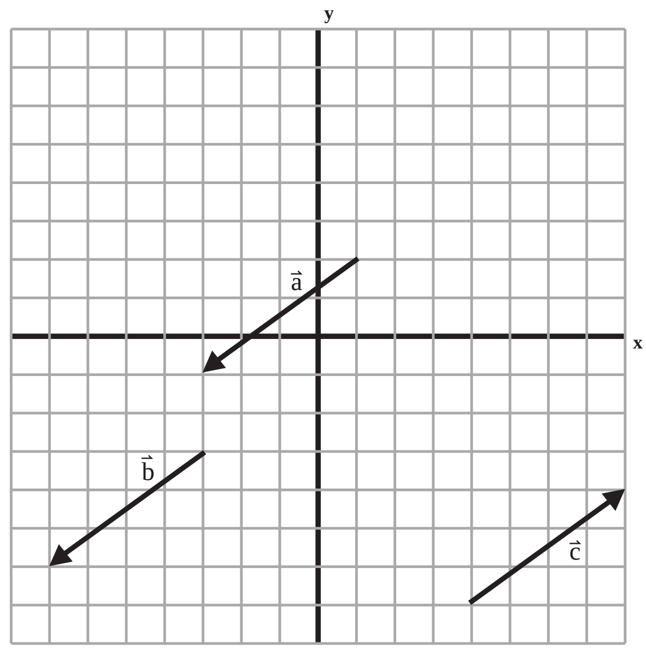


Figure 1

(f) Placing the tail of the vector at $\langle 5, 2, 0\rangle$ draw the vector $\vec{p} = \langle -7, 3, 0\rangle$ on the grid in Figure 1. Label it $\vec{p}$.

MAKE SURE THE TIP AND TAIL OF EACH VECTOR YOU DRAW ARE CLEARLY DISTINGUISHABLE.

(g) Placing the tail of the vector at $\langle -5, 8, 0\rangle$ draw the vector $-\vec{p}$ on the grid in Figure 1. Label it $-\vec{p}$.

(h) What are the components of the vector $\vec{d}$, in Figure 2?

$\vec{d}$ = < ____ , ____ , ____ > m

(i) If $\vec{e} = -\vec{d}$, what are the components of $\vec{e}$?

$\vec{e}$ = < ____ , ____ , ____ > m

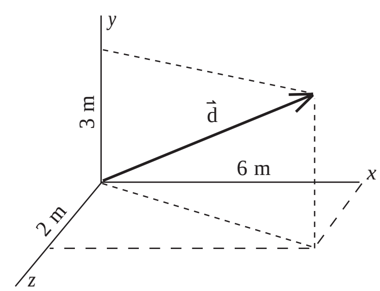


Figure 2

(j) If the tail of vector $\vec{d}$ were moved to location $\langle -5, -2, 4\rangle$ m, where would the tip of the vector be located?

Tip location would be < ____ , ____ , ____ > m

(k) If the tail of vector $-\vec{d}$ were placed at location $\langle -1, -1, -1\rangle$, where would the tip of the vector be located?

Tip location would be < ____ , ____ , ____ > m

Expressing vector relations symbolically:

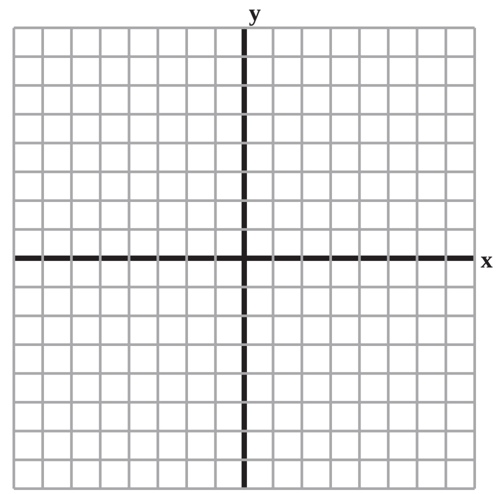
(l) Consider a vector $\vec{u} = \langle u_x, u_y, u_z\rangle$, and another vector $\vec{p} = \langle p_x, p_y, p_z\rangle$. If $\vec{u} = \vec{p}$, then which of the following statements must be true? Some, all, or none of the following may be true:

(i) $u_x = p_x$ (ii) $u_y = p_y$ (iii) $u_z = p_z$ (iv) The direction of $\vec{u}$ is the same as the direction of $\vec{p}$

True statements: ____

II. Position vectors, relative position vectors, adding and subtracting vectors

Imagine that you have a baseball and a tennis ball at different locations. The center of the baseball is at $\langle 3, 5, 0\rangle$ m, and the center of the tennis ball is at $\langle -3, -1, 0\rangle$ m. On the graph below, do the following:



(a) Draw dots at the locations of the center of the baseball and the center of the tennis ball.

(b) Draw the position vector of the baseball, which is an arrow whose tail is at the origin and whose tip is at the location of the baseball. Label this position vector $\vec{B}$. Clearly show the tip and tail of each arrow.

(c) Complete this equation: $\vec{B}$ = < ____, ____, ____> m.

(d) Draw the position vector of the tennis ball. Label it $\vec{T}$.

(e) Complete this equation: $\vec{T}$ = < ____, ____, ____> m.

(f) Draw the relative position vector for the tennis ball relative to the baseball. The tail of this vector is at the center of the baseball, and the tip of the vector is at the center of the tennis ball. Label this relative position vector $\vec{r}$.

(g) Complete the following equation by reading the coordinates of $\vec{r}$ from your graph:

$\vec{r}$ =< ____ , ____ , ____ > m

(h) Calculate the following vector difference:

$\vec{T} - \vec{B} = \langle (T_x - B_x), (T_y - B_y), (T_z - B_z)\rangle$ = < ____ , ____ , ____ > m.

(i) Is the following statement true? $\vec{r} = \vec{T} - \vec{B}$ __________

(j) Write two other equations relating the vectors $\vec{B}$, $\vec{T}$, and $\vec{r}$:

$\vec{B}$ = ____________________________ $\vec{T}$ = ____________________________________

Note that a relative position vector can be calculated as FINAL location (head) minus INITIAL location (tail).

III. Magnitudes of vectors

The magnitude of a vector is its length, and is denoted by an absolute value symbol around a vector. The magnitude of a vector is equal to the square root of the sum of the squares of its components: $|\vec{A}| = \sqrt{(A_x)^2 + (A_y)^2 + (A_z)^2}$.

The magnitude of a vector is always a positive number (note that a length cannot be negative).

(a) Calculate the magnitudes of the vectors $\vec{B}$, $\vec{T}$, and $\vec{r}$.

$|\vec{B}|$ = _________ m $|\vec{T}|$ = ____________ m $|\vec{r}|$ = _____________ m

(b) In section II you found that $\vec{r} = \vec{T} - \vec{B}$. Calculate the difference of the magnitudes $|\vec{T}| - |\vec{B}|$:

$|\vec{T}| - |\vec{B}|$ = _______________m. Compare this with $|\vec{T} - \vec{B}| = |\vec{r}|$ = __________ m

It is important to note that adding and subtracting magnitudes of vectors is NOT THE SAME as adding and subtracting vectors. The sum or difference of magnitudes is NOT EQUAL to the magnitude of the vector sum or vector difference. Study the graph above to see why this is so.

(c) What is the vector whose tail is at $\langle 9.5, 7, 0\rangle$ m and whose head is at $\langle 4, -13, 0\rangle$ m? < _______ , _______ , ______ > m

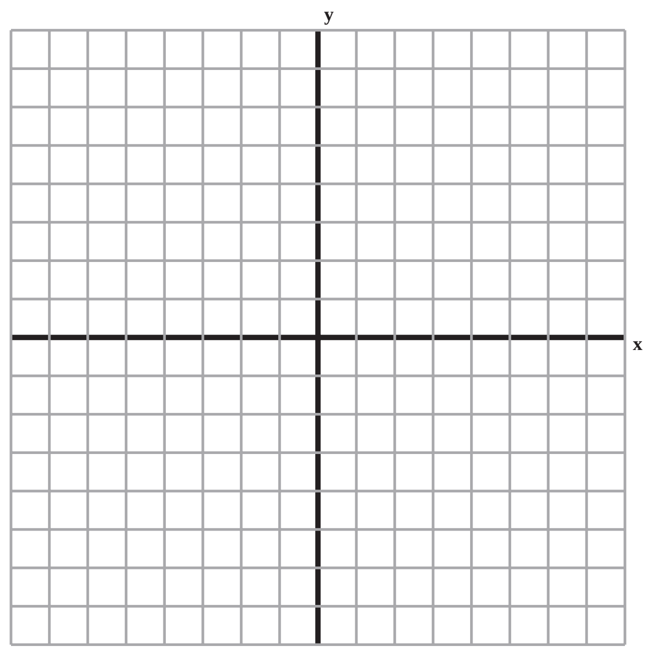
(d) What is the magnitude of this vector? ________________ m

(e) A man is standing on the roof of a building with his head at the position $\langle 12, 30, 13\rangle$ m. He sees the top of a tree, which is at the position $\langle -25, 35, 43\rangle$ m. What is the relative position vector that points from the man's head to the top of the tree?

< _______ , _______ , ______ > m

(f) What is the distance from the man's head to the top of the tree? ___________ m.

IV. Multiplying and dividing vectors by scalars



(a) On the graph at the right, draw the vector $\vec{f} = \langle -2, 4, 0\rangle$, putting the tail of the vector at $\langle -3, 0, 0\rangle$. Label the vector $\vec{f}$.

(b) Calculate the vector $2\vec{f}$, and draw this vector on the graph, putting its tail at $\langle -3, -3, 0\rangle$, so you can compare it to the original vector. Label the vector $2\vec{f}$.

$2\vec{f}$ = < ____ , ____ , ____ > m

How does the magnitude of $2\vec{f}$ compare to the magnitude of $\vec{f}$?______________________

How does the direction of $2\vec{f}$ compare to the direction of $\vec{f}$? ____________________

(c) Calculate the vector $\vec{f}/2$, and draw this vector on the graph, putting its tail at $\langle -3, -6, 0\rangle$, so you can compare it to the other vectors. Label the vector $\vec{f}/2$.

$\frac{\vec{f}}{2}$ = < ____ , ____ , ____ > m

How does the magnitude of $\vec{f}/2$ compare to the magnitude of $\vec{f}$? _________

How does the direction of $\vec{f}/2$ compare to the direction of $\vec{f}$? __________

(d) Does multiplying a vector by a scalar change the magnitude of the vector? ____________

(e) Does multiplying a vector by a scalar change the direction of the vector? ______________

V. Unit vectors

Any vector can be factored into the product of its magnitude times a unit vector which specifies its direction. The magnitude of a unit vector is 1. The unit vector corresponding to a vector $\vec{g}$ is written with a "hat" over it, like this: $\hat{g}$.

(a) On the graph above, draw the vector $\vec{g} = \langle 4, 7, 0\rangle$ m. Put the tail of the vector at the origin.

(b) Calculate the magnitude of $\vec{g}$: $|\vec{g}|$ = ___________ m

(c) The unit vector $\hat{g} = \frac{\vec{g}}{|\vec{g}|}$. Calculate $\hat{g}$ and $|\hat{g}|$: $\hat{g}$ = < _________ , _________ , ________ > $|\hat{g}|$ =__________

(d) On the graph draw $\hat{g}$. Put the tail of the vector at $\langle 1, 0, 0\rangle$ so you can compare $\hat{g}$ and $\vec{g}$.

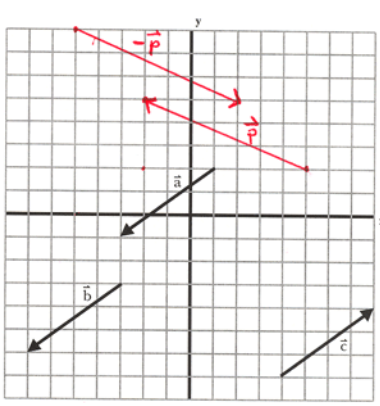
(e) How does the direction of $\hat{g}$ compare to the direction of $\vec{g}$? _____________________

(f) Calculate the product of the magnitude $|\vec{g}|$ times the unit vector $\hat{g}$:

$(|\vec{g}|)(\hat{g})$ = (______) < _________ , _________ , ________ > m = < ____ , ____ , ____ > m

This is an example of factoring a vector into its (scalar) magnitude times its (vector) direction: $\vec{g} = (|\vec{g}|)(\hat{g})$.

Answers to exercises. Check these answers only after you have worked out the exercises yourself!



Section I:

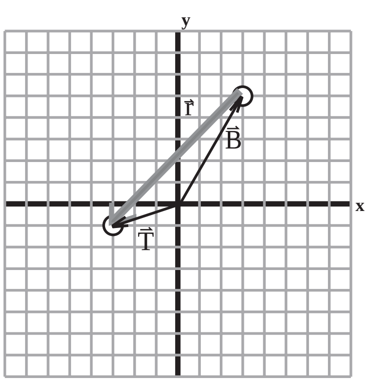
(a) $\langle -4, -3, 0\rangle$ (b) $\langle -4, -3, 0\rangle$ (c) true

(d) $\langle 4, 3, 0\rangle$ (e) true

h) $\langle 6, 3, 2\rangle$ (i) $\langle -6, -3, -2\rangle$ (j) $\langle 1, 1, 6\rangle$

(k) $\langle -7, -4, -3\rangle$

(l) All four statements are true



Section II:

(c) $\langle 3, 5, 0\rangle$ (e) $\langle -3, -1, 0\rangle$ (g) $\langle -6, -6, 0\rangle$

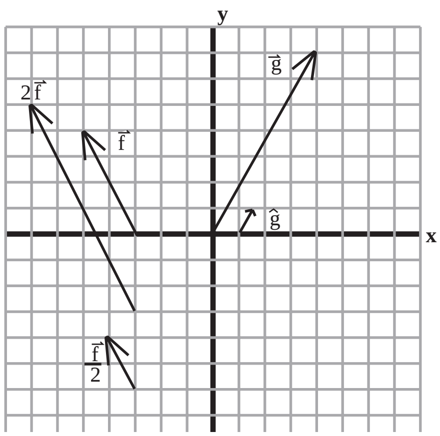
(h) $\langle -6, -6, 0\rangle$ (i) yes

(j) $\vec{T} - \vec{r}$ (k) $\vec{r} + \vec{B}$

Section III:

(a) 5.83 , 3.16 , 8.48 (b) -2.67, 8.48

(c) $\langle -5.5, -20, 0\rangle$ (d) 20.7 (e) $\langle -37, 5, 30\rangle$, (f) 47.9



Section IV:

(b) $\langle -4, 8, 0\rangle$, twice as large, same direction

(c) $\langle -1, 2, 0\rangle$, half as large, same direction

(d) yes (e) no, unless the scalar is negative (which reverses the direction)

Section V:

(b) 8.06 (c) $\langle 0.496, 0.868, 0\rangle$, $0.997 \approx 1$

(e) same (f) $\langle 4, 7, 0\rangle$