Chapter 8
Multidimensional Arrays

8.1 Introduction

- Thus far, you have used one-dimensional arrays to model linear collections of elements. You can use a two-dimensional array to represent a matrix or a table. For example, the following table that describes the distances between the cities can be represented using a two-dimensional array.

<table>
<thead>
<tr>
<th></th>
<th>Chicago</th>
<th>Boston</th>
<th>New York</th>
<th>Atlanta</th>
<th>Miami</th>
<th>Dallas</th>
<th>Houston</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chicago</td>
<td>0</td>
<td>983</td>
<td>787</td>
<td>714</td>
<td>1375</td>
<td>967</td>
<td>1087</td>
</tr>
<tr>
<td>Boston</td>
<td>983</td>
<td>0</td>
<td>214</td>
<td>1102</td>
<td>1763</td>
<td>1723</td>
<td>1842</td>
</tr>
<tr>
<td>New York</td>
<td>787</td>
<td>214</td>
<td>0</td>
<td>888</td>
<td>1549</td>
<td>1548</td>
<td>1627</td>
</tr>
<tr>
<td>Atlanta</td>
<td>714</td>
<td>1102</td>
<td>888</td>
<td>0</td>
<td>661</td>
<td>781</td>
<td>810</td>
</tr>
<tr>
<td>Miami</td>
<td>1375</td>
<td>1763</td>
<td>1549</td>
<td>661</td>
<td>0</td>
<td>1426</td>
<td>1187</td>
</tr>
<tr>
<td>Dallas</td>
<td>967</td>
<td>1723</td>
<td>1548</td>
<td>781</td>
<td>1426</td>
<td>0</td>
<td>239</td>
</tr>
<tr>
<td>Houston</td>
<td>1087</td>
<td>1842</td>
<td>1627</td>
<td>810</td>
<td>1187</td>
<td>239</td>
<td>0</td>
</tr>
</tbody>
</table>

```c
double[][] distances = {
    {0, 983, 787, 714, 1375, 967, 1087},
    {983, 0, 214, 1102, 1763, 1723, 1842},
    {787, 214, 0, 888, 1549, 1548, 1627},
    {714, 1102, 888, 0, 661, 781, 810},
    {1375, 1763, 1549, 661, 0, 1426, 1187},
    {967, 1723, 1548, 781, 1426, 0, 239},
    {1087, 1842, 1627, 810, 1187, 239, 0},
};
```
8.2 Two-Dimension Array Basics

- You can use a two-dimensional array to represent a matrix or a table.
- Occasionally, you will need to represent n-dimensional data structures. In Java, you can create n-dimensional arrays for any integer n.

8.2.1 Declaring Variables of Two-Dimensional Arrays and Creating Two-Dimensional Arrays

- Here is the syntax for declaring a two-dimensional array:
  
  ```
  dataType [][] arrayRefVar;
  or
  dataType arrayRefVar[][];  // This style is correct, but not preferred
  ```

- As an example, here is how you would declare a two-dimensional array variable matrix of int values
  
  ```
  int [][] matrix;
  or
  int matrix[][];  // This style is correct, but not preferred
  ```

- You can create a two-dimensional array of 5 by 5 int values and assign it to matrix using this syntax:
  
  ```
  matrix = new int[5][5];
  ```

![FIGURE 8.1 The index of each subscript of a two-dimensional array is an int value, starting from 0.](image)
Caution

- It is a common mistake to use matrix[2,1] to access the element at row 2 and column 1.
- In Java, each subscript must be enclosed in a pair of square brackets.
- You can also use an array initializer to declare, create and initialize a two-dimensional array.
  For example,

```java
int[][] array = {
    {1, 2, 3},
    {4, 5, 6},
    {7, 8, 9},
    {10, 11, 12}
};
```

```java
text
int[][] array = new int[4][3];
array[0][0] = 1; array[0][1] = 2; array[0][2] = 3;
array[1][0] = 4; array[1][1] = 5; array[1][2] = 6;
array[2][0] = 7; array[2][1] = 8; array[2][2] = 9;
array[3][0] = 10; array[3][1] = 11; array[3][2] = 12;
```
8.2.2 Obtaining the Lengths of Two-Dimensional Arrays

```
int[ ][ ] x = new int[3][4];
```

x.length is 3
x[0].length is 4, x[1].length is 4, x[2].length is 4

**FIGURE 8.2** A two-dimensional array is a one-dimensional array in which each element is another one-dimensional array.
8.2.3 Ragged Arrays

- Each row in a two-dimensional array is itself an array. So, the rows can have different lengths. Such an array is known as a **ragged array**.

```java
int[][] triangleArray = {
    {1, 2, 3, 4, 5},
    {2, 3, 4, 5},
    {3, 4, 5},
    {4, 5},
    {5}
};
```

- If you don’t know the values in a ragged array in advance, but know the sizes, say the same as before, you can create a ragged array using the syntax that follows:

```java
int [][] triangleArray = new int[5][];
triangleArray[0] = new int[5];
triangleArray[1] = new int[4];
triangleArray[2] = new int[3];
triangleArray[3] = new int[2];
triangleArray[4] = new int[1];
```
8.3 Processing Two-Dimensional Arrays

- Suppose an array matrix is declared as follows:

```
int [ ] [ ] matrix = new int [10][10];
```

- Here are some examples of processing two-dimensional arrays:
  - (Initializing arrays with input values) The following loop initializes the array with user input values:

```
java.util.Scanner input = new Scanner(System.in);
System.out.println("Enter " + matrix.length + " rows and " +
    matrix[0].length + " columns: ");
for (int row = 0; row < matrix.length; row++) {
    for (int column = 0; column < matrix[row].length; column++) {
        matrix[row][column] = input.nextInt();
    }
}
```

  - (Initializing arrays with random values) You can now assign random values to the array using the following loop:

```
for (int row = 0; row < triangleArray.length; row++)
    for (int column = 0; column < triangleArray[row].length; column++)
        triangleArray[row][column] = (int) (Math.random( ) * 1000);
```

  - (Printing arrays)

```
for (int row = 0; row < matrix.length; row++) {
    for (int column = 0; column < matrix[row].length; column++) {
        System.out.print(matrix[row][column] + " ");
    }
    System.out.println();
}
```

  - (Summing all elements)
  - (Summing elements by column)
  - (Which row has the largest sum?)
8.4 Passing Two-Dimensional Arrays to Methods

- You can pass a two-dimensional array to a method just as you pass a one-dimensional array.
- Listing 8.1 gives an example with a method that returns the sum of all the elements in a matrix.

**LISTING 8.1 PassTwoDimensionalArray.java**

```java
import java.util.Scanner;

public class PassTwoDimensionalArray {
    public static void main(String[] args) {
        int[][] m = getArray(); // Get an array
        // Display sum of elements
        System.out.println("\nSum of all elements is " + sum(m));
    }

    public static int[][] getArray() {
        // Create a Scanner
        Scanner input = new Scanner(System.in);

        // Enter array values
        int[][] m = new int[3][4];
        System.out.println("Enter " + m.length + " rows and " + m[0].length + " columns: ");
        for (int i = 0; i < m.length; i++)
            for (int j = 0; j < m[i].length; j++)
                m[i][j] = input.nextInt();
        return m;
    }

    public static int sum(int[][] m) {
        int total = 0;
        for (int row = 0; row < m.length; row++)
            for (int column = 0; column < m[row].length; column++)
                total += m[row][column];
        return total;
    }
}
```

Enter 3 rows and 4 columns:
1 2 3 4
5 6 7 8
9 10 11 12

Sum of all elements is 78
8.5 Case Study: Grading a Multiple-Choice Test

- **Objective:** write a program that grades multiple-choice test.
- **Suppose there are eight students and ten questions, and the answers are stored in a two-dimensional array.**
- **Each row records a student’s answers to the questions, as shown in the following array:**

<table>
<thead>
<tr>
<th>Students’ Answers to the Questions:</th>
<th>Key to the Questions:</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 1 2 3 4 5 6 7 8 9</td>
<td>0 1 2 3 4 5 6 7 8 9</td>
</tr>
<tr>
<td>Student 0  A B A C C D E E A D</td>
<td>Key</td>
</tr>
<tr>
<td>Student 1  D B A B C A E E A D</td>
<td>D B D C C D A E A D</td>
</tr>
<tr>
<td>Student 2  E D A C B E E A D</td>
<td></td>
</tr>
<tr>
<td>Student 3  C B A E D C E E A D</td>
<td></td>
</tr>
<tr>
<td>Student 4  A B D C C D E E A D</td>
<td></td>
</tr>
<tr>
<td>Student 5  B B E C C D E E A D</td>
<td></td>
</tr>
<tr>
<td>Student 6  B B A C C D E E A D</td>
<td></td>
</tr>
<tr>
<td>Student 7  E B E C C D E E A D</td>
<td></td>
</tr>
</tbody>
</table>

**LISTING 8.2 GradeExam.java**

```java
public class GradeExam {
    /** Main method */
    public static void main(String args[]) {
        // Students' answers to the questions
        char[][] answers = {

        // Key to the questions

        // Grade all answers
        for (int i = 0; i < answers.length; i++) {
            // Grade one student
            int correctCount = 0;
            for (int j = 0; j < answers[i].length; j++) {
                if (answers[i][j].equals(keys[j]))
                    correctCount++;
            }
            System.out.println("Student "+ i + ", "+ keys[j] + "+ correct count is "+ correctCount);
        }
    }
}
```
<table>
<thead>
<tr>
<th>Student</th>
<th>Correct Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
</tr>
</tbody>
</table>
8.6 Case Study: Finding a Closest Pair

- The GPS navigation system is becoming increasingly popular.
- The system uses the graph and geometric algorithms to calculate distances and map a route.
- The section presents a geometric problem for finding a closest pair of points.

![Diagram of points and coordinates]

FIGURE 8.3 Points can be represented in a two-dimensional array.

LISTING 8.3 FinNearestPoints.java

- Given a set of points, the closest-pair problem is to find the two points that are nearest to each other.

```java
import java.util.Scanner;

public class FindNearestPoints {
    public static void main(String[] args) {
        Scanner input = new Scanner(System.in);
        System.out.print("Enter the number of points: ");
        int numberOfPoints = input.nextInt();

        double[][] points = new double[numberOfPoints][2];
        System.out.print("Enter "+ numberOfPoints +" points: ");
        for (int i = 0; i < points.length; i++) {
            points[i][0] = input.nextDouble();
            points[i][1] = input.nextDouble();
        }

        int p1 = 0, p2 = 1; // Initial two points
        double shortestDistance = distance(points[p1][0], points[p1][1],
                                            points[p2][0], points[p2][1]); // Initialize shortestDistance

        // Compute distance for every two points
        for (int i = 0; i < points.length; i++) {
            for (int j = i + 1; j < points.length; j++) {
                double distance = distance(points[i][0], points[i][1],
                                             points[j][0], points[j][1]); // Find distance
                // Update p1 and p2 if shorter distance is found
            }
        }
    }
}
```

CMPS161 Class Notes (Chap 08) Page 10 /17 Dr. Kuo-pao Yang
if (shortestDistance > distance) {
    p1 = i; // Update p1
    p2 = j; // Update p2
    shortestDistance = distance; // Update shortestDistance
}

// Display result
System.out.println("The closest two points are " +
    "(" + points[p1][0] + ", " + points[p1][1] + ") and (" +
    points[p2][0] + ", " + points[p2][1] + ")");

/** Compute the distance between two points (x1, y1) and (x2, y2)*/
public static double distance(
    double x1, double y1, double x2, double y2) {
    return Math.sqrt((x2 - x1) * (x2 - x1) + (y2 - y1) * (y2 - y1));
}

Enter the number of points: 8
Enter 8 points: -1 3 -1 1 1 2 0.5 2 -1 3 3 4 2 4 -0.5
The closest two points are (1.0, 1.0) and (2.0, 0.5)
8.7 Case Study: Sudoku

- This section presents an interesting problem of a sort that appears in the newspaper every day: Sudoku
- Sudoku is a 9 X 9 grid divided into smaller 3 X 3 boxes (also called regions or blocks) as shown in Figure 8.4(a).
- Some cells, called fixed cells, are populated with numbers from 1 to 9.
- The objective is to fill the empty cells, also called free cells, with numbers 1 to 9 so that every row, every column, and every 3 X 3 box contains the numbers 1 to 9 as shown in Figure 8.4 (b).

![Sudoku Grid](image)

(a) Puzzle  
(b) Solution

FIGURE 8.4 The Sudoku puzzle in (a) is solved in (b).

LISTING 8.4 CheckSudokuSolution.java

```java
import java.util.Scanner;

public class CheckSudokuSolution {
    public static void main(String[] args) {
        // Read a Sudoku solution
        int[][] grid = readASolution();

        System.out.println(isValid(grid) ? "Valid solution" : "Invalid solution");
    }

    /** Read a Sudoku solution from the console */
    public static int[][] readASolution() {
        // Create a Scanner
        Scanner input = new Scanner(System.in);

        System.out.println("Enter a Sudoku puzzle solution:");
        int[][] grid = new int[9][9];
        for (int i = 0; i < 9; i++)
```
for (int j = 0; j < 9; j++)
grid[i][j] = input.nextInt();

return grid;
}

/** Check whether a solution is valid */
public static boolean isValid(int[][] grid) {
    // Check whether each row has numbers 1 to 9
    for (int i = 0; i < 9; i++)
        if (!is1To9(grid[i])) // If grid[i] does not contain 1 to 9
            return false;

    // Check whether each column has numbers 1 to 9
    for (int j = 0; j < 9; j++) {
        int[] column = new int[9];
        for (int i = 0; i < 9; i++)
            column[i] = grid[i][j];

        if (!is1To9(column)) // If column does not contain 1 to 9
            return false;
    }

    // Check whether each 3 by 3 box has numbers 1 to 9
    for (int i = 0; i < 3; i++)
        for (int j = 0; j < 3; j++) {
            int k = 0;
            int[] list = new int[9]; // Get all number in the box to list
            for (int row = i * 3; row < i * 3 + 3; row++)
                for (int column = j * 3; column < j * 3 + 3; column++)
                    list[k++] = grid[row][column];

            if (!is1To9(list)) // If list does not contain 1 to 9
                return false;
        }

    return true; // The fixed cells are valid
}

/** Check whether the one-dimensional array contains 1 to 9 */
public static boolean is1To9(int[] list) {
    // Make a copy of the array
    int[] temp = new int[list.length];
    System.arraycopy(list, 0, temp, 0, list.length);

    // Sort the array
    java.util.Arrays.sort(temp);

    // Check if list contains 1, 2, 3, ..., 9
    for (int i = 0; i < 9; i++)
        if (temp[i] != i + 1)
            return false;
```java
    return true; // The list contains exactly 1 to 9
  }
}

Enter a Sudoku puzzle solution:
9 6 3 1 7 4 2 5 8
1 7 8 3 2 5 6 4 9
2 5 4 6 8 9 7 3 1
8 2 1 4 3 7 5 9 6
4 9 6 8 5 2 3 1 7
7 3 5 9 6 1 8 2 4
5 8 9 7 1 3 4 6 2
3 1 7 2 4 6 9 8 5
6 4 2 5 9 8 1 7 3

Valid solution
```
8.8 Multidimensional Arrays

- Occasionally, you will need to represent n-dimensional data structures. In Java, you can create n-dimensional arrays for any integer n.
- The following syntax declares a three-dimensional array variable scores, creates an array, and assigns its reference to scores:

  ```java
double[ ][ ][ ] x = new double[2][3][4];
```

  - `x.length` is 2
  - `x[0].length` is 3, `x[1].length` is 3
  - `x[0][0].length` is 4, `x[0][1].length` is 4, `x[0][2].length` is 4,
  - `x[1][0].length` is 4, `x[1][1].length` is 4, `x[1][2].length` is 4

- For example, you may use a three-dimensional array to store exam scores for a class of 6 students with 5 exams, and each exam has 2 parts (multiple-choice and essay).
  ```java
double [ ][ ][ ] scores = new double[6][5][2];
```

  - You can also use the short-hand notation to create and initialize the array as follows:
    ```java
double [ ][ ][ ] scores = {
        {7.5, 20.5}, {9.0, 22.5}, {15, 33.5}, {13, 21.5}, {15, 2.5},
        {4.5, 21.5}, {9.0, 22.5}, {15, 34.5}, {12, 20.5}, {14, 9.5},
        {6.5, 30.5}, {9.4, 10.5}, {11, 33.5}, {11, 23.5}, {10, 2.5},
        {6.5, 23.5}, {9.4, 32.5}, {13, 34.5}, {11, 20.5}, {16, 7.5},
        {8.5, 26.5}, {9.4, 52.5}, {13, 36.5}, {13, 24.5}, {16, 2.5},
        {9.5, 20.5}, {9.4, 42.5}, {13, 31.5}, {12, 20.5}, {16, 6.5}};
    ```

  - `scores[i][j][k]` represents which student, which exam, and whether it's multiple-choice or essay.
8.8.2 Case Study: Guessing Birthdays

- The program can guess your birth date.

\[ 19 = 10011 \text{ in binary} \]
\[ 7 = 111 \text{ in binary} \]
\[ 23 = 11101 \text{ in binary} \]

Note: 19 is 10011 in binary. 7 is 111 in binary. 23 is 11101 in binary

\[
\begin{array}{c}
10000 \\
10 \\
+ 1 \\
10011 \\
\end{array} + \begin{array}{c}
00110 \\
10 \\
+ 1 \\
00111 \\
\end{array} = \begin{array}{c}
10000 \\
1000 \\
+ 1 \\
11101 \\
\end{array}
\]

\[
19 + 7 = 23
\]

LISTING 8.6 GuessBirthdayUsingArray.java

```java
import java.util.Scanner;

public class GuessBirthdayUsingArray {
    public static void main(String[] args) {
        int day = 0; // Day to be determined
        int answer;

        int[][][] dates = {
            {{1, 3, 5, 7},
            {9, 11, 13, 15},
            {17, 19, 21, 23},
            {25, 27, 29, 31}},

            {{2, 3, 6, 7},
            {10, 11, 14, 15},
            {18, 19, 22, 23},
            {26, 27, 30, 31}},

            {{4, 5, 6, 7},
            {12, 13, 14, 15},
            {20, 21, 22, 23},
            {28, 29, 30, 31}},

            {{8, 9, 10, 11},
            {12, 13, 14, 15},
            {24, 25, 26, 27},
            {28, 29, 30, 31}},

            {{16, 17, 18, 19},
            {20, 21, 22, 23},
            {24, 25, 26, 27},
            {28, 29, 30, 31}}
        };
    }
}
```
// Create a Scanner
Scanner input = new Scanner(System.in);

for (int i = 0; i < 5; i++) {
    System.out.println("Is your birth day in Set" + (i + 1) + "?");
    for (int j = 0; j < 4; j++) {
        for (int k = 0; k < 4; k++)
            System.out.printf("%4d", dates[i][j][k]);
        System.out.println();
    }
    System.out.print("Enter 0 for No and 1 for Yes: ");
    answer = input.nextInt();
    if (answer == 1)
        day += dates[i][0][0];
}
System.out.println("Your birth day is " + day);

<table>
<thead>
<tr>
<th>Is your birth day in Set1?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1  3  5  7</td>
</tr>
<tr>
<td>9 11 13 15</td>
</tr>
<tr>
<td>17 19 21 23</td>
</tr>
<tr>
<td>25 27 29 31</td>
</tr>
</tbody>
</table>

Enter 0 for No and 1 for Yes: 1

<table>
<thead>
<tr>
<th>Is your birth day in Set2?</th>
</tr>
</thead>
<tbody>
<tr>
<td>2  3  6  7</td>
</tr>
<tr>
<td>10 11 14 15</td>
</tr>
<tr>
<td>18 19 22 23</td>
</tr>
<tr>
<td>26 27 30 31</td>
</tr>
</tbody>
</table>

Enter 0 for No and 1 for Yes: 1

<table>
<thead>
<tr>
<th>Is your birth day in Set3?</th>
</tr>
</thead>
<tbody>
<tr>
<td>4  5  6  7</td>
</tr>
<tr>
<td>12 13 14 15</td>
</tr>
<tr>
<td>20 21 22 23</td>
</tr>
<tr>
<td>28 29 30 31</td>
</tr>
</tbody>
</table>

Enter 0 for No and 1 for Yes: 0

<table>
<thead>
<tr>
<th>Is your birth day in Set4?</th>
</tr>
</thead>
<tbody>
<tr>
<td>8  9 10 11</td>
</tr>
<tr>
<td>12 13 14 15</td>
</tr>
<tr>
<td>24 25 26 27</td>
</tr>
<tr>
<td>28 29 30 31</td>
</tr>
</tbody>
</table>

Enter 0 for No and 1 for Yes: 0

<table>
<thead>
<tr>
<th>Is your birth day in Set5?</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 17 18 19</td>
</tr>
<tr>
<td>20 21 22 23</td>
</tr>
<tr>
<td>24 25 26 27</td>
</tr>
<tr>
<td>28 29 30 31</td>
</tr>
</tbody>
</table>

Enter 0 for No and 1 for Yes: 1
Your birth day is 19