ISAT 645 Review 1

February 20, 2020

Directions: Organize, show your work, write clearly, and use correct syntax.

1. The coverage for Test 1 is the list of homework exercises that have been given since the beginning of the semester.

2. There are two dice where one is a fair die, and the other is a loaded die. The loaded die has four 1’s and two 2’s. The two dice are tossed and the outcome recorded.
   a) Let \( X \) be a random variable that evaluates the sum of the outcome. 
      Find the expected value of \( X \). 
      \[ E[X] = \frac{29}{6} \] 
   b) Let \( Y \) be a random variable that indicates the outcome of the loaded die. 
      Evaluate the conditional probability \( P[X = 3, 4 \mid Y = 2] \), i.e., the probability that the sum is either 3 or 4 given that the outcome of the loaded die is 2. 
      Answer: \( \frac{1}{3} \)

3. A fair coin is tossed two times, and the two outcomes are recorded. Let \( Y \) be a random variable that counts the number of heads in two tosses. Let \( X \) denote the number of heads in the first toss. Define the random variable \( Z = XY \). Find the mean and variance of \( Z \).
   Answer: \[ E[Z] = \frac{3}{4}, \text{Var}[Z] = \frac{11}{16} \]

4. Jenny is thinking about opening a bake shop. She believes that any small business should be started only if there is a good chance of making a profit. Jenny can open a small shop, a large shop, or no bake shop at all. If Jenny builds a large bake shop, she estimates a profit of 30 units if the market is favorable, and a loss of 20 units if the market is unfavorable. A small bake shop will return an estimated profit of 10 units in a favorable market, and a loss of 6 units if the market is unfavorable.
   Jenny is thinking of hiring a marketing firm for a fee of 2. The marketing firm makes forecasts of the market. She estimates that there is a \( \frac{3}{5} \) probability that the firm will make a forecast of a favorable market. In addition, there is a \( \frac{3}{4} \) probability that the market will be favorable given a favorable forecast from the firm. However, there is only a \( \frac{3}{10} \) probability of a favorable market if the marketing firm makes a forecast that is an unfavorable market.
   If the marketing firm is not hired, Jenny believes there is a 50-50 chance for favorable market.
   a) Determine the expected profit if the marketing firm is not hired. 
      Answer: 5 units from a large bake shop
   b) Evaluate the expected profit if the marketing firm is hired. 
      Answer: 8.5 units

5. A student takes a multiple choice test with 3 problems. Each problem in the test lists five possible answers, but there is only one correct answer. Suppose a student randomly guesses the answers.
   (a) What is the probability that the student obtains at most 2 correct answers? 
      Answer: \( \frac{124}{125} \)
   (b) What is the probability that the student obtains zero correct answers? 
      Answer: \( \frac{64}{125} \)
   (c) What is the probability that the student obtains at least one correct answer? 
      Answer: \( \frac{61}{125} \)

6. Five coins are inside an urn. Of these, four are fair coins, and the other is loaded with two heads. A statistical experiment consists of choosing two coins with replacement, then tossing both coins. If the outcome has two heads, find the probability that at least one coin tossed was loaded. 
   Answer: \( \frac{5}{9} \)

7. Five coins are inside an urn. Of these, four are fair coins, and the other is loaded with two heads. A statistical experiment consists of choosing two coins without replacement, then tossing both coins. If the outcome has two heads, find the probability that at least one coin tossed was loaded. 
   Answer: \( \frac{4}{7} \)