Directions: Show your work, and write clearly. Simplify the final answers.

1. Sketch the graphs of $x + y = 2$, $z = 3$, and $x^2 + z = 4$.

2. Determine the unit vectors $v$ in the Cartesian plane that are tangent to the graph of $f(x) = \tan(x)$ at the point where $x = \frac{\pi}{3}$.

   \[
   \text{answer: } \pm \left( \frac{1}{\sqrt{17}}, \frac{4}{\sqrt{17}} \right)
   \]

3. Let $f(x, y) = e^{2x} \cos(xy)$. Evaluate the directional derivative of $f(x, y)$ at the point $(x, y) = (1, \frac{\pi}{3})$, and in the direction of the unit vector $(\cos(\frac{\pi}{3}), \sin(\frac{\pi}{3}))$.

   \[
   \text{answer: } -e^{2/12} \left( 3 + \sqrt{3}\pi \right)
   \]

4. Find an equation of the plane that is tangent to the surface $2xy^2 - 4\sqrt{z} = -6$ at the point $P(1, -1, 4)$.

   \[
   \text{answer: } 2x - 4y - z = 2
   \]

5. Determine the center and radius of the sphere given by $x^2 + y^2 + z^2 - 10x + 6y + 2z + 26 = 0$.

   \[
   \text{center: } (5, -3, -1)
   \]

   \[
   \text{radius: } 3
   \]

6. Find the vectors $v$ in the Cartesian plane with length $\|v\| = 3$, and that are tangent to the graph of $f(x) = 4\sin(2x)$ at the point where $x = \frac{\pi}{12}$.

   \[
   \text{answer: } \pm \left( \frac{3}{2}, \frac{12\sqrt{3}}{7} \right)
   \]

7. Determine the gradient $\nabla f(x, y, z)$ of $f(x, y, z) = ze^{y/x}$.

   \[
   \text{answer: } \nabla f(x, y, z) = \left\{ -\frac{ye^{y/x}}{x^2}, \frac{ze^{y/x}}{x}, e^{y/x} \right\}
   \]

8. Determine the angle between vectors $v = (2, -2, 1)$ and $w = (4 + \sqrt{6}, -(4 + \sqrt{6}), 2 - 4\sqrt{6})$.

   \[
   \text{Show your work, organize, and write clearly.}
   \]

   \[
   \text{answer: } \frac{\pi}{3}
   \]

9. Find an equation of the plane that is tangent to the surface $z = x \cos(x) + 2\sin(y) + 1 - \sqrt{3}$ at the point $P(0, \frac{\pi}{4}, 1)$.

   \[
   \text{answer: } x + y - z = \frac{\pi}{3} - 1
   \]

10. Let $v = (2, 1, 4)$ and $w = (-1, 2, 2)$ be two vectors. Evaluate $\frac{v \cdot w}{\|w\|^2} (v \times w)$.

   \[
   \text{answer: } (-\frac{16}{9}, \frac{64}{9}, \frac{40}{9})
   \]

11. Determine the curl of the vector field $f(x, y, z) = (\sin(2y) + \tan(z), \cos(y), xy)$.

   \[
   \text{answer: } (x, \sec^2(z) - y, -2 \cos(2y))
   \]

12. Evaluate $\int_0^{\pi/2} \int_0^1 y \sin(xy) dx dy$

   \[
   \text{answer: } \frac{\sqrt{2}}{2} + \frac{\pi}{4} - 1
   \]