Math 312 Review 1

The assigned homework exercises form the coverage for Test 1.
The following items are additional practice problems.

Directions: Organize, show your work, and write clearly.

1. Plot the points $A(2, 1, 1)$, and $B(4, -3, 5)$. Label the coordinate axes.
   Then determine an equation of the sphere where points $A$ and $B$ are diametrically opposite points in the sphere.

2. Sketch the graph of the equation in 3-space. Label the coordinate axes.
   (a) $2x + z = 4$
   (b) $y = z^2 + 1$
   (c) $y = \sqrt{x}$

3. Determine the vectors of length $\sqrt{39}$ that are tangent to the graph of $f(x) = \sec(x)$ at the point where $x = \frac{\pi}{3}$.
   answer: $(\sqrt{3}, 6)$

4. Determine the partial derivatives of $f(x, y) = \ln(x)e^{x/y}$.
   answer: $\frac{\partial f}{\partial x} = \frac{e^{x/y}}{x} + \frac{e^{x/y}\ln(x)}{y}$ and $\frac{\partial f}{\partial y} = -\frac{xe^{x/y}\ln(x)}{y^2}$

5. The intersection of the planes $x + y - z = 0$ and $2x - y + z = 3$ is a line in 3-space.
   Find a set of parametric equations of the line.
   possible answer: $x = 1$, $y = t$, $z = 1 + t$

6. A plane contains the point $A(1, -2, 3)$, and vector $v = (\cos(\frac{\pi}{3}), \sin(\frac{\pi}{6}), \tan(\frac{\pi}{3}))$ is perpendicular to the plane.
   Determine the standard equation of the plane. The plane’s standard equation is of the form $ax + by + cz = d$
   answer: $x + y - 2\sqrt{3}z = -1 - 6\sqrt{3}$

7. Determine the gradient $\nabla f(x, y, z)$ of $f(x, y, z) = ze^{y/x}$.
   answer: $\nabla f(x, y, z) = \left( -\frac{yz e^{y/x}}{x^2}, \frac{ze^{y/x}}{x}, e^{y/x} \right)$

8. Determine the angle between vectors $v = (2, -2, 1)$ and $w = (4 + \sqrt{6}, -4 - \sqrt{6}, 2 - 4\sqrt{6})$.
   answer: $\frac{\pi}{3}$

9. Find an equation of the plane that is tangent to the
   surface $z = x\cos(x) + 2\sin(y) + 1 - \sqrt{3}$ at the point $P(0, \frac{\pi}{4}, 1)$.
   answer: $x + y - z = \frac{\pi}{4} - 1$

10. Let $v = (2, 1, 4)$ and $w = (-1, 2, 2)$ be two vectors. Evaluate $\frac{v \cdot w}{||w||^2}(v \times w)$.
    answer: $(-\frac{16}{3}, -\frac{64}{9}, \frac{40}{9})$

11. Determine the projection of vector $w = (-2, 3)$ onto vector $v = (3, 4)$.
    answer: $\left(\frac{18}{25}, \frac{24}{25}\right)$