Math 200
Second Derivative Test
Homework Exercises

March 16, 2020
The Second Derivative Test

Let $y = f(x)$ be a function and suppose $f'(c) = 0$.

1. If $f''(c)$ is positive, then $f(c)$ is a relative minimum value of $f(x)$.

2. If $f''(c)$ is negative, then $f(c)$ is a relative maximum value of $f(x)$.

3. If $f''(c) = 0$, the Second Derivative Test is not applicable.
Example 1

Let $f(x) = x^3 - x^2 + 2$.

Determine the relative extrema (i.e., relative maximum values and relative minimum values) of $f(x)$.

1. First, we set and solve $f'(x) = 0$ as follows.

   
   
   
   \[
   f'(x) = 3x^2 - 2x
   \]

   \[
   = x(3x - 2).
   \]

   We find $f'(x) = 0$ when $x = 0$ or $x = \frac{2}{3}$.

2. Since $f'(x) = 3x^2 - 2x$, we find

   \[
   f''(x) = 6x - 2.
   \]
Example 1, continued

We apply the Second Derivative Test.

We substitute the critical numbers found in step 1 into \( f''(x) = 6x - 2 \).

3.1 We find \( f''(0) = 6(0) - 2 = -2 \).

Since \( f''(0) \) is negative, by the Second Derivative Test, we obtain \( f(0) \) is a relative maximum value of \( f(x) \).

Recall, \( f(x) = x^3 - x^2 + 2 \). Then \( f(0) = 2 \) is a relative maximum value of \( f(x) \).
Example 1, continued

We substitute the other critical number found in step 1 into \( f''(x) = 6x - 2 \).

3.2 We find

\[
f'' \left( \frac{2}{3} \right) = 6 \left( \frac{2}{3} \right) - 2 = 4 - 2 = 2.
\]

Since \( f''(\frac{2}{3}) \) is positive, by the Second Derivative Test, we obtain \( f(\frac{2}{3}) \) is a relative minimum value of \( f(x) \).

Moreover, we find

\[
f \left( \frac{2}{3} \right) = \left( \frac{2}{3} \right)^3 - \left( \frac{2}{3} \right)^2 + 2
\]

\[
= \frac{8}{27} - \frac{12}{27} + \frac{54}{27}
\]

\[
= \frac{50}{27}.
\]
Example 1, answers

relative maximum value of $f(x)$: $f(0) = 2$

relative minimum value of $f(x)$: $f \left( \frac{2}{3} \right) = \frac{50}{27}$
Homework exercises

Solve the following exercises in Section 3.4, page 172, # 23, 24, 33, 35, 37