Math 200 Final Review

Directions: Organize, show, and write your work clearly.

1. Apply the limit process to find the derivative of the function \( f(x) = x^2 \).
   answer: \( 2x \)

2. Apply the limit process in evaluating \( \int_1^2 x^2 \, dx \)
   answer: \( \frac{7}{3} \)

3. Evaluate the derivative of the function. Simplify, and do not leave negative exponents in the final answer.
   \( a) \ f(x) = \ln(\sec(x) + \tan(x)) \)
   answer: \( \sec(x) \)

   \( b) \ f(x) = \ln \left( \frac{e^{2x} - 1}{e^{2x} + 1} \right) \)
   answer: \( \frac{2e^{2x}}{e^{2x} - 1} \)

   \( c) \ f(x) = \frac{2x^2 - 2}{x^2 + 1} \)
   answer: \( f'(x) = \sqrt{x} \sec^2(x) + \frac{\tan(x)}{2\sqrt{x}} \)

   \( d) \ f(x) = \sqrt{x} \tan(x) \)
   answer: \( f'(x) = \frac{1}{\sqrt{x}} - x^{2/3} \)

   \( e) \ f(x) = \frac{2}{3}x^{2/3} - \frac{3}{5}x^{5/3} \)
   answer: \( f'(x) = 6\sin^3(\sqrt{x}) \cos(\sqrt{x}) e^{4\sin^3(\sqrt{x})} \)

   \( f) \ y = e^{\sin^3(\sqrt{x})} \)
   answer: \( f'(x) = -3x^4 \csc^2(x^3) + 2x \cot(x^3) - 1 \)

   \( g) \ y = x^2 \cot(x^3) - x \)
   answer: \( y = -x + 5 \)

4. Find the slope-intercept form of the line that is tangent to the function at the indicated point \( P \).
   \( a) \ h(x) = \frac{4}{x-1}, \ P(3, h(3)) \)
   answer: \( y = -x + 5 \)

   \( b) \ f(x) = \csc^2(x), \ P(\frac{\pi}{6}, f(\frac{\pi}{6})) \)
   answer: \( y = -8\sqrt{3}x + \frac{12 + 4\sqrt{3}}{3} \)

5. The edges of a cube are increasing at a rate of 2 inches per hour. How fast is the volume of the cube increasing when the surface area of the cube is 64 square inches?
   answer: 64 cubic inches per hour

6. Determine the open intervals where \( f(x) = -2x^3 + 3x^2 + 36x + 2 \) is increasing or decreasing. Include the sign chart for \( f'(x) \). Then apply the First Derivative Test to find the relative extreme values of \( f(x) \). Sketch the shape of the graph of \( f(x) \).
   increasing on \((-2, 3)\)
   decreasing on \((-\infty, -2)\) and on \((3, \infty)\)
   rel. max value: 83
   rel. min value: -42

7. Determine the open intervals where \( f(x) = x^{5/3} - \frac{1}{3}x^{8/3} \) is concave upward or concave downward. Include the sign chart for \( f''(x) \). Then find the points of inflection of \( f(x) \).
   concave up on \((0, 8)\)
   concave down on \((-\infty, 0)\) and on \((8, \infty)\)
   points of inflection: \((0, 0)\) and \((8, 24)\)

8. Determine the relative extreme values of the function.
   Apply the Second Derivative Test. Explain how you are applying the test.
   \( a) \ f(x) = 2x^3 - 3x^2 - 36x - 2 \)
   relative maximum value: \( f(-2) = 42 \)
   relative minimum value: \( f(3) = -83 \)
(b) \( f(x) = \frac{2x}{x^2 + 1} \)

relative maximum value: \( f(1) = 1 \)
relative minimum value: \( f(-1) = -1 \)

9. Evaluate the integral.

(a) \( \int x(3\sqrt{x} - 2)^3 \, dx \)

answer: \(-\frac{24}{3} x^{5/2} + 3x^3 + 2x^2 + C\)

(b) \( \int_1^0 \left( \frac{2}{3x\sqrt{x}} - \frac{2}{x^2} \right) \, dx \)

answer: \(-\frac{8}{9}\)

(c) \( \int_{\pi/9}^{\pi/6} \sin(3x) \, dx \)

answer: \(\frac{1}{6}\)

(d) \( \int_0^{12} x\sqrt{x^2 + 25} \, dx \)

answer: \(\frac{2072}{3}\)

(e) \( \int_0^3 x^2 \, dx \)

answer: \(\frac{511}{6}\)

(f) \( \int_0^{\pi/6} \sin^2(x) \cos(x) \, dx \)

answer: \(\frac{1}{2}\)

(g) \( \int_\frac{1}{2}^{\pi/4} \csc(\pi x) \cot(\pi x) \, dx \)

answer: \(\frac{3-2\sqrt{3}}{3\pi}\)

(h) \( \int_0^{\pi/3} \frac{\sin(3x)}{\sqrt{1 + \cos(3x)}} \, dx \)

answer: \(\frac{2\sqrt{2}}{3}\)

(i) \( \int_{1/6}^{1/3} \cot(\pi x) \, dx \)

answer: \(\frac{\ln(3)}{2\pi}\)

(j) \( \int_{\pi/8}^{\pi/6} \sec^2(2x)e^{\tan(2x)} \, dx \)

answer: \(\frac{1}{2}(e^{\sqrt{3}} - e)\)

(k) \( \int \frac{dx}{x^2 - 2x + 5} \)

answer: \(\frac{1}{2} \arctan\left(\frac{x-1}{2}\right) + C\)

(l) \( \int_2^3 \frac{dx}{\sqrt{4x - x^2}} \)

answer: \(\frac{\pi}{6}\)

10. Determine the area of the indicated region. Shade and include a sketch of the region.

(a) A region is bounded by the graphs of \( y = 4\sqrt{x} \) and \( y = x \).

answer: \(\frac{128}{3}\)

(b) A region is bounded by the graphs of \( y = 2x - 1 \), \( y = 5 \), and \( x = 0 \).

answer: 9

(c) A region is bounded by the graphs of \( y = 2x - 6 \) and \( y = x^2 - 2x - 3 \).

answer: \(\frac{4}{3}\)

11. Solve the problems about a free-falling object. A ball is thrown upward from an initial height of 64 feet above the ground. If the ball reaches a maximum height of 100 feet from the ground, determine the initial velocity at which the ball was thrown.

answer: 48 ft/sec

12. Sketch and shade the region \( R \) bounded by the graph of \( y = 2\sqrt{x} \), \( x = 0 \), \( x = 4 \), and \( y = 0 \).

(a) A solid is generated when the region \( R \) is revolved about the \( x \)-axis. Evaluate the volume of the solid.

(b) A solid is generated when the region \( R \) is revolved about the \( y \)-axis. Evaluate the volume of the solid.

13. The base of a solid is bounded by the graphs of \( y = x + 1 \), \( y = 2x + 3 \), \( x = 0 \), and \( x = 2 \).

The cross sections of the solid perpendicular to the \( x \)-axis are squares. Find the volume of the solid.

answer: \(\frac{56}{3}\)