Steady Currents; non-Ohmic Materials

**Introduction**

We have seen that if a voltage difference is applied across a conducting material, a current results. That is, if we create an electric field inside a material containing charges that are free to move, we see that those charges are accelerated by the field and a motion of charges (a current) results. We have seen that the current generated is proportional to the applied voltage. While this seems reasonable, we will see that this is actually surprising.

**Why we’re all amazed that \( V=IR \).**

We all took resistors, placed a voltage difference, \( \Delta V \), across them and simultaneously measured the current, \( I \), which passed through them. We then plotted the current as a function of the voltage difference (sometimes referred to as the “voltage drop”, since charges “fall” through them) and observed a straight line. Such dependence is called an ohmic one, and a system that shows such behavior is called an ohmic conductor. But is ohmic behavior to be expected?

Model a resistor as a long cylinder (i.e., a cylinder that is much longer than its diameter) filled with small conducting particles. When I place the two ends of the resistor at a different potentials (i.e., a potential difference, or voltage, is applied), an electric field will be created, whose average magnitude is given by \( E=\Delta V/L \), where \( L \) is the length of the cylinder. The small conducting particles will respond to the field by accelerating, and the acceleration will be \( a=F/m=eE/m \), where \( e \) is the charge on the particles and \( m \) is their mass. So the velocity, in the absence of other influences, will be \( v=v_0+at=(eE/m) t \), where I’ve assumed that the particles are at rest before the voltage is applied. The current will be made up of all the charges in the resistor.
moving in the direction of the applied electric force. Refer to the figure: If the conductor is filled with a number density \( N \) of conducting particles (\( N \) is number per unit volume), then as they all move under the influence of the field, the number of particles passing the shaded area in time \( \Delta t \) will be \( NA \Delta L = Nav \Delta t \), where \( \Delta L = v \Delta t \) is the distance between the two shaded planes, \( A \) is the area of the cylinder and \( v \) is the speed of each of the charged particles. The charge passing in time \( \Delta t \) will be \( e \) (the charge on each particle) times the number passing and the charge passing per unit time (which is defined as the current) will be the charge passing divided by \( \Delta t \), or \( I = eNAv = (e^2AN\tau/mL)\Delta V \). This looks vaguely like Ohm’s law (\( I = \Delta V/R \)) until you notice that the coefficient that relates \( I \) to \( \Delta V \) is linear in time. In other words, this analysis predicts that current will rise linearly with time as long as the constant voltage difference is applied. This is absolutely not what we see. After a virtually instantaneous (much too fast for us to measure with our crude implements) rise from zero, the current settles down to a constant value for a given voltage drop. So we’ve obviously left something out of our analysis above. In fact, what we’ve left out is what the resistor is made of.

A resistor doesn’t just contain tiny charged particles to provide current, it also contains resistor atoms for the tiny charged particles to bang into. So let’s try to model that. Imagine that the electric field can only accelerate the charged particles for a short time \( \tau \) before they bang into something (resistor atoms) and grind to a halt; then get accelerated again, and so on. The velocity of the particles will look like:

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\text{That is, it will increase linearly in time while the field is accelerating the particle (t=0 to t=\tau), and then will drop to zero, and then repeat. For times long compared to the accelerating time \( \tau \), the particle’s velocity along the resistor will look like the average value, indicated by the horizontal line. Since the acceleration is presumed constant, the average speed is } (v_0+v_f)/2 = a\tau/2 = eE/2m \tau, \text{ where I’ve used the previous expression for the acceleration. Putting this in for } v \text{ in the previous expression for current, we get } I = (e^2AN\tau/2mL)\Delta V. \text{ This expression predicts Ohm’s law as long as the coefficient relating } I \text{ to } \Delta V \text{ is constant. Since } e, m \text{ and } 2 \text{ are constants of nature, no problem there. In one of those flukes that so often save a physicist’s bacon, even if the resistor is not a perfect cylinder (A/L not constant), variations here just introduce factors of } \sqrt{2} \text{ or } \pi \text{ or other numbers, preserving the basic form of the equation. So this analysis predicts Ohm’s law so long as the product } N\tau \text{ is a constant. So, our experiments on I-V curves are consistent with this analysis, and we conclude from them that } N\tau \text{ is, in fact, a constant for the resistors we}
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studied. And this analysis also makes testable predictions. For example, it predicts that the resistance of a cylinder varies linearly as \( L \) and inversely as \( A \). (Your book says that this is true—it gives the expression \( R = \rho L/A \), where \( \rho \) is called the “resistivity”. We can see that the resistivity is \( \rho = 2m/(e^2N\tau) \).) It’s possible to test this rather than just take the book’s word (although we know that they couldn’t print it if it wasn’t true). But given the pressures of time, we will move on to consider what are called non-ohmic conductors. These are conductors that don’t satisfy Ohm’s law (and so some of the assumptions we’ve made above fail for these kind of conductors—as we do the lab, think about which ones they are). The non-ohmic conductors we will consider are diodes and light bulbs. The task is the same: vary applied voltage, measure current and plot the two on an X-Y graph. We can keep the same definition of resistance (slope of the \( V \) versus \( I \) graph) and simply realize the \( R \) will vary with \( I \) (and so with \( V \)). By connecting the data on the plot with a smooth line and measuring the slope at several points, we can plot \( R \) as a function of \( I \) (or \( V \)). Also, you can use the techniques from the first lab to determine the relationship between \( I \) and \( V \) for these non-ohmic resistors.

Aside: This model also allows us to determine the power used up in the resistor. In time \( \tau \), the particles will each gain an energy \( \frac{1}{2}mv_f^2 \) and will then give it up in a collision with the resistor atoms. The total number of particles doing this will be \( NAL \) and so the power dissipated in the resistor will be:

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P = \frac{NAL\left(\frac{1}{2}mv_f^2\right)}{\tau} = \frac{NALm}{2\tau}\left(\frac{e^2\tau\Delta V}{mL}\right)^2
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It isn’t too tough to put this is the form \( P=I^2R \), which is to be expected. This doesn’t so much prove that this model is correct, as much as it assures us that the model is not inconsistent with the known laws of electricity and magnetism.

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**Task 1 – Constructing an I-V curve**

Wire a single resistor as shown in the figure (and as you’ve done before). You will again measure \( V \) and \( I \) while systematically changing \( V \) with the power supply. You will then plot \( I \) as a function of \( V \) (such a plot is called an “I-V curve”). You may expect to see that it is NOT linear, but as indicated in the previous lab and the notes above, you may still determine the
resistance, R.

**Task 2) – Constructing an I-V curve**

Repeat task 1) for a different non-ohmic resistor.

**Conclusions**

Prepare a report that includes the data you measured (I-V plots and the graphs of resistance obtained). Discuss the shape of the I-V curve you obtained using the diode as compared to the light bulb, and compare each to the ohmic resistors you measured previously. What does the shape of the curves tell you about the assumptions we made earlier? Ensure domestic tranquility.