Notes on measuring Moment of Inertia (or better, Rotational Inertia):

The experiment described in the lab notes is more or less as we did it (the major difference is we varied the angle), but the analysis was different in that we included a frictional force. So these notes are mostly on the analysis of the measurements.

We want to use energy to analyze the data, since using Newton's law requires us to add torque (which we have not discussed in class yet). So we need to identify the forces and the work done. I identify three forces affecting the object rolling down the ramp: a force of contact with the ramp (that, as usual, I find it convenient to divide into a perpendicular part and a parallel part; that is, a “normal” force and a “frictional” force) and a gravitational force. (Can you think of any that we left out? Can you estimate the effect and justify leaving them out?)

The gravitational force is a conservative force, so rather than calculate the work done, I imagine a potential energy associated with that force. The “perpendicular” or “normal” force does zero work because it is perpendicular. So the work energy theorem becomes:

\[ \Delta (K + U) = W_{NC} \]  

The only non-conservative force that does any work is friction, so \( W_{NC} \) is:

\[ W_{NC} = \vec{F}_{fric} \cdot \Delta \vec{r} = |\vec{F}_{fric}| |\Delta \vec{r}| \cos \theta = (\mu N) L \cdot (\sin \theta) = -\mu mgL \sin \theta \]  

Note that I used the rule of thumb that \( F_{fric} = \mu N \) (we won't prove this in today's lab, but in general, you would need to) and that \( N = mg \cos \theta \) (since the rolling object does not accelerate perpendicular to the ramp). Setting the zero of potential energy at the top of the ramp (NOTE: In class, we set \( U=0 \) at the bottom of the ramp – I do it differently here to reinforce that it can't matter), we can write the initial mechanical energy as:

\[ (K + U)_i = \frac{1}{2} m v_i^2 + mg (h) = 0 \]  

(since \( v \) and \( y \) are zero at the beginning) and the final mechanical energy as:

\[ (K + U)_f = \frac{1}{2} m v_f^2 + \frac{1}{2} I \omega^2 + mg (h) = 0 \]  

where \( v \) is the final translational speed, \( \omega \) is the final angular speed, and the object is at an altitude \( h \) below the beginning point. As discussed in class, IF (and only if) the object is rolling without slipping, \( v \) and \( \omega \) are related by \( v = \omega R \) so the final mechanical energy can be written:

\[ (K + U)_f = \frac{1}{2} m v_f^2 \left[ 1 + \frac{I}{MR^2} \right] - mgL \sin \theta = 0 \]  

(Note that I wrote \( h = L \sin \theta \).) Putting all this together in Equation (2), we have:

\[ \frac{1}{2} m v_f^2 \left[ 1 + \frac{I}{MR^2} \right] - mgL \sin \theta = -\mu mgL \cos \theta \]  

A little algebra and I can (almost) write this in a form to analyze the data:

\[ \frac{v^2}{2gL} = \frac{1}{1 + \frac{I}{MR^2}} \sin \theta - \frac{\mu}{1 + \frac{I}{MR^2}} \cos \theta \]  

The problem is that this is hard to verify. But since the angle will be small (less than about 5º), we can use the approximation we've used before and linearize the equation:

\[ \frac{v^2}{2gL} = \frac{1}{1 + \frac{I}{MR^2}} \theta - \frac{\mu}{1 + \frac{I}{MR^2}} \]  

(8)