1. **(15 points)** Use the expression we derived in class \( \frac{F_D}{W} = C_D \left( \frac{\rho_{\text{object}}}{\rho_{\text{fluid}}} \right) \left( \frac{D}{L_{\text{object}}} \right) \) to estimate how far a ping-pong ball must fall before the force of Rayleigh drag is significant. (You can borrow a ping-pong ball if you need. The parameters in the equation were defined in class.)

\[ m = 2.58 \text{ g}, \quad 2.65 \text{ g} \]
\[ d = 4.0 \text{ cm}, \quad 4.0 \text{ cm} \]
\[ \rho_{\text{obj}} = \frac{M}{\frac{4}{3} \pi R^3} = \frac{8.8 \times 10^{-3} \text{ kg}}{4 \times (2 \times 10^{-2} \text{ m})^3} = 80 \text{ kg/m}^3 \]

\[ \frac{F_D}{W} = C_D \left( \frac{\rho_{\text{fluid}} \cdot D}{\rho_{\text{obj}} \cdot L_{\text{object}}} \right) \], \text{we require} \quad D > \left( \frac{\rho_{\text{obj}}}{\rho_{\text{fluid}}} \times C_D \right) L

\[ D > \left( \frac{1.2}{80} \right) \left( \frac{1}{0.5} \right) (4 \text{ cm}) = 0.1 \text{ cm} = 1 \text{ mm} ! \]
2. **(30 points)** At the front of the room, a spring-weight system is set up with three forces that apparently add to zero. (Do NOT manipulate the system).

\[ F_1 \text{, Force } 1 = 6.7 \text{ N} \]

\[ F_2 \text{, Force } 2 = 13.2 \text{ N} \]

\[ F_3 \text{, Force } 3 = 40.7 \text{ N} \]

A) Determine the weight and mass of the hanging object.

\[ \vec{W} = -(\vec{F}_1 + \vec{F}_2) = -\frac{1}{2} \left( \langle -5.57, 3.35 \rangle \text{ N} + \langle 6.4, 1.96 \rangle \text{ N} \right) \]

\[ = \langle 0.83, 5.31 \rangle \text{ N} = 5.37 \text{ N} @ 261^\circ \]

\[ m = \frac{\vec{W}}{g} = 0.55 \text{ kg} \]

B) If possible, use the data you took here to estimate the error in the mass. If it's not possible, explain why it can't be done.

Since \( W_x \) should be zero, the error can be estimated as \( \pm 0.8 \text{ N} \).

This translates to \( \pm 0.08 \text{ kg} \).

The actual measured mass is 500.7 g.

\[ \Theta \text{ Since the angles add to } 359.5^\circ \]

\[ \text{Can the error be explained by a } 0.5^\circ \text{ error in the angles?} \]
3. (20 points) IF YOU DID THE MODELLING LAB WITH GLOWSCRIPT: The following is Glowscript code to simulate the motion of a ball in a gravitational field (it should look familiar) including code to add Stoke's drag: \( \vec{F} = 6 \pi \eta \vec{R}(-\vec{v}) \). There is one syntax error that will crash the code and other logic errors so that the code does not correctly model a drag force.

```
GlowScript 2.1 VPython
#floor=box(pos=pos(0,0,0), size=pos(2,0.2,4))
ball=sphere(pos=pos(-1,0,0), radius=0.02, color=color.red, make_trail=True)

ball.m=0.1 #kg
v0=4.0 #m/s
theta=60 #degrees
PI=3.14159265
g=vec(0,-9.8,0) #N/kg
eta=1#kg/m/s

ball.p=ball.m*v0*vec(cos(theta*pi/180),sin(theta*pi/180),0)

t=0
dt=0.01

while ball.pos.y>=0:
    rate(100)
    Fnet = ball.m*g-6*PI*eta*ball.radius*(-v0)
    ball.p = ball.pos+Fnet*dt
    ball.pos = ball.pos+ball.pos*dt/ball.m
    t=t+dt

print(ball.pos.y)
```

A) Explain specifically what is wrong with the code.

```
syntax error in defining Fnet = (vector) - (scalar)
bball.p update uses ball.pos
ball.pos += ball.pos dt incorrectly
```

B) What should you do to correct the code?

```
... = 6*PI*eta*ball.radius*(-ball.p/ball.m)  
```

(above)
4. (20 points) IF YOU DID THE MODELLING LAB WITH A SPREADSHEET: Shown is a screenshot of a spreadsheet intended to model a projectile subject to its weight \( \vec{W} = mg (\text{down}) \) and Stoke's drag \( \vec{F} = 6\pi \eta R (-\vec{v}) = mg \left( \frac{\vec{v}}{\vec{v_T}} \right) [-\vec{v}] \).

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Stokes air drag</td>
<td></td>
<td></td>
<td>rhoAIR=</td>
<td>1.2</td>
<td>ALL UNITS ARE SI</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>( v_0 = )</td>
<td>45</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>( \theta_0 = )</td>
<td>45</td>
<td>gee</td>
<td>9.8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Radius=</td>
<td>0.04</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>mass=</td>
<td>0.0027</td>
<td>( \sqrt{T} = )</td>
<td>0.0351</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>time(sec)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>31.8198</td>
<td>31.8198</td>
<td>0</td>
</tr>
</tbody>
</table>

A) Show that, if \( \vec{v_T} \) is the speed at which the magnitude of the drag force is equal to the magnitude of the weight, that the magnitude of the drag force \( 6\pi \eta R \vec{v} \) is equal to \( mg \left( \frac{\vec{v}}{\vec{v_T}} \right) \).

\[
\vec{F} = \vec{W}; \quad 6\pi \eta R \vec{v_T} = mg \Rightarrow 6\pi \eta R = \frac{mg}{\vec{v_T}}
\]

Then \( |\vec{F}| = 6\pi \eta R \vec{v} = \frac{mg \vec{v}}{\vec{v_T}} \frac{\vec{v}}{\vec{v_T}} = mg \left( \frac{\vec{v}}{\vec{v_T}} \right) \) \( \checkmark \)

B) What should be typed in cell E7 to model this motion?

\[
\alpha_x = \frac{\vec{F}_x}{m} = \frac{1}{m} \cdot mg \left( \frac{\vec{v}}{\vec{v_T}} \right) \left( \frac{-\vec{v_x}}{\vec{v}} \right) = -g \frac{\vec{v_x}}{\vec{v_T}}
\]

\[
\alpha = -\frac{3}{E} \times \frac{D7}{E5} \quad (9)
\]
5. (10 points) Suppose that the rolling friction lab is repeated, and a graph of acceleration as a function of \( \sin \theta \) shows a straight line with a slope of \((10 \pm 1.5) \frac{m/s}{s}\) and intercept of \((-0.22 \pm 0.03) \frac{m/s}{s}\).

A) Use this to find the most reliable estimate of the rolling friction coefficient, \( \mu \). (You may use the model we discussed in class: \( a = g \sin \theta - \mu g \).

\[
\mu = -\frac{\frac{\mu g}{g}}{\text{slope}} = -\frac{-0.22}{10} = 0.022
\]

B) Find the most reliable estimate of the error in the coefficient, \( \sigma_\mu \).

\[
\sigma_\mu = \mu \sqrt{\left(\frac{\sigma_{\mu \text{in}}}{\mu}\right)^2 + \left(\frac{\sigma_{\mu \text{out}}}{\mu}\right)^2} = 0.022 \left[\left(\frac{0.03}{0.022}\right)^2 + \left(\frac{1.5}{10}\right)^2\right]^{1/2}
\]

\[
= 0.022 \times 0.20 = 0.0044
\]
6. **(10 points)** A neutron in a nucleus usually lasts forever, but if you leave them by themselves, a lone neutron decays into a proton and an electron in about 15 minutes. Shown are the results of two methods of measuring the lifetime. Do they agree? Explain.

<table>
<thead>
<tr>
<th>Method</th>
<th>Value [sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Beam</td>
<td>887.7 ± 3.1 (from 2013)</td>
</tr>
<tr>
<td>B. Bottle</td>
<td>877.7 ± 0.7 (from 2018)</td>
</tr>
</tbody>
</table>

\[
\text{Beam} \pm 2\sigma = (881.5, 893.9) \text{ s}
\]

\[
\text{Bottle} \pm 2\sigma = (876.3, 879.1) \text{ s}
\]

The results differ by more than 2\(\sigma\). This is referred to as "tension" in the measurements.

Do they differ by more than 5\(\sigma\)?
7. (15 points) Plot this data in a way that allows you to determine whether the acceleration is constant. If it is, find the acceleration and estimate the error in the acceleration.

<table>
<thead>
<tr>
<th>Distance [m]</th>
<th>0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time [sec]</td>
<td>0</td>
<td>0.27</td>
<td>0.54</td>
<td>0.81</td>
<td>1.1</td>
</tr>
<tr>
<td>$T^2$</td>
<td>0</td>
<td>0.07</td>
<td>0.29</td>
<td>0.66</td>
<td>1.2</td>
</tr>
</tbody>
</table>

$D \propto T^2$ not linear

so if accelerating, $\ddot{a}$ is not constant.