1. (20 points) The figure shows a plot of the function: 

\[ R = \frac{v_0^2 \sin \theta \cos \theta}{g} \left[ 1 + \sqrt{1 + \frac{2gH}{v_0^2 \sin^2 \theta}} \right] \]

that we used in class to model a projectile. Also shown are two representations of \( R(\theta) \) in which Rayleigh drag: 

\[ F_D = \frac{1}{2} C_D \rho A v^2 \]

has been added. Which curve (A or B) best represents the effect of drag? Explain your answer.

The drag force reduces the range by reducing \( u_x \). Both show this. BUT in the case that \( u_x = 0 \) (launch angle of 1.57 rad), the range is NOT changed (it's already zero) so \( A \) is best.
2. **(20 points)** At the front of the room, a spring-weight system is set up with three forces that apparently add to zero. (Do NOT manipulate the system).

A) Determine the weight and mass of the hanging object.

\[
\begin{aligned}
F_1 &= 0.94 \text{N} \langle \cos 12^\circ, \sin 12^\circ \rangle = \langle 0.919, 0.195 \rangle \text{N} \\
F_2 &= 1.6 \text{N} \langle \cos 145^\circ, \sin 145^\circ \rangle = \langle -1.3106, 0.9177 \rangle \text{N} \\
\overrightarrow{W} &= -(\overrightarrow{F}_1 + \overrightarrow{F}_2) = -\langle -0.3916, 1.1127 \rangle \text{N} \\
\overrightarrow{W} &= \langle 0.3916, -1.1127 \rangle \text{N} \\
W &= \frac{|\overrightarrow{W}|}{g} = \frac{1.1796 \text{N}}{9.8 \text{N/kg}} = 0.120 \text{ kg}
\end{aligned}
\]

B) If possible, use the data you took *here* to estimate the error in the mass. If it's *not* possible, explain why it can't be done.

I can't measure the error (even repeated measurements are difficult to do for one person - multiple people could legitimately do independent measurements).

I could guess that the error in angles \(\pm 1^\circ\) (so error in \(\sim \frac{1}{100} \sim 1\%\)) or the error in force is \(\sim \frac{0.2 \text{N}}{1 \text{N}} \sim 20\%\).

But these are only guesses.
3. **(20 points)** Consider these four values of the time to travel a given distance:
{1.4, 1.5, 1.5, 1.7}

A) Find the average time and the error in the time. Show your work.

\[
t_{\text{avg}} = \frac{1.4 + 1.5 + 1.5 + 1.7}{4} = 1.525 \text{ s}
\]

\[
\sigma_t^2 = \frac{(1.4 - 1.525)^2 + 2 \times (1.5 - 1.525)^2 + (1.7 - 1.525)^2}{3}
\]

\[
= 0.0158 \text{ s}^2 \Rightarrow 
\]

\[
\sigma_t = 0.1258 \text{ s} \quad (= 8.25\%)
\]

B) Calculate the error in the squared time, \( \sigma_{t^2} \). Show your work.

\[
\sigma_{t^2} = 2 t_{\text{avg}} \sigma_t
\]

\[
= 2(1.525)(0.1258)
\]

\[
= 0.3837 \text{ s}^2 \quad (= 16.5\%)
\]
4. (20 points) In the lab about rolling friction, we developed the net force as 
\[ F_{\text{NET}} = mg \sin \theta - \mu mg \cos \theta \]. We then developed the relationship we used to analyze the data:
\[ a = g \sin \theta - \mu g \]. Explain how we got from the first equation to the second.

Use \( a = \frac{F_{\text{Net}}}{m} \) to get

\[ a = g \sin \theta - \mu g \cos \theta \]  

For small angles, \( \cos \theta \approx 1 \)

so

\[ a \approx g \sin \theta - \mu g \]
5. (20 points) Left on its own, a neutron will decay spontaneously, turning into a proton, an electron and a neutrino. The lifetime, measured by various techniques, is shown in the table. Indicate which of these parameters agree with one another (that is, which are statistically the same within experimental error).

(For example, you might say that A and B agree with each other, and C and D agree, but that each pair disagrees with the other pair – THIS IS JUST AN EXAMPLE).

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Value [minutes]</th>
<th>± 1 SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. ILL beam lifetime</td>
<td>14.89 ± 0.09</td>
<td>(14.98, 14.80)</td>
</tr>
<tr>
<td>B. NIST beam lifetime</td>
<td>14.76 ± 0.06</td>
<td>(14.81, 14.70)</td>
</tr>
<tr>
<td>C. MamBo material bottle</td>
<td>14.69 ± 0.04</td>
<td>(14.73, 14.65)</td>
</tr>
<tr>
<td>D. Permanent magnet trap</td>
<td>14.64 ± 0.03</td>
<td>(14.67, 14.61)</td>
</tr>
</tbody>
</table>

"Agree" means range of possible results overlap
so A, B agree; B, C agree; C, D agree
A, C disagree; A, D disagree
B, D disagree
