Mapping Electric Fields

Introduction

Using the technique developed in the previous lab to map the electric potential, we will see how it is possible to map the electric field. It is important to note, however, that it is potential (and more specifically, potential difference) that is being measured.

Recall that the electric field is a vector field; that is, associating with each point in space a vector, the electric field, indicates the presence of the electric field. So at each point in space, it is necessary to indicate up to three quantities; the magnitude and direction of the field (two quantities to specify direction) or the three Cartesian components. (In this lab, we will be mapping fields in two dimensions, so there are only two components to each field vector.) To determine these components, we will need a means by which to convert the measured potential difference into an electric field.

Recall that the electric potential field was defined analogously to the gravitational potential energy. And remembering that the gravitational force was then equal to the gradient of the potential energy, we can note that the electric field will be given by gradients in the electric potential. (Remember that a gradient is a rate of change of a quantity with respect to a variable measuring position – so it is just a special name for a certain kind of rate of change – indicating the slope of a steep hill as the “grade” refers to just this idea). That is, we can write the electric field as:

\[ E_s = -\frac{\Delta V}{\Delta s} \]  

where I have said that the component of the field in the direction \( s \) is equal to the negative gradient of the potential in that direction. So, I can consider the ordinary Cartesian components:

\[ E_x = -\frac{\Delta V}{\Delta x}, \quad E_y = -\frac{\Delta V}{\Delta y}, \quad E_z = -\frac{\Delta V}{\Delta z} \]  

Alternately, I can consider the maximum gradient. That is, for a fixed \( \Delta s \), \( E_s \) will be maximum (i.e., \( E_s \) will be the total field) when I find the maximum voltage change.

So, the way to map an electric field is to measure the potential difference (\( \Delta V \)) for a small but finite displacement (\( \Delta s \)). If this \( \Delta s \) is chosen along a specified axis, you get the field component along that axis. Alternately, if the direction of \( \Delta s \) is chosen so that \( \Delta V \) is a maximum, you have the total field. You will do both.

Task 1)

First take your equipotential maps from the previous lab and draw the electric field lines corresponding to the equipotential surfaces you measured. Since the electric field is in the direction of maximum gradient, and the equipotential surfaces correspond to the minimal gradient (i.e., \( \Delta V = 0 \)), it can be shown that the electric field is always perpendicular to the equipotential surface. To determine which direction to choose, (there are two directions associated with the perpendicular to a surface) note the negative sign. That means that the field points from high potential to low potential.
**Task 2)**

Establish an electric field as was previously done in the electric potential lab. Using the voltmeter, construct a vector field map of the electric field for three electrode configurations (point potentials, parallel lines, whatever). You do this by separating the two voltmeter probes by a known distance (and holding this fixed!!), and then measuring the voltage difference (be careful! difference is given on the voltmeter as the difference between the ground terminal and the + terminal). The direction of the field will be indicated by the greatest negative voltage difference (why negative?), and the magnitude of the field is given approximately by:

\[ E_s \equiv -\frac{\Delta V}{\Delta s} \]  

(3)

where \( \Delta s \) is the distance between the ends of the probes, and \( \Delta V \) is the voltage difference read from the voltmeter. Draw a vector on the paper pointing in the direction of the field whose length is proportional to the field strength (so that when you get done you can tell at a glance where areas of high and low fields are, and the patterns indicated).

You should take enough readings that you build up an intelligible picture of the electric field.

**Task 3)**

Finally, choose a number (at least five, but enough to draw reasonable conclusions) of the field vectors you drew and compare the calculated x- and y-components with the measured ones. Calculate the x- and y-components by establishing x- and y-axes and measuring the angle of the field vector with respect to these axes. Measure the components by referring to Eq. 2 and realizing that the electric field component along a particular direction is the voltage difference measured along that direction, divide by the distance between the probes (along that direction).

**Conclusion**

Compare the electric field lines you drew using the equipotential surfaces with the vector field diagrams you measured in this lab. Compare the measured components with the calculated components. Draw insightful (not inciteful) conclusions. Reduce taxes.