1. (10 points) Recall the projectile lab, for which we developed a model of:

\[ R = \sqrt{4H \sin \theta \ D^0.5} \]

Find the predicted range if a sphere of mass 0.01 kg and radius 1 cm is rolled a distance 20 cm on the ramp and allowed to fall 1.0 m. Use \( \theta = 1.5^\circ \)

\[ R = \left[ 4(1.0 m) \sin 1.5^\circ \right]^{0.5} (0.2 m)^{0.5} \]

\[ = 0.3236 \text{ m}^{\frac{1}{2}} \quad 0.447 \text{ m}^{\frac{1}{2}} = 0.1444 \text{ m} \]
2. (20 points) At the front of the room, a spring-weight system is set up with three forces that apparently add to zero. (Do NOT manipulate the system).

A) Find the horizontal components. Does the result make sense? Explain

\[ R_x = (0.95 \cos 19 + 1.18 \cos 146) \text{N} \]
\[ = (0.8982 + (-0.9783)) \text{N} \]
\[ = -0.080 \text{N} \]
\[ = \text{Small but should be zero} \]

B) Find the vertical components. Does the result make sense? Explain

\[ R_y = (0.95 \sin 19 + 1.18 \sin 146 + 0.98 \sin 270) \text{N} \]
\[ = (0.3093 + 0.6600 + (-0.98)) \text{N} \]
\[ = -0.011 \text{N} \]
\[ = \text{Small but should be zero} \]

\[ R_y = (5.0 \sin 48 + 3.8 \sin 163 + 4.9 \sin 270) \text{N} \]
\[ = (3.716 + 1.111 + (-4.9)) \text{N} \]
\[ = -0.073 \text{N} \]
\[ = \text{Small but should be zero} \]
3. (15 points) In the energy in collisions lab, you rolled a spring loaded cart down a shallow ramp and measured how far back up the ramp it rolled after bouncing off a wall. Consider a cart (mass of 250 g) with an extra 500 g loaded (so total mass is 750 g) that is allowed to roll 35 cm down a ramp tilted at 2 deg. Suppose it bounces back up 20 cm. Set the zero level for potential energy at the point the cart just touches the wall.

A) Calculate the starting gravitational potential energy.

\[ U = m g \Delta y = m g D \sin \theta \]
\[ = (0.75 \text{ kg})(9.8 \frac{N}{kg})(0.35 \text{ m}) \sin 2^\circ \]
\[ = 0.090 \text{ J} \]

B) Calculate the final gravitational potential energy.

\[ U_f = m g \Delta y = m g D \sin \theta \]
\[ = (0.75 \text{ kg})(9.8 \frac{N}{kg})(0.20 \text{ m}) \sin 2^\circ \]
\[ = 0.051 \text{ J} \]
4. (20 points) Suppose that the rolling friction lab is repeated, and a graph of acceleration as a function of \( \sin \theta \) shows a straight line with a slope of \((10 \pm 1.5) \frac{m/s}{s}\) and intercept of \((-0.22 \pm 0.03) \frac{m/s}{s}\).

A) Use this to find the most reliable estimate of the rolling friction coefficient, \( \mu \). (You may use the model we discussed in class: \( a = g \sin \theta - \mu g \).)

\[
\mu = \frac{-\frac{\text{Intercept}}{g}}{\text{Slope}} = - \frac{(-0.22 \frac{m/s}{s})}{(10 \frac{m/s}{s})} = 0.022
\]

B) Find the most reliable estimate of the error in the coefficient, \( \sigma_\mu \).

\[
\mu_{\text{max}} = -\frac{(-0.25)}{(8.5)} = 0.029
\]
\[
\mu_{\text{min}} = -\left(\frac{-0.19}{11.5}\right) = 0.016
\]

\[
\sigma_\mu = \frac{0.029 - 0.016}{2} = 0.0114 = 0.007
\]

So, \( \mu = (0.022 \pm 0.007) \).
5. **(20 points)** The Hubble constant is a parameter that tells how fast the Universe is expanding. The table shows several measurements of this parameter. Indicate which of these parameters agree with one another (that is, which are statistically the same within experimental error). (For example, you might say that A and B agree with each other, and C and D agree, but that each pair disagrees with the other pair – THIS IS JUST AN EXAMPLE).

<table>
<thead>
<tr>
<th>OBSERVER</th>
<th>Value [km/s/Mpc]</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. H0LiCOW collaboration</td>
<td>72.5 ± 2.2</td>
<td>74.7 - 70.3</td>
</tr>
<tr>
<td>B. Planck Mission</td>
<td>67.66 ± 0.42</td>
<td>68.08 - 67.24</td>
</tr>
<tr>
<td>C. Hubble Space Telescope</td>
<td>73.45 ± 1.66</td>
<td>75.11 - 71.79</td>
</tr>
<tr>
<td>D. LIGO Collaboration</td>
<td>70.0 ± 10</td>
<td>80.0 - 60.0</td>
</tr>
</tbody>
</table>

The range for A does **not** overlap the range for B, so

- **A** **disagrees** with **B**
- **A** **agrees** with **C**
- **A** **agrees** with **D**
- **B** **disagrees** with **C**
- **B** **agrees** with **D**
- **C** **agrees** with **D**
6. **(15 points)** Plot this data in a way that allows you to determine whether the acceleration is constant. If it is, find the acceleration and estimate the error in the acceleration.

<table>
<thead>
<tr>
<th>Distance [m]</th>
<th>0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time [sec]</td>
<td>0</td>
<td>0.54</td>
<td>1.1</td>
<td>1.6</td>
<td>2.2</td>
</tr>
<tr>
<td>$t^2$ [sec^2]</td>
<td>0</td>
<td>0.2916</td>
<td>1.21</td>
<td>2.56</td>
<td>4.84</td>
</tr>
</tbody>
</table>

Possibly linear, **BUT** does not intercept zero.

So **NOT** constant $a$.

(but downward curve seen.)