Exam I  PHYS 430 Introduction to Astrophysics  
Spring 2021  D Norwood

Some stuff to think about: Read all the questions first and do the ones you find easier first. Use sunscreen. Feel free to ask if something is unclear or you feel you need other information - I won’t tell you how to work the problem, but I’m happy to clarify. TELL ME WHO YOU ARE. Above all, relax and say your mantra. Trust me on the sunscreen.

1) [15 points] A method I proposed to repeat Eratosthenes measurement of the diameter of the Earth was to measure the elevation (angle above the HORIZON) of the SUN at noon on the solstice, drive as far as you can NORTH, spend the night and repeat the measurement at noon the next day.  
a) What is the point of choosing the solstice as the day on which to make the measurement?  
b) If you drive 600 mi (= 960 km) and measure the difference in the elevation to be 8.9° (= 0.155 radians), what does this result give for the radius of the Earth?

2) [25 points] In class, we discussed the possibility that the Earth is a large, flat plate rather than a large sphere (with an incredibly small deviation from sphericity :). You can use Gauss’ law equation to show that the gravitational field away from the edges is $\vec{g} = 4\pi G \rho t \text{(down)}$, where $G$ is Newton’s gravitational constant, $\rho$ is the density of the plate and $t$ is it’s thickness.  
a) Knowing that $\vec{g} = 9.8 \text{ N/kg (down)}$, assume that the Earth is made of rock ($\rho \approx 3000 \text{ kg/m}^3$) and calculate it’s thickness.  
b) In class, I made a dumb joke about walking to the edge of the Earth and falling off. It’s not just a dumb joke, it’s dumb physics. Explain what would really happen if you walked off the ‘edge’ of a flat Earth. (HINT: What force keeps you on the surface of the Earth?) Draw a sketch if it helps the explanation. (Draw the edge of the flat plate if it helps.)

3) [15 points] Recall that the “redshift parameter” is defined by  
$$z = \frac{\lambda_{\text{OBS}} - \lambda_{\text{REST}}}{\lambda_{\text{REST}}}$$  
and can be written  
$$z = \sqrt{\frac{1 + \beta}{1 - \beta}} - 1$$  
and, for $\beta$ small, as  
$$z \approx \beta$$  
By watching the motion of sunspots, it was discovered that the equator of the Sun has a rotational period of 24.5 Earth days ($\approx 2 \cdot 10^6 \text{ sec}$). This is confirmed by measuring the Doppler shift of radiation from the edges of the Sun (one of which moves away from the Earth and the other towards). Use this period and the radius of the Sun ($R_{\text{SUN}} \approx 7 \cdot 10^8 \text{ m}$) to calculate the speed of one edge of the Sun, and find the fractional change in wavelength you must measure to confirm this speed.
4) [25 points] We presented several arguments that the Earth is spherical. One is that the strength of the gravitational field will decrease with altitude above a spherical Earth, but not above a large, flat Earth.

a) [TAKE HOME PART ONLY] Show that the ratio of the strength of the gravitational field a height, \( h \), above sea level (\( h=0 \)) is approximately:
\[
\frac{g(h)}{g(0)} \approx 1 - \frac{2h}{R},
\]
where \( R \) is the radius of the Earth (= 6378 km).

b) The gravitational field at Huascaran, Peru (elevation \( h=6662 \text{ m} \)) is measured to be\(^1\) \( g(h)=9.76392 \text{ N/kg} \) and \( g(0)=9.80665 \text{ N/kg} \). How does this data compare with the prediction from part a)?

5) [20 points] Recall that to discuss orbits, we wrote an “effective” potential energy, consisting of the conserved rotational kinetic energy (which we thought of as the “centrifugal barrier”) and the actual potential energy due to an attractive force.

a) Sketch the effective potential for a Hooke’s law force (\( V(r)=\frac{1}{2}kr^2 \)).

b) Draw in several values for the total energy, \( E \), and use that to discuss the types of orbits allowed by this potential. What is dramatically different about this force than the inverse square force we discussed in class?

c) [TAKE HOME PART ONLY] What is the minimum energy allowed and what type of orbit is it? (HINT: How did your calculus teacher show you to find the minimum of a function?) [\textbf{ans: I got} \( E=L\sqrt{\left(\frac{k}{m}\right)} \)]
(1) a) The solstice is the day at which the elevation of the Sun at noon is a maximum. Being a maximum, it changes very little each day near the maximum.

\[
\phi = \Theta_1 - \Theta_2 = 0.155 \\
R = \frac{S}{\phi} = \frac{(960 \text{ km})}{(0.155)} = 6200 \text{ km}
\]

b) 

\[
\begin{align*}
\text{As you approach the edge, you have to "climb" over and around the eddy to get to the other side.}
\end{align*}
\]

(2) a) 

\[
\frac{9.8 \text{ N}}{m_g} = 4\pi (6.67 \times 10^{-11} \text{ m}^3/\text{kg}^2) (3000 \text{ km/s})^2
\]

\[
\Rightarrow t = \frac{9.8}{4\pi (6.67 \times 10^{-11}) (3000)} \text{ m} = 3900 \text{ km}
\]
\[ v = rw = (7.1 \times 10^8 \text{ m})(\frac{2\pi}{2.1 \times 10^6 \text{ s}}) = 2200 \text{ m/s} \]

\[ \beta = \frac{2200}{3.1 \times 10^8} = 7.3 \times 10^{-6} = \frac{d\lambda}{\lambda} \]

\[ 4a) \quad g(R+h) = \frac{GM}{(R+h)^2} = \frac{GM}{R^2} \left(1 + \frac{h}{R}\right)^{-2} \]

\[ = \frac{GM}{R^2} \left(1 - \frac{2h}{R}\right) = g(R) \left(1 - \frac{2h}{R}\right) \]

\[ \frac{g(R+h)}{g(R)} = \frac{g(h)}{g(0)} = 1 - \frac{2h}{R} \]

Why can't you find the \( g \) a distance \( h \) below the surface using \( g(R-h) = \frac{GM}{(R-h)^2} \)?

\[ b) \quad \frac{g(6662 \text{ m})}{g(10)} = \frac{9.76892}{9.80665} = 0.9956 \text{ exp.} \]

\[ > 1 - \frac{2(6662 \text{ km})}{6400 \text{ km}} = 0.9979 \text{ theo} \]

- So in the ballpark
b) There is a circular orbit that, as before, has minimum $E$ for a given $L$, or maximum $L$ for a given $E$. Any orbit above this is bound with a pericapsis and an apocapsis. There are NO unbound orbits.

\[ E = \frac{1}{2} kr^2 + \frac{l^2}{2mr^2} \]
\[ E' = -F = kr - \frac{l^2}{mr^3} = 0 \]

so $r^2 = \frac{L}{\sqrt{km}}$. Then

\[ E = \frac{1}{2} \frac{k}{\sqrt{km}} \cdot \frac{L}{\sqrt{km}} + \frac{l^2}{2m} \cdot \frac{\sqrt{km}}{L} = L \sqrt{\frac{k}{m}} \]

\[ \text{UNITS?} \]

How do you prove it's a minimum?