REDSHIFT PARAMETER – with fewer screwups…

Unlike what I showed you in class, the redshift parameter, z, is defined as:

\[ z = \frac{\lambda_{\text{OBS}} - \lambda_{\text{EMIT}}}{\lambda_{\text{OBS}}} \]

(The -1 shows up later – mea culpa). The subscript OBS refers to the observed wavelength of EM radiation and EMIT refers to the wavelength emitted by the source, so this is the fractional change in wavelength caused by the motion of the source as observed in a stationary frame and defined so that \( z=0 \) if a source is at rest wrt the observer. Note that the expression I wrote in class for the Doppler shift was for a MOVING observer observing fixed sources (because I wanted to talk about using the Doppler shift to prove the Earth was moving through a set of fixed stars). In this case, though, we consider the expansion of the Universe (more on that later) as viewed from an Earth at rest. So to use the expression we have for the Doppler shift, consider that in that expression, the sources that seem to be moving away are behind (so at \( \theta = 0 \)). The the Doppler shift expression becomes:

\[ \bar{\omega} = \omega \left( 1 - \beta \cos \theta \right) = \omega \gamma (1 - \beta) = \omega \frac{\sqrt{\left(1 - \beta\right)^2}}{\sqrt{1 - \beta^2}} = \omega \frac{\sqrt{\left(1 - \beta\right)(1 + \beta)}}{(1 - \beta)} = \omega \sqrt{\frac{1 - \beta}{1 + \beta}} \]

(Be sure you can argue this is a red shift, as expected for a source moving away). To proceed, we must either rewrite the redshift parameter, z, in terms of frequency (it’s in terms of wavelength above), or the Doppler shift in terms of wavelength (it’s in terms of frequency above). I’ll do the former; you do the latter (that is, rewrite the equation above, showing the shifted frequency in terms of speed as an equation showing the shift in wavelength in terms of speed). Then put that expression into the redshift parameter to get the final result. Should be the same.)

To rewrite z in terms of frequency, I can either substitute for wavelength using \( c = \frac{\lambda \omega}{2 \pi} \) or recognize that, since I have wavelength in numerator AND denominator, I can simply use \( \lambda \propto 1/\omega \).

Then the redshift parameter becomes:

\[ z = \frac{\lambda_{\text{OBS}} - \lambda_{\text{EMIT}}}{\lambda_{\text{OBS}}} = \frac{1/\bar{\omega} - 1/\omega}{1/\omega} = \frac{\omega - \bar{\omega}}{\omega} \]

From the expression for the Doppler shift above, \( \omega - \bar{\omega} = \omega - \omega \gamma + \omega \gamma \beta \). So we can substitute it all into the redshift parameter expression:

\[ z = \frac{\omega (1 - \gamma + \gamma \beta)}{\omega \gamma (1 - \beta)} = \frac{1}{\gamma (1 - \beta)} - 1 = \frac{1 + \beta}{1 - \beta} - 1 \]

(There’s the -1 that I mistakenly wrote at the beginning). Finally, if \( \beta \) is small, we can use the Taylor series approximation:

\[ z = \sqrt{\frac{1 + \beta}{1 - \beta}} - 1 = (1 + \beta)^{1/2} - 1 \approx (1 + \frac{1}{2} \beta) - 1 = 1 + \beta + \left(\frac{\beta}{2}\right)^2 - 1 \approx 1 + \beta \]