Classically, the particle moves along a line of constant momentum. Its location at any time is the point \((p, x) = (p, \frac{p}{m^2})\).

In QM, the uncertainty principle means that the point \((p, \frac{p}{m^2})\) becomes a region \((dp, dx)\) with \(dp \cdot dx > \hbar\).

\(E = \frac{p^2}{2m} + \sqrt{kx} = \frac{k^2}{2mx^2} + \sqrt{k} x^{\frac{1}{2}} = cm^2t\)

\(A) \frac{dE}{dx} = \left(\frac{k^2}{2m}\right) \left(-2x^3\right) + \sqrt{k} \left(\frac{1}{2}\right) x^{\frac{1}{2}} = 0\)

\[-\frac{k^2}{m} + \sqrt{k} \cdot \frac{1}{2} x^{\frac{1}{2}} = 0\]

\[x^{\frac{1}{2}} = \frac{2k^2}{m \sqrt{k}}\]

Stationary energy @ \(x = \left(\frac{2k^2}{m \sqrt{k}}\right)^{\frac{3}{5}}\)

\[E = \frac{k^2}{2m} \left(\frac{m \sqrt{k}}{2k^2}\right)^{\frac{3}{5}} + \sqrt{k} \left(\frac{2k^2}{m \sqrt{k}}\right)^{\frac{1}{5}}\]

\[= \frac{1}{4} \left(\frac{2k^2}{m}\right)^{\frac{1}{5}} + \left(\frac{2k^2}{m}\right)^{\frac{1}{5}} = \frac{5}{4} \left(\frac{2k^2}{m}\right)^{\frac{1}{5}}\]

\(\text{Units?}\)
2B) Alternatively, you could use dimensional analysis directly.

\[ k \cdot J^2 = \frac{kg^2 \cdot m^4}{m^8} = \frac{kg^2 \cdot m^3}{m^7} \]

\[ k \cdot J \cdot S = kg \cdot m^2 \cdot S \]

\[ E = t^2 \cdot k \cdot m \cdot c \]

\[ E_{kg} = J = kg \cdot a^2 \cdot S^2 - kg \cdot b^2 \cdot S^2 + 4bc \]

\[ a + 2b + c = 1 \]
\[ 2a - 3b = 2 \]
\[ -a + 4b = -2 \]

\[ a = 2 + 4b = 2 - \frac{8}{3} = \frac{2}{3} \]
\[ 2(2 - 4b) + 3b = 2 \]
\[ 4 - 8b + 3b = 2 \]
\[ 5b = 2 \]
\[ b = \frac{2}{5} \]

\[ c = -\frac{1}{5} \]

\[ E = t^{2/5} \cdot k^{2/5} \cdot m^{-1/5} = \left( \frac{k^2}{m} \right)^{1/5} \]
A) \[ E = \frac{p^2}{2m} + \sqrt{kx} = \sqrt{k} x_{\text{max}} \]

\[ x_{\text{max}} = \frac{E^2}{k} \]

B) starting with \[ -\frac{\hbar^2}{2m} y''(x) + \sqrt{kx} y(x) = E y(x) \]

\[ y''(x) = -\frac{2m}{\hbar^2} (E - \sqrt{kx}) y(x) \]

the WF starts at (0,0) and I assume initial slope is positive.

(What if it's initial slope is negative?)

up to the classical turning point, the curvature is negative.

As the WF approaches the classical TP, the curvature decreases (still negative) past the TP, the curvature is positive and increases as \( x \) increases.

At large \( x \), WF must approach zero.
The only difference is that the energy phase term \( \pm \omega t \) can be positive or negative. It's hard to rationalize negative energy.

The origin is that the energy in relativity enters as \( E^2 \):

\[
E^2 = (pc)^2 + (mc^2)^2
\]

And so the time derivative is second order:

\[
E^2 \rightarrow \left( \frac{i\hbar \partial}{\partial t} \right)^2
\]