Exam I  PHYS 331 Math Methods  Fall 2019 D Norwood

Some stuff to think about: Read all the questions first and do the ones you find easier first. Use sunscreen. Feel free to ask if something is unclear or you feel you need other information - I won't tell you how to work the problem, but I'm happy to clarify. TELL ME WHO YOU ARE. Above all, relax and say your mantra. Trust me on the sunscreen.

1) The equation that relates Celsius to Fahrenheit temperature is: \( F = \frac{9}{5}C + 32 \).

Using it, you can find, for example, that body temperature is about 98°F or 37°C. (Or that 40°C is either hot (Celsius) or cold (Fahrenheit).) That is, in general, Fahrenheit and Celsius temperatures are numerically different. But there is one temperature for which the value is the same. Find it. That is, what is \( x \) if \( x°F = x°C \)?

2) Use the definition of the derivative to find the derivative of \( x^2 \).

3) Determine the first three terms of the Taylor series expansion of \( f(x) = \ln(x) \).

Recall that Taylor showed that, under certain physically convenient circumstances, the series \( f(a) + f'(a)(x - a) + \frac{f''(a)}{2!}(x - a)^2 + \ldots = \sum_{n=0}^\infty f^{(n)}(a)/(x - a)^n \) converges to \( f(x) \). HINT: Using \( a = 1 \) is a convenient choice.

4) In a normal, Euclidean space, the distance between two points is given by \( r^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 \).

Use this and the figure shown to derive the law of cosines: \( c^2 = a^2 + b^2 - 2ab \cos \theta \).

5) Use a trig substitution to evaluate:

\[
\int_{0}^{1} \frac{dx}{\sqrt{1 - x^2}}.
\]
\[ X = \frac{9}{5} X + 32 \]

\[ (1 - \frac{9}{5})X = 32 = -\frac{4}{5}X \]

\[ X = -\frac{5}{4}(32) = -40 \]

\[ c^2 = (b - a \sin \Theta)^2 + (0 - a \cos \Theta)^2 \]

\[ = b^2 - 2ab \sin \Theta + a^2 \sin^2 \Theta + a^2 \cos^2 \Theta \]

\[ = b^2 + a^2 - 2ab \sin \Theta \]

\[ (\text{this happened because I mislabelled the coordinates, should have been} \ (a \cos \Theta, a \sin \Theta)) \]
2. \[ (x^2)' = \frac{(x+\Delta x)^2 - (x^2)}{\Delta x} \]

\[ = \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x} \]

\[ = 2x + \Delta x \]

\[ \lim_{\Delta x \to 0} \frac{2x + \Delta x}{\Delta x} = 2x \]

3. \[ f(1) = \ln(1) = 0 \]

\[ f'(1) = \frac{1}{x}, \quad \frac{1}{1} = 1 \]

\[ f''(1) = -\frac{1}{x^2} = -1 \]

\[ f(x) = \ln x \approx 0 + 1(x-1) + \left(\frac{-1}{2}\right)(x-1)^2 \]

\[ = x - 1 - \frac{1}{2}(x^2 - 2x + 1) \]

\[ = 2x - \frac{3}{2} - \frac{1}{2}x^2 \]
3(x+1)^4 \text{ note that the next term will have } (x-1)^3 = x^3 - 3x^2 + 3x - 1 \\
\text{i.e., more constant and linear (and quadratic) terms, so this series converges, but}
\text{mind-bendingly slowly. What happens if you instead do something equivalent - expand ln(1+x) \\
around } x = 0 ? \text{ [Try it ...]}

\text{5) Use } x = \sin \theta: \\
\left| \lim_{x \to 0} \frac{\ln(1+x)}{x} = \ln'(x) \right| \\
= \frac{\pi}{2} - \theta = \frac{\pi}{2}